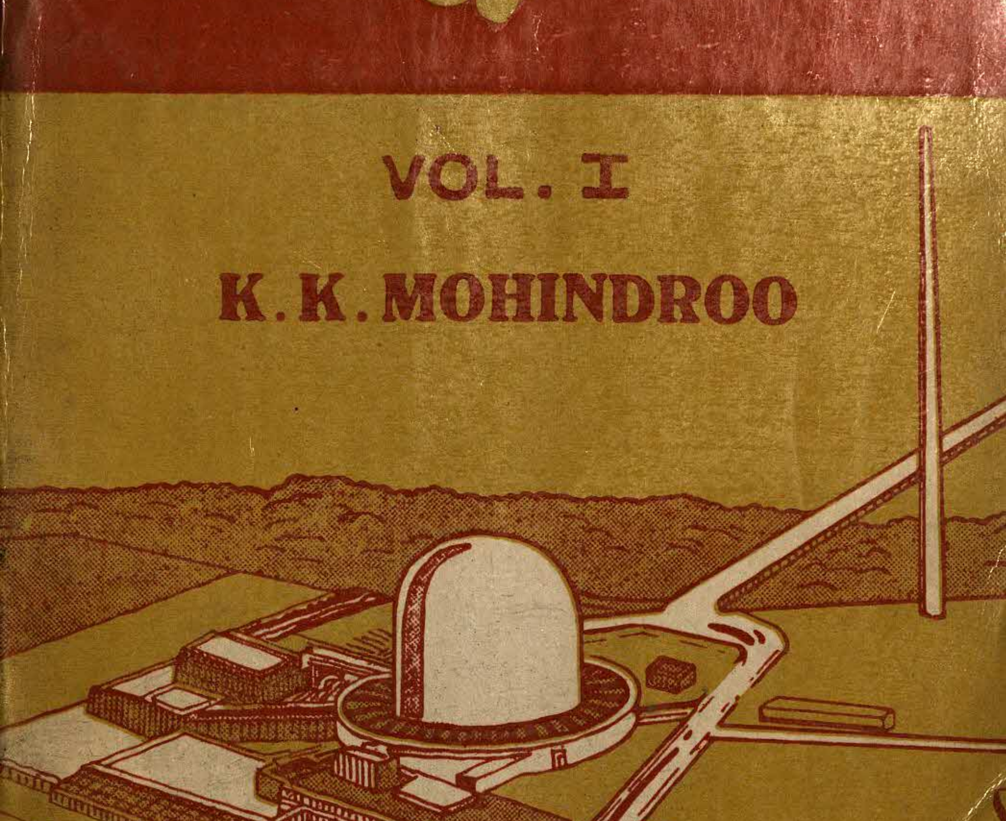
Basic Concepts of Physics

VOL. I

K. K. MOHINDROO



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BASIC CONCEPTS OF PHYSICS

(VOL. I)

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Preface to the Fourth Edition

The first three editions and subsequent reprinted editions of this book have been used by my fellow teachers and students of class XI. The experience gained in this way has suggested certain improvements and corrections which have been included in this edition. In some instances, the new material is intended to cope with questions frequently raised by the students. Other additions have been necessary to bring the subject upto date.

In this edition I have added a set of questions taken from recent Senior School Certificate Examinations (All-India and Delhi Scheme) conducted by the Central Board of Secondary Education, New Delhi.

I would like to pay my tribute to the work done by my own students. They have been a fertile source of ideas of which many are included in this edition

I am also thankful to several of my friends and fellow teachers from Public Schools, Central Schools, Government and Aided Schools for their valuable suggestions and criticisms for the book. Report about any shortcomings and suggestions for the improvement of the book are always welcome.

My special appreciation and thanks are to the publishers M/S PITAMBAR PUBLISHING COMPANY for bringing out the book in the present nice form.

K. K. MOHINDROO

PREFACE

For the last few years there has been a growing awareness in the country that if our students are to be given useful knowledge of contemporary Physics, a fresh approach has to be made to cope with the exponential growth of the subject. The Boards of Education of various states and union territories in India have introduced new and advanced courses in Physics in the Senior Secondary School curricula which would cultivate scientific temper and scientific thinking. It is with this aim in view that the present book, '*Basic Concepts of Physics*' intended to be a course in Physics for the academic stream of the 10+2 pattern of education, has been written. The book has been written after long and direct contact of over 20 years with the students of Physics. I had the privilege to come across the actual problems confronting students. A sincere and strenuous effort has been put in.

The present belief is to convey to the student from the very beginning the basic concepts of Physics so that the student develops *confidence* to follow advanced courses in Science and Technology and is enabled to make a smooth transition from the school level courses to the graduate courses. **The main emphasis in the book is, therefore, on the basic concepts.**

The two problems, the heterogeneous background of students and the limited time available to the teacher, have inspired the writing of this text and it is believed that the presentation will materially aid the teacher in coping with these difficulties. In order to reduce the text to manageable size, considerable soul searching was required. My guiding principles in this respect were the following :

1. The students are of normal intelligence and background.

2. Students taking such a course as this require a firm working grasp of the basic principles of Physics.

We cannot teach all Secondary School general science in a course at this level. Therefore, we must restrict the course, in essential body at least to the major principles of Physics and their more important applications. *You will see that enough material of* *practical importance is there to maintain student interest.* In fact, since this is the *terminal course* in Physics for some of the students, a special effort has been made to prepare the student for the practical application of the principles of Physics.

For convenience, the book is split into two volumes divided into units in order to adequately cover all the units prescribed in the latest syllabus of Central Board of Secondary Education, New Delhi.

Within each unit there are several other elements which serve both pedagogical and motivational purposes. All definitions within the text appear in bold face. To engage the student's attention, there will occasionally be highlighted sentences. Every unit also contains many worked out examples of problems with all steps shown to help students *master problem-solving techniques*. A summary with each important concept numbered is included in each unit. *Good thought-provoking Questions and Problems* are essential in this course, and so concluding each unit is a set of questions about the concepts covered in the text, followed by an extensive set of problems. It is my firm belief that a course at this level should not be a memory course. Although some of the simpler problems are of ordinary drill variety, the majority require a good understanding of principles for their solution. The working of many problems is essential to a true mastery of the subject. In my opinion, the better student should work nearly all the problems in the text. *Questions and problems have been framed which would keep student's mind always active—an approach which is quite averse to the traditional one.*

In the effort to minimize confusion over units, MKS and SI system is used throughout the text. However, because the *c.g.s.* system remains in use by many groups including scientists and engineers, it is used occasionally in examples and problems.

The symbols used in the book have been selected mainly in conformity with the recommendations of N.C.E.R.T.

The other special features of this book are :

(*i*) Language is simple and lucid.

(*ii*) Diagrams drawn are clear, neat, well-illustrated and accurate.

(*iii*) Mathematical derivations have been made logical and easily understandable.

(*iv*) Use of vectors, differentiation and integration has been preferred throughout the text as required by the C.B.S.E.

(*v*) The style of the book and the quality of content is such that it would lead the student to better understanding and he would gain self-confidence and *would abhor hankering after cheap books which give memory course only.* **The students would be able to face the examinations with confidence.**

(*vi*) Greater emphasis has been laid on basic concepts.

(*vii*) The book will help the students to be more independent and self-sufficient. It will increase the student's ability and willingness to inquire for themselves. Above all, it will increase the interest in and enjoyment of Physics.

The aim of the book is, however, not to replace the teacher. The teacher in the class room can always train a young pupil on proper lines ; the present book is to serve as an aid for this purpose.

I am thankful to several of my colleagues, friends and students (former and present) for their valuable suggestions, criticism and help in the preparation of this book. I shall feel greatly obliged to those readers who will bring to my notice the errors and shortcomings of the book and forward to me their suggestions for its improvement.

—AUTHOR

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$\left(C_p - C_v = \frac{R}{J}\right)$

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UNIT I

MEASUREMENT

1·1 LENGTH AND TIME :

Scientific investigation and study springs from a desire to discover and to understand the working of the world and of the heavenly bodies comprising the universe.

Complete understanding is the ultimate goal but this seems to be farther away with each forward step we take. Man always regards the unknown as a challenge—an unquenchable thirst for knowledge seems to be an essential part of his character.

The scientist takes part in this quest for knowledge which has been going on from early times and which is now being pursued at an ever increasing tempo with its consequent greater and greater effect upon the entire populace of the world. No matter whether or not science is to our vocation, it will nevertheless, exert a dominating influence upon our lives, and for this reason alone we should at least know its language and its limitations. We should be able to appreciate the modern trends and should try to understand, as far as possible, just where Science is leading us.

The phenomena of interest to scientists cover an enormous range of magnitude from atomic sizes to the scale of the universe, and from tiny fractions of a second to thousands of millions of years.

Scientists of the seventeenth century were particularly interested in astronomy, and it is therefore easy to understand why some of them regarded the universe as a collection of isolated bodies separated by vast regions of empty space. The appearance of the sky on a clear night illustrates this (Fig. 1.1). The stars are points of light whose diameters are clearly very much smaller than the distances between them and most of the space in the sky appears to be empty. The distance from the sun to the nearest star, Alpha Centauri, is about thirty million times the diameter of the sun !

Newton was not aware of this fact, since the first measurement of the distance of a star was not made until the year 1838, but he was aware of the distances involved in the Solar System.

We have learnt in earlier classes about the solar system, of which our planet Earth is a member. The radius of the earth is 6.4×10^6m and its natural satellite, the moon, is at a distance 3.85×10^8m from the earth. The average distance from the earth to the sun, about 149×10^6 km or $1{\cdot}49 \times 10^{11}$ m is one of the important measuring units used by astronomers as the Astronomical Unit (*AU*). In terms of this Unit, the average distance from the sun to the planet Pluto is close to 40 astronomical units or about 5900×10^6 km or $5{\cdot}9 \times 10^{12}$ m.

The great distances between the members of our solar system, however, are dwarfed by both the enormous size of *Milky Way*, the galaxy of which our solar system is a rather insignificant part, and by the vast expanse of the Universe. Even the astronomical

(Fig. 1·1) A portion of the night sky. "Isolated bodies separated by vast regions of empty space."

unit is not satisfactory, and consequently, a still larger unit of distance was formulated-the *light year*. A light year is the distance travelled by light in one year, about 9.5×10^{12} km ($9 \cdot 5 \times 10^{15}$ *m*).

Some idea of the size of the Milky Way may be grasped from its dimensions. It is estimated that the diameter of our galaxy is 10^5 light years and that it is some 1500 light years thick. Our sun is about 3×10^4 light years from the centre of the Milky Way and travels around the galactic centre in about 2×10^8 years.

A galaxy is a group of stars. There are many billions of stars in our galaxy. The distance between one galaxy and a neighbouring galaxy varies from a few tenth to several million light years. The galaxies are found to be receding from the earth at speeds increasing with their distances from the earth. Assuming that the limiting speed of these galaxies is the same as the speed of light, the theoretical limit of the size of the universe is 10 billion light years.

At the other extreme, we now know, that all objects—the air the rocks ; the oceans ; and the myriad forms of complicated living organisms that teem over the surface of the earth—can be visualized as sufficiently complicated structure built out of atoms, and that the atom itself is a structure composed of the three fundamental particles, electrons ; protons and neutrons. The protons and neutrons are packed tightly together into the nucleus, which occupies a region of diameter of about 10^{-14} m at the centre of the atom. The electron has a diameter of about 10^{-15} m and is about 10^{-10} m from the nucleus. The atom is, therefore, 10,000 times larger than the nucleus and about 100,000 times larger than the electron. The radius of a hydrogen atom is 5×10^{-11} m while the effective radius of a proton is 1.2×10^{-15} m. Thus we find that on the atomic and sub-atomic scale, the distances are very very minute.

The recurrence of the day and night and the seasons gave an idea of the passage of time. There is a clock built in the human system. The heart goes on beating from birth to death. The pulse was used for measuring short time intervals. From the position of the sun and the heavenly bodies above the sky, man used to fix the time of the day and night, respectively.

1·2 MEASUREMENT :

The above method of describing the universe involves two basic types of measurement—measurement of length and measurement of time.

Length :

The related ideas of length, distance and space arise from man's need to move from one place to another. The word space describes the extension of our environment, the fact that it is possible to move physically and conceptually from one region to another. Distance implies a measurement of space ; when we talk of a short distance or a long distance, we make a relative judgement of the effect of moving or of what is involved in moving from one point in space to another. *The idea of length is an elaboration of the idea of distance, i.e. it is simply an interval of distance.* When we speak of distance we admit in principle that extensions in space can be measured, but when we speak of length we imply that we have developed a method of making the measurement. Thus we use units of length to give magnitude to our description of distance.

Given a standard unit of length, we can measure many distances directly by laying objects representing the unit end to end. Suppose, we want to know the length of a drawing room. Using a metre scale, we find out how many times the drawing room is as long as the metre. If it is, say, 8 times the metre, the length of the drawing room is 8 metres. Often, however, we need to measure very large distances or distances to inaccessible points. In such cases, the method of measurement by direct comparison cannot be applied. Indirect methods of measurement, therefore, become necessary. We can use methods based on geometry and trigonometry. Heights of mountains, poles, buildings, widths of bodies of water and the distance to the moon and other celestial objects can be measured by methods called *triangulation* and *Parallax*.

A few examples will illustrate the points :

Height of a building, (*Distance Known*). Let AB be a tall building observed from ground level at point C (Fig. 1·2) Distance

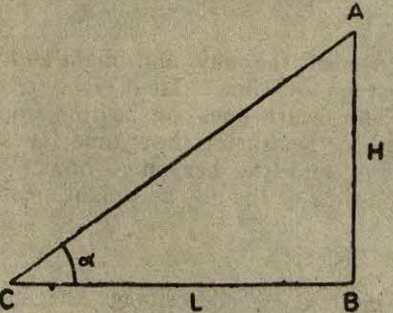

Fig. 1·2

CB on the ground can be measured. If the angle of elevation α be measured using some mechanical device, say a range finder or sextant, the height AB=H can be easily evaluated using the trigonometric relations.

$$\tan\alpha = \frac{AB}{BC} = \frac{H}{L} \quad \text{where } L = BC$$

$$\text{or} \quad \boxed{\text{Height } H = L \tan\alpha} \qquad \ldots(1)$$

Height of a Hill (*Distance Unknown*) :

If the point B (Fig. 1.3) of the hill (AB) is inaccessible *i.e.* if you cannot measure the distances DB or CB even then you can determine its height (H).

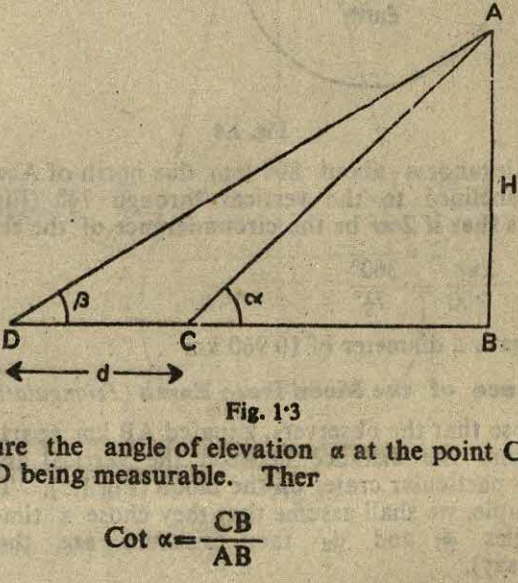

Fig. 1·3

Measure the angle of elevation α at the point C and β at the point D, CD being measurable. Ther

$$\text{Cot } \alpha = \frac{CB}{AB}$$

$$\text{Cot } \beta = \frac{DB}{AB}$$

$$\text{or Cot } \beta - \text{Cot } \alpha = \frac{DB}{AB} - \frac{CB}{AB} = \frac{CD}{AB} = \frac{d}{H}$$

$$\text{or} \quad \boxed{\text{Height } (AB = H) = \frac{d}{\text{Cot } \beta - \text{Cot } \alpha}} \qquad \ldots(2)$$

Inaccessible horizontal distance can be measured similarly. The line of observation BC is called the *base line*.

Measurement of diameter of Earth :

Eratosthenes observed that at one point in Egypt, near Aswan, on the longest day of the year, the sun penetrated to the full depth of a deep well at noon. Thus the sun was in the vertical direction with respect to the horizon. On the same day at the noon time, at Alexandria about 800 km due north of Aswan, the sun rays were inclined to the vertical through $7\frac{1}{2}°$ (Fig. 1.4) Geometry shows that if $2\pi r$ be the circumference of the earth then,

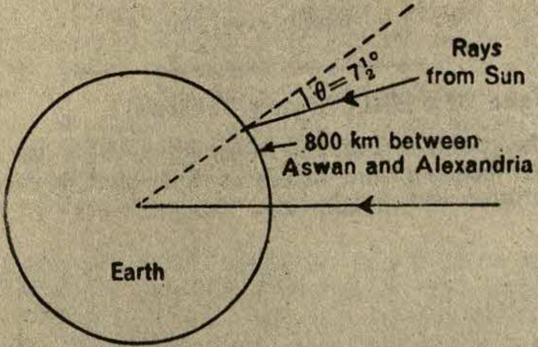

Fig. 1.4

$$\frac{2\pi r}{800} = \frac{360°}{7\frac{1}{2}°}$$

This gave a diameter of 10 960 km

Distance of the Moon from Earth (*Triangulation Method*)

Suppose that the observers, situated AB km apart, simultaneously measure the distance between the vertical and the line of sight to a particular crater on the moon (Fig 1.5). To make the example simple, we shall assume that they chose a time when the zenith angles ϕ_1 and ϕ_2 they measure are the *same i.e.* $\phi_1 = \phi_2 = \phi$ (say).

Since $\phi_1 = \alpha + \theta$

$$\phi_2 = \alpha + \theta$$

$$\therefore \quad \phi_1 + \phi_2 = 2\phi = 2(\alpha + \theta)$$

or $\alpha = \phi - \theta$

If AB distance is measured then

$$\theta = \frac{AB/2}{\text{Radius of earth (R)}}$$

Thus knowing AB, R and measuring Zenith angle ϕ, angle α is determined.

$$\text{Now } \tan \alpha = \frac{AC}{CM} = \frac{R \sin \theta}{CM}$$

If CM $>$ R or OC, then CM $\approx$ OM

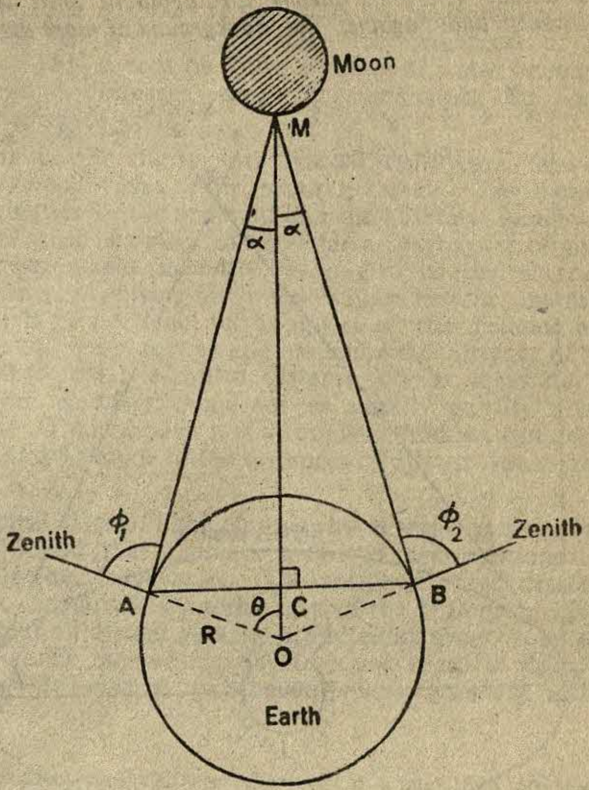

Fig. 1.5

Hence
$$\tan \alpha = \frac{R \sin \theta}{OM}$$

or
$$\boxed{OM = \frac{R \sin \theta}{\tan \alpha}} \qquad ...(3)$$

Thus the distance of the moon OM can be determined with considerable accuracy.

Distances to some of the nearer stars (closer than 100 light years) can be determined by the *Parallax Method*, using the diameter of the earth's orbit about the sun as the base line, as suggested in Fig (1.6). The longest base line we can use without leaving the earth is, the distance from one point on the earth's orbit around the sun to the point reached by the earth a half year later. The star S is the first viewed from point A ; six months later, it is viewed from point B. The remote stars shown in the figure form a kind of fixed wall on which the shift of the nearer star S is noted. (*Parallax means the apparent shift in position of a heavenly body against the background of more distant stars when viewed from different positions*). The magnitude of this shift plus the value of the earth's orbital diameter allow computation of the distance to the star S. As is clear from the figure (1.6) if ϕ is the angle of parallax, then

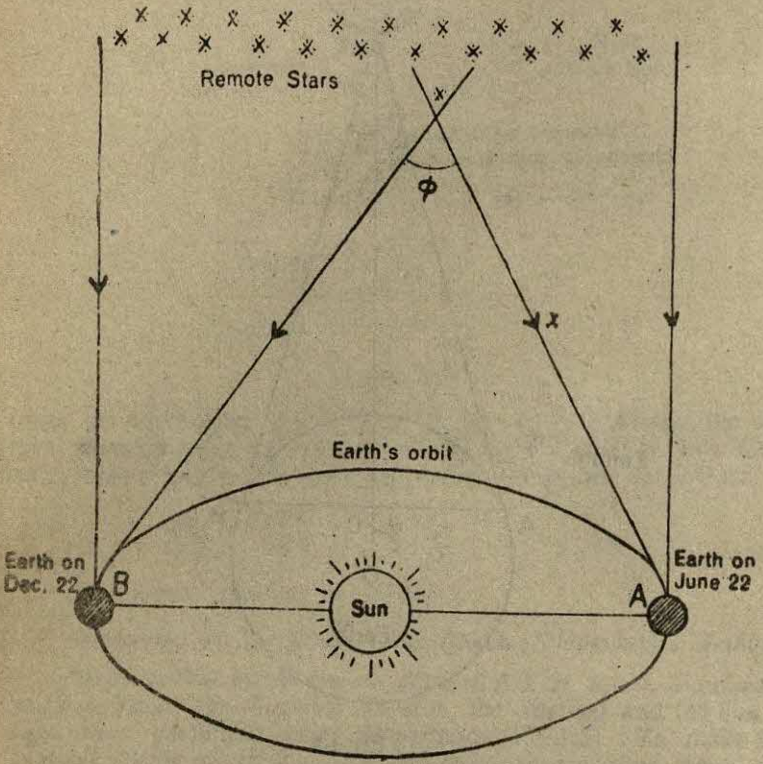

Fig. 1·6 (Parallax Method)

$$\boxed{\tan\phi \approx \phi = \frac{AB}{x}} \quad \text{or} \quad \boxed{x = \frac{AB}{\phi}} \quad \text{since } \theta \text{ is very small} \qquad \ldots(4)$$

Where AB is the diameter of earth's orbit and x is the distance of the star.

The limit on stellar distances that can be found with this base line is found to be about 10^{18} m. Thus the distances to only a few hundred nearby stars can be determined by the parallax method. Distances to the more remote stars are measured by other indirect methods *i.e.* methods involving their brightness or the Doppler Shift. This is because as the distance of a star increases, the parallax angle ϕ decreases considerably and thus, the above method cannot be relied upon.

If the distance of a star is more than 100 light years then the *Spectroscopic Method* yields more accurate result. The method involves determination of the apparent and the absolute luminosity of the star and then figuring out how far a nearby star of given absolute luminosity would have to be to give us the observed apparent luminosity. Applying the inverse square law (*i.e.* Intensity of light is inversely proportional to the square of the distance between the source and the observer) and assuming the distance of the nearby star from the earth is known, distance of far away star is determined. The two stars may not be exactly equally strong sources of light. Even though it is a rough method, we can determine the order of magnitude of the distances of distant stars from the earth.

If the spectral lines of light emitted by distant galaxies are observed, it is found that there is a shift in their positions and hence a change in their frequencies. The shift was first detected by Hubble who explained it on the basis of the ***Doppler effect* and concluded that all distant galaxies are receding away from the earth. Further studies proved that farther a galaxy is, the faster it is receding away. It indicates that the *universe is expanding*. This

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****Doppler Effect**

This refers to effects on the apparent frequency of a wave train produced by (*i*) motion of the source toward or away from the Stationary Observer and (*ii*) motion of the observer towards or away from the stationary source, the motion in each case being with reference to the (supposed to be stationary) medium.

In particular, for light, Doppler Effect is generally taken as the apparent change in the wavelength of light or frequency produced by the motion in the line of sight of either the source of light or the observer. In terms of frequency, it can be shown from the theory of Relativity that

$$\nu_o = \nu_s \sqrt{\frac{C-V_r}{C+V_r}}$$

Where ν_o = observed frequency

ν_s = frequency of the source

C = velocity of light

V_r = velocity of the source moving away from the observer.

If the source is approaching the observer, then we have to change the sign of V_r.

In an intense study, Hubble observed that the light emitted by stars of different galaxies suffers a *red shift* when observed from the Earth. According to the Doppler Principle, the light emitted by a moving object appears more red (red light being of lower frequency than the other colours) when it is receding from the observer (or the observer is receding from it).

receding of galaxies concluded from the change in the frequencies of their spectral lines is called *Hubble's Principle.*

For measurement of very small distances such as those within molecules and atoms, only indirect methods can be adopted. For example, we can apply Avogadro's Hypothesis to determine the size of the atom or we can determine the size of the gold nucleus by alpha-particles scattering experiments performed by Rutherford. The diameter of the gold nucleus found by Rutherford's α—scattering experiment is of the order of 10^{-14} *m.*

The order of magnitude of lengths from astronomical to the sub-atomic distances over which measurements can be made are given below.

TABLE 1·1

Order of Magnitude of Distances

(The figures have been rounded off to the nearest power of ten)

Body or Distance measured	*Order of length in metres*
Distant Quasar (Farthest Photograph galaxy) (Quasi-stellar Radio Source)	10^{26}
Great Nebula in Andromeda (nearest galaxy)	10^{22}
Diameter of our galaxy	10^{20}
Nearest Star (Alpha Centauri)	10^{17}
Mean distance of the planet Pluto from the sun	10^{13}
Radius of the Sun	10^{9}
Mean distance from the Earth to the moon	10^{8}
Radius of the Earth	10^{7}
Distance between Delhi & Ambala	10^{5}
Length of the Football field	10^{2}
Height of man	10^{0}
Diameter of Pencil	10^{-2}
Thickness of a sheet of paper	10^{-4}
Diameter of a red blood corpuscle	10^{-5}
Size of a virus	10^{-8}
Size of an oil molecule	10^{-9}
Estimated Diameter of a hydrogen atom	10^{-10}
Estimated size of a large nucleus	10^{-14}
Estimated diameter of Proton	10^{-15}

Time :

What do we mean by time ? Regardless of what we are doing or where we are located, we are always aware of a phenomenon referred to as the passage of time. We note the phenomenon in various ways; for example; by the apparent motion of the sun, the seasons, the heartbeat and the calendar. Our concept of measurement of length was based on our intuitive notion of distance; we will assume that we have also an intuitive feeling for the measurement of time.

A phenomenon that repeats itself can be used as a measure of time for example, the oscillations of a pendulum and a coiled spring etc. The measurement consists of counting the repetitions. The rotation of the earth on its axis is a repetitive phenomenon occuring in nature. This rotation determines the length of the day and was used as a standard of time from very early times. The day is subdivided into hours, minutes and seconds. Fortunately, all systems of measuring time are related to the second.

Table 1·2. gives the order of magnitude of time intervals that can be measured.

TABLE 1·2

Order of Magnitude of Time Intervals

**Order of time Intervals in seconds*	*An event of that Order*
10^{17}	Age of the earth
10^{15}	Time elapsed since the appearance of prehistorical animals.
10^{13}	Time elapsed since man appeared on the earth.
10^{11}	Age of the Pyramids of Gaza (Egypt).
10^{9}	Human life expectancy.
19^{7}	Time for the Earth to revolve once round the sun.
10^{5}	Time for the Earth to rotate once on its axis.
10^{3}	Time taken by light from the sun to reach the earth.
10^{2}	One minute.
10^{0}	Time between heart beats.
10^{-2}	Time for an electric fan to make one complete rotation.
10^{-4}	Time for the vibration of a high frequency audible sound note.
10^{-8}	Life time of an excited state of atom.
10^{-11}	Time for light to pass through a glassplate.
10^{-15}	Time for an electron to revolve once around a proton in an atom of Hydrogen.
10^{-22}	Time for a single revolution of a proton in a nucleus.

***The figures have been rounded off to the nearest power of ten.**

1·3 SYSTEMS OF UNITS AND SI UNITS :

Physics has been defined as the study of properties of matter, of the interactions between different portions of matter and ultimately of the effects of these interaction upon our senses. Thus in Physics, precision in measurement is very essential. Now measurement inevitably involves comparison with a chosen *standard* or *unit* of similar kind, so that the first essential step to be taken is the selection of a suitable standard or unit in accordance with the nature of the physical quantity to be measured, and the second, to determine its value in terms of the chosen unit. In other words, to form an exact idea of the magnitude of a physical quantity, it is necessary to express,

(*i*) the standard or unit in which the quantity is measured ;

and (*ii*) the number of times the quantity contains that stanaard or unit.

For instance in expressing the age of a child we must first settle upon a certain period of the time as unit. If we choose a month as the unit, the age may be stated as being, say 6 months where month is the name of the *unit* chosen and 6 is the number of times the age of the child contains that unit. If, however, we had selected a day as the unit of time, the age would be 180 days ; but it must be remembered that in all cases the name of the unit must be mentioned along with the number of times the unit is contained in that quantity. *Thus, the larger the unit, the smaller the numerical value of that quantity ; and the smaller the unit, the larger its value or, the numerical value of a quantity is inversely proportional to the magnitude of the unit selected as the standard.* It follows, therefore, that the product of the numerical value of the quantity and the magnitude of the unit in which it is expressed is a constant.

Thus $$6 \times 1 = 180 \times \frac{1}{30}$$

or in general, if n_1 and n_2 be the numerical values of a given physical quantity, corresponding the the units x_1 and x_2 respectively, we have

$$\boxed{n_1 x_1 = n_2 x_2}$$

For measuring different kinds of quantities, we must obviously have different kinds of units. If these be selected in any arbitrary manner they will be quite urelated to each other, and their use will create difficulties and complications in actual practice. Thus it is desirable that as *few* different kinds of units as possible should be introduced into our measurements. Different kinds of units are, therefore, all based on some *fundamental units*, so as to be interdependent and properly related to each other. The guiding principle in their choice is as follows :

(*i*) they are well defined and of a suitable size ;

(*ii*) they are easily reproducible at all places ;

(*iii*) they are imperishable *i.e.* they are not subject to any secular changes (*i.e.*, to changes with time).

(*iv*) they do not readily or appreciably vary with varying physical conditions like temperature, pressure etc ; and if they do, their manner of variation is perfectly, correctly known.

(*v*) they should be accepted internationally.

The *fundamental units chosen*, and internally employed are those of **mass**, **length** and **time**. The reason why these alone are chosen as the fundamental units, and not any others, would seem to be that they represent our elementary scientific notions and cannot be derived from one another ; nor can they be resolved into anything more basic or fundamental. All other units in Mechanics can be derived from them and are, therefore, called *derived units*. For example, the units of *area* and *volume* are derived units, for they can both be derived from the unit of length, the former being the area of a square, and the latter, the volume of a cube, *each of unit length*. Similarly, the unit of velocity is a derived unit and is the velocity of a body which covers unit distance, or length, in unit time. Again the unit of acceleration, force, work ; momentum, energy, density, etc. are all derived units.

A coherent system of units is a system based on a certain set of 'basic units' from which all derived units are obtained by multiplication or division without introducing numerical factors.

The *MKS* system is a coherent system of units in mechanics. Here M stands for metre (a unit of length) K for *kilogram* (a unit of mass) and *S* for *second* (a unit for time). The units of all other quantities in mechanics can be obtained from the above three units by their simple multiplication and division.

Though the metric system spread to several countries in the form of the CGS system (Cetimetre, Gramme, Second System) after the French Revolution, over the years much thought has been given to extending and improving the original metric system. In 1960 the General Conference of Weights and Measures recommended a coherent system of units named "*Systeme International d' Unite's*". abbreviated '**SI**' in all languages. SI is thus the extension and refinement of the traditional metric system and is intended to replace all other systems of units now existing in all branches of science, engineering and technology, and at all levels of industry and commerce. The SI is like a common currency for all these fields.

In SI system, there are seven *fundamental* and two *supplementary* units so as to cover all fields of Physics. The seven fundamental units are :

(*i*) **metre** (for *length*)

(*ii*) **kilogram** (for *mass*)

(*iii*) **second** (for *time*)

(*iv*) **kelvin** (for *thermodynamic temperature* in the field of thermodynamics)

(*v*) **candela** (for *luminous intensity* in the field of Photometry)

(*vi*) **ampere** (for *current* or *current intensity* in the field of electricity).

(*vii*) **mole** (for amount of substance)

The *two supplementary* units are ;

(1) **radian** (for *plane angle*) and

(2) **steradian** (for *solid angle*)

The various SI units have been explained below :

(a) Fundamental Units :

(*i*) **metre (m) :**

The many units of length in use today reflect some of the interesting history of measurement. Long before the various units were known, another unit of length, the *Cubit*, was used by the Egyptians. The cubit is approximately the length of the forearm from the end of the elbow to the tip of the middle finger. Other units have also been based on parts of the body. The *Span* is the spread of the hand from the tip of the thumb to the tip of the little finger.

In the metric system, the standard unit of length is the *metre.* Originally a metre was intended to be $\frac{1}{10\,000\,000}$ of the distance from the equator to the north-pole (along a meridian passing through Paris) ; so the distance could be redetermined if the original standard were lost. After the length representing the standard metre had been marked off on a metal bar, it was found that small error had been made. This error was considered too small to warrant recalibration of the standard metre. Consequently, the meridian quadrant through Paris has a length of 10 002 288 metres, and *the metre was redefined as the distance at zero degree celsius (0°C) between two parallel lines ruled on a platinum-iridium bar.*

This bar is preserved in the International Bureau of Weights and Measures, which was established by the Metric Convention of 1875 and is located near Paris, France. A large number of

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such bars, all compared as accurately as possible with the first standard, were made. Each member nation of the convention was allowed to draw two bars by lot. Copy of the standard metre is available at the National Physical Laboratory, New Delhi.

In 1960, a new definition of the standard metre was adopted. *The standard metre is now specified as being exactly equal to 1,650,763·73 wave-lengths in vacuum, of the radiation corresponding to the transition between the levels* $2p_{10}$ *and* $5d_5$ *of the Krypton isotope of atomic mass number 86* (Kr^{86}_{36}).

This number of wavelengths corresponds, of course, to the length of the previous standard ; but it is, as far as scientists know, unchangeable, always reproducible, and not subject to either the effects of aging and deterioration or possible destruction. The particular wavelength chosen is uniquely characteristic of Krypton 86 and is very sharply defined. This isotope can be obtained with great purity, relatively easily and cheaply.

Multiples and submultiples of the metre are used for convenience. The smaller units, the submultiples, are obtained by dividing the standard unit, the metre, into tenths, hundredths, thousandths, etc. The decimal system employed is the same one most countries now use for their currency. One tenth of a metre is a *decimetre* (*dm*), one hundredth is a *centimetre* (*cm*), and one thousandth is a *millimetre* (*mm*). To obtain a large unit, a multiple of the metre is used. This unit is equal to 1000 m and called the *kilometre* (*km*).

Table *1·3* shows how large and small units are formed.

TABLE 1·3

Metric Units of Length

Multiple and submultiple Equivalents

1 kilometre	$=10^3$ metre (m)
1 metre	=the standard unit
1 decimetre (dm)	$=10^{-1}$ m
1 centimetre (cm)	$=10^{-2}$ m
1 millimetre (mm)	$=10^{-3}$ m
1 micron (μ)	$=10^{-6}$ m
1 angstrom (Å)	$=10^{-10}$ m

Because the metric system uses the base ten, the change from one unit to another is very easy. Some of the commonly used metric prefixes are given in Table 1·4. These prefixes can be used for multiples and submultiples of *any unit*.

TABLE 1·4

Metric Prefixes

Prefix	*Abbreviation*	*Meaning*
tera	T	× 10^{12}
giga	G	× 10^{9}
mega	M	× 10^{6}
kilo	k	× 10^{3}
deci	d	× 10^{-1}
centi	c	× 10^{-2}
milli	m	× 10^{-3}
micro	μ	× 10^{-6}
nano	n	× 10^{-9}
pico	p	× 10^{-12}
femto	f	× 10^{-15}
atto	a	× 10^{-18}

(*ii*) **kilogram (kg)** :

The standard of mass is the kilogram (kg) which is defined as the mass of the international prototype of the kilogram—the standard block of platinum-iridium alloy (90% Platinum and 10% iridium) kept in the International Bureau of Weights and Measures in Sevres, near Paris. The diameter of the cylinder is equal to its height. Prototype copies of the kilogram have also been supplied to various countries for national reference and one such copy is also available at the National Physical Laboratory, New Delhi.

The kilogram was originally intended by Borda to be the mass of one cubic decimetre of water at 4°C. which is now known to be only approximately true. The gram is 10^{-3} kg and the metric ton is 10^{3} kg.

1 metric ton = 10 quintals
1 quintal = 100 Kilograms
1 kilogram = 1000 grams
1 gram = 10 decigrams
= 100 centigrams
= 1000 milligrams

This fundamental unit is not defined in atomic terms at present. But with improvements in physical measurements of atomic quantities even this standard seems destined, in time, to be replaced by a simply reproducible definition based on the masses of atoms.

(*iii*) **second (s)** :

On account of the diurnal rotation of the earth, the heavenly bodies, including the sun, appear to rise on the horizon in the east and set in the west and reappear again on the next day to complete the cycle exactly in the same manner. When the sun appears to attain the highest position, it is said to be in the *meridian*. *The interval of time that elapses between any two successive passages of the sun across the meridian is called the apparent solar day.* Now, for various reasons, the length of the apparent solar day does not remain constant but varies from day to day. If the lengths of all the solar days in a year be added together and the sum is divided by the number of days, we obtain what is known as the *mean solar day*. This is divided into 24 *hours*, each *hour* into 60 *minutes* and each *minute* into 60 *seconds*. The **second** or rather, the **mean solar second** as it is called, and which is $\frac{1}{24 \times 60 \times 60} = \frac{1}{86400}$ of a *mean solar day*, is taken as the unit of time.

There are 365·2420 mean solar days in one tropical year (Tropical signifies the time required for the earth to make one trip around the sun). In other words, the earth makes 365.2420 rotations on its own axis while making one complete revolution in its orbit around the sun. Investigations indicated that the annual revolution of the earth about the sun was more constant and regular than the earth's daily rotation on its own axis. Consequently, the standard unit of time was changed and, until recently was defined :

1 second $= \frac{1}{31\ 556\ 925{\cdot}9747}$ part of the tropical year beginning in 1900.

This standard second differed from $\frac{1}{86\ 400}$ part of the mean solar day by less than 1 part in 100 000 000.

Nevertheless, even this standard was found to change at the rate of about **half** of a second per century. Time defined in terms of the rotation of the earth is called Universal Time (U.T.). Time defined in terms of the Earth's orbital motion is called Ephemeris Time (E. T.).

Quartz Crystal Clocks based on the electrically sustained natural periodic vibrations of a Quartz Wafer serve well as secondary time standards. The best of these have kept time for a year with a maximum error of 0.02 second. The Quartz clock with an accuracy of 1 in 10^8 s is at National Physical Laboratory, New Delhi.

To meet the need for a better time standard, *atomic clocks* have been developed in several countries, using periodic atomic vibrations as a standard.

A particular type of atomic clock, based on a characteristic frequency associated with the *Cesium atom*, has been in continuous operation at the National Physical Laboratory in England since

1955. A cesium atom behaves like a tiny magnet. It experiences a sideways deflection as it moves through a non-uniform magnetic field. The amount and the direction of the deflection depend on the strength of this magnet and on the orientation of the axis of the magnet in this field. *Thus the cesium atom acts like a pendulum.*

In 1964, the second, based on the cesium clock was adopted as an International Standard by the twelfth General Conference of Weights and Measures meeting in Paris. According to this-*the second is the duration of 9 192 631 770 periods of radiation corresponding to the unperturbed transition between the two hyper fine levels (4,0 – 3,0) of the ground state* ($^3S_{1/2}$) *of caesium atom (133)*

This action increases the accuracy of time measurements to 1 part in 10^{11}, an improvement over accuracy associated with the astronomical methods of about 200. If two cesium clocks are operated at this precision, and if there are no other sources of error, the clocks will differ only by one second after running 5000 years.

Atomic clocks are still in a phase of rapid development and it was for this reason that "Cesium Second" was adopted only temporarily. For example the *hydrogen laser* gives promise of producing a clock with an error of only one second in 33 000 000 years.

It is interesting that the length of the second approximates the time interval between two human heart beats. This coincidence is not fundamental but it does provide an excellent means for estimating *short intervals of time.*

Astronomers, however, use a time scale called *Sidereal Time.* This time is determined by the rotation of the earth relative to the fixed stars (sidereal means stars). Sidereal time is independent of he earth's revolution about the sun. Because of the earth's motion in its orbit, sunrise occurs nearly four minutes later each day as observed by the stars. A sidereal day has only 23 hours, 56 minutes and 4·091 seconds of mean solar time. As a result, there are approximately 366·25 sidereal days in a tropical year.

(*iv*) kelvin (K) :

Ordinarily, we measure temperature in degree centigrade (°C). In this scale, the two fixed temperatures taken are ice-point 0°C and the *steam point* (100 °C) and the interval between these two is divided into 100 equal parts. Each degree on a centigrade scale is exactly one-hundredth part of the interval between these two fixed points. This scale evidently depends upon the thermal expansion of mercury used as the thermometric substance.

But there is another scale in use called the *Kelvin Scale* or *Thermodynamic Scale.* One unique feature of this scale is that it has a natural lower fixed point which corresponds to—273·16°C (*i.e.* 273.16 degrees centigrade below the ice-point). This temperature on the Kelvin Scale is referred to as 0 K. The ice point (0 C) on

Kelvin or Thermodynamic Scale is 273·16 K so that one degree interval on the centigrade scale is the same as a degree on Kelvin scale. The temperature *t* on centigrade scale can be expressed into that on Kelvin scale by the relation **$T=(t+273{\cdot}16)$ K** where T denotes the temperature on the Kelvin Scale.

The kelvin, unit of thermodynamic temperature is the fraction 1/273·16 of the thermodynamic temperature of the triple point of water.

It may be noted that according to SI system the triple point of water is taken as a fundamental fixed point having a temperature 273·16 K by definition.

(*v*) **candela (cd)** :

It is the luminous intensity of a given source of light under specific conditions. It is defined as the luminous intensity, in a given direction of a source which emits monochromatic radiation of frequency 540×10^{12} Hz and of which the radiant intensity in that direction is 1/683 W/Sr. The candela is approximately 0·98 of the original international candle—a sperm candle, which weighs one sixth of a pound, is 7/1 inch in diameter and burns at the rate of 120 grains (7.78 gm) of wax per hour.

(*vi*) **ampere (A)** :

It is defined as that constant current which, when flowing in two straight parallel conductors of infinite length, and of negligible area of cross-section, and placed one metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.

(*vii*) **mole (mol)** :

The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0·012 kg of carbon 12.

(*b*) **Supplementary Units :**

(*a*) **radian (rad)**

It is the plane angle between two radii of a circle which cuts off on the circumference an arc equal to the length of the radius. It should be noted from Fig. (1.7) that

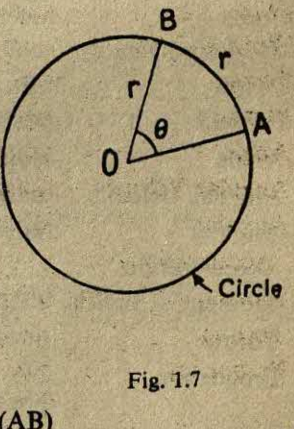


Fig. 1.7

$$\theta\ (\text{in radians})=\frac{\text{arc (AB)}}{\text{radius (OA)}}. \qquad \ldots(5)$$

(2) steradian (sr) :

It is the solid angle which, with its vertex at the centre of a sphere, cuts off an area of the surface of the sphere, equal to that of a square having sides of length equal to the radius of the sphere.

If ΔS is the area cut off on the surface of a sphere of radius r, the solid angle at the centre of the sphere is given by

$$\Omega=\frac{\Delta S}{r^2} \qquad -(6)$$

The following table—1.5 gives the SI units of various physical quantities.

Table 1.5

Quantity	**U N I T S**	
	S.I. Name	*Abbreviation*
Length	metre	m
Mass	kilogram	kg
Ttime	second	s
Area	sq metre	m^2
Volume	cubic metre	m^3
Density	kilogram per metre cube	$kg\ m^{-3}$
Velocity	metre per second	ms^{-1}
Acceleration	metre per second squared	ms^{-2}
Momentum	newton second kilogram metre per second	NS $kg\ ms^{-1}$
Force	newton	N
Work and Energy	joule	J
Power	watt	W
Pressure	pascal	Pa
Angle	radian	rad
Angular Velocity	radian per second	rad/s
Angular Acceleration	radian per second squared	rad/s^2
Moment of Inertia	kilogram metre squared	$kg\ m^2$
Torque	newton metre	Nm
Frequency	No. of Oscillations pre sec or hertz	s^{-1} Hz
Surface Tension	newton per metre	Nm^{-1}
Current	ampere	A

1·4 DIMENSIONS AND DIMENSIONAL EQUATIONS

The units of mass, length and time are denoted by the capital letters, M, L and T, which merely indicate their nature and not their magnitude. And, since the unit of area is the product of two unit lengths, we have the unit of area represented by L×L or L^2 : and similarly the unit of volume, being the product of three unit lengths, is represented by L×L×L or L^3. We express this by saying that the unit of area is of two dimensions in length, and the unit of volume, of three dimensions in length. Since neither the unit of area nor that of volume depends upon mass and time, their dimensions are said to be zero in both mass and time and we may, therefore, represent these units as $M^0 L^2 T^0$ and $M^0 L^3 T^0$, respectively.

Thus the dimensions of a derived unit may be defined as the powers to which the fundamental units of mass, length and time must be raised so as to completely represent it. For example,

$$\text{Velocity} = \frac{\text{distance or length}}{\text{Time}} = \frac{L}{T} = L^1 T^{-1},$$

and hence the dimensions of the unit of velocity are 1 in length and −1 in time. Since it is independent of mass, its dimension in mass is *zero*, and we may, therefore, represent it by $M^0 L^1 T^{-1}$.

Again, since acceleration $= \frac{\text{Velocity}}{\text{Time}}$ the dimensions of the unit of acceleration are $\frac{M^0 L^1 T^{-1}}{T} = M^0 L^1 T^{-2}$, and so on.

It will thus be seen that the dimensions of a physical quantity are obtained by simply defining it in terms of those physical quantities whose dimensions in mass, length and time are known-the value of a derived unit depending upon the values of the fundamental units from which it is derived.

A *Dimensional formula* is a compound expression, showing how and which of the fundamental units enter into the unit of a physical quantity. As explained above, the dimensional formula for velocity is $[M^0 L^1 T^{-1}]$ and for acceleration it is $[M^0 L^1 T^{-2}]$. But when we put it in the form.

Velocity $= M^0 L^1 T^{-1}$, it is called a *dimensional equation for velocity*.

Thus a dimensional equation is an expression which expresses the physical quantity in terms of fundamental units of mass, length and time. For instance, equations of the form

$$\text{Force} = M^1 L^1 T^{-2}$$

$$\text{Pressure} = M^1 L^{-1} T^{-2}$$

$$\text{Moment of Inertia} = M^1 L^2 T^0$$

are called dimensional equations.

The dimensional equations of a few physical quantities are given below :

1. Area$=(\text{length})^2=M^0\,L^2\,T^0$
2. Volume$=(\text{length})^3=M^0\,L^3\,T^0$
3. Velocity$=\dfrac{\text{Length}}{\text{Time}}=\dfrac{L}{T}=L^1\,T^{-1}=M^0\,L^1\,T^{-1}$
4. Acceleration$=\dfrac{\text{Velocity}}{\text{Time}}=\dfrac{L^1\,T^{-1}}{T}=L^1\,T^{-2}=M^0\,L^1\,T^{-2}$
5. Momentum$=$mass$\times$velocity $=M\times\dfrac{L}{T}=M^1\,L^1\,T^{-1}$
6. Force$=$mass$\times$acceleration $=M\times\dfrac{L}{T^2}=M^1\,L^1\,T^{-2}$
7. Work or Energy$=$Force$\times$distance
   $=[M^1\,L^1\,T^{-2}]\times[L^1]=M^1\,L^2\,T^{-2}$
8. Couple or (Torque)$=$Force$\times$length of arm
   $=[M^1\,L^1\,T^{-2}]\times[L^1]=M^1\,L^2\,T^{-2}$
9. Kinetic Energy$=\frac{1}{2}$ mass$\times$(Velocity)^2
   $=M^1\times[L^1\,T^{-1}]^2=M^1L^2T^{-2}$
10. Potential energy$=$mass$\times$acceleration due to gravity $\times$distance $=M^1\times L^1\,T^{-2}\times L^1=M^1\,L^2\,T^{-2}$.
11. Power$=\dfrac{\text{Work}}{\text{Time}}=\dfrac{M^1\,L^2\,T^{-2}}{T}=M^1\,L^2\,T^{-3}$
12. Density $=\dfrac{\text{Mass}}{\text{Volume}}=\dfrac{M}{L^3}=M^1\,L^{-3}\,T^0$
13. Specific gravity$=$a mere ratio$=$*no dimensions.*
14. Pressure $=\dfrac{\text{Force}}{\text{area}}=\dfrac{M^1\,L^1\,T^{-2}}{L^2}=M^1\,L^{-1}\,T^{-2}$
15. Stress$=\dfrac{\text{Force}}{\text{area}}=\dfrac{M^1\,L^1\,T^{-2}}{L^2}=M^1\,L^{-1}\,T^{-2}$
16. Strain$=$a mere ratio and hence it is dimensionless.
17. Elasticity $=\dfrac{\text{Stress}}{\text{Strain}}=M^1\,L^{-1}\,T^{-2}$
18. Surface Tension$=\dfrac{\text{Force}}{\text{Length}}=\dfrac{M^1\,L^1\,T^{-2}}{L^1}$
    $=M^1\,T^{-2}=M^1\,L^0\,T^{-2}$

19. Surface Energy$=\dfrac{\text{Work}}{\text{area}}=\dfrac{M^1\,L^2\,T^{-2}}{L^2}$

$=M^1\,T^{-2}=M^1\,L^0\,T^{-2}$

20. Angle$=\dfrac{\text{Arc}}{\text{Radius}}=\dfrac{L}{L}$

=a mere ratio. It is thus dimensionless.

21. Angular Velocity$=\dfrac{\text{Angle}}{\text{Time}}=\dfrac{1}{T}=T^{-1}=M^0\,L^0\,T^{-1}$

22. Angular Acceleration $=\dfrac{\text{change in angular velocity}}{\text{Time}}=\dfrac{T^{-1}}{T}$

$=T^{-2}=M^0\,L^0\,T^{-2}$

23. Moment of Inertia = Mass × (radius of gyration)^2

$=M^1\,L^2=M^1\,L^2\,T^0$

24. Frequency $=\dfrac{\text{number of vibrations}}{\text{Time}}=\dfrac{1}{T}=T^{-1}=M^0L^0T^{-1}$

25. Coefficient of Viscosity$=\dfrac{\text{Force/area}}{\text{Velocity/distance}}$

$$=\frac{M^1\,L^1\,T^{-2}/L^2}{L^1\,T^{-1}/L^1}=M^1\,L^{-1}\,T^{-1}$$

26. Gravitational Constant$=G=\dfrac{F\times d^2}{m_1\,m_2}=\dfrac{M^1\,L^1\,T^{-2}\times L^2}{M^2}$

$=M^{-1}\,L^3\,T^{-2}$

27. Specific Heat = Joule per kg per K rise in temperature

= Energy × (mass)$^{-1}$ × (Tem. diff.)$^{-1}$

$=M^1\,L^2\,T^{-2}\times M^{-1}\times K^{-1}$

$=M^0\,L^2\,T^{-2}\,K^{-1}$

28. Angular Momentum = Moment of Inertia × Angular Velocity

$=M^1\,L^2\times T^{-1}=M^1\,L^2\,T^{-1}$

29. Temperature$=M^0\,L^0\,T^0\,K^1$

30. Heat Energy$=M^1\,L^2\,T^{-2}$

31. Thermal Conductivity$=\dfrac{\text{Heat}\times\text{distance}}{\text{Area}\times\text{Temp. diff.}\times\text{Time}}$

$$=\frac{[M^1L^2T^{-2}]\times[L]}{(L^2)\times[K]\times[T]}=M^1\,L^1\,T^{-3}\,K^{-1}$$

32. Latent Heat$=\dfrac{M^1L^2T^{-2}}{M}=M^0L^2T^{-2}$

33. Force Constant$=\dfrac{\text{Force}}{\text{Displacement}}=M^1\,L^0\,T^{-2}$

34. Planck's Constant = Energy × Time = $M^1\ L^2\ T^{-1}$

35. Boltzmann Constant = $\frac{\text{Energy}}{\text{Temperature}} = M^1\ L^2\ T^{-2}\ K^{-1}$

36. Radius of Gyration = Length = L^1

The student is no doubt aware that in Physics we come across two types of quantities, *viz. variables* and *constants* which may both be dimensional or dimensionless. Thus we have

(*i*) *Dimensional variables*—these are quantities like acceleration, velocity, force etc. which the Physicist has to deal with, at every step.

(*ii*) *Dimensional constants*—Quantities which have a constant value and yet have dimensions. For instance, Gravitational constant (G) and the velocity of light in Vacuo (c) etc.

(*iii*) *Non-dimensional variables*—Quantities which are variable and yet have no dimensions *e.g.*, specific gravity, strain or an angle etc.

(*iv*) *Non-dimensional constants*—these are mere numbers like 3, 2, π, etc.

Thus, *numerics & pure numbers* have no dimensions in Mass, Length & Time.

Uses of Dimensional equations

A careful examination of the dimensional equations of the various physical quantities involved in a relation *i.e.* an analysis of their dimensions, is of great help to us in more ways than one. Its two chief uses are the following :—

(a) Checking the Accuracy of a given Physical Equation

This depends upon what is called the **principle of homogeneity of dimensions**, *according to which the dimensions of all the terms on the two sides of an equation must be the same*. This follows at once from the fact that it is not possible to compare two physical quantities of different natures, and that only quantities of the *same* nature can be added up together, their resultant being also of the *same* nature. We can add mass to mass to get the resultant mass or velocity to velocity to get the resultant velocity, we cannot add velocity and mass or volume and time etc. *If, therefore, in a given relation the terms on either side have the same dimensions, the relation is a correct one*, but if they have not, there is a flaw somewhere which must be diligently sought out.

Let us take a couple of *examples* :—

(1) *To check the accuracy of the relation*, $t=2\pi\sqrt{l/g}$ *for a* simple pendulum.

[*A.I.S.S.C. 1986*]

Here the term t on the *left hand side* has only *one dimension in time* or the dimension of t is [T], its dimensions in both mass and length being *zero*.

And, on the *right hand side* 2π has no dimensions, being just a number; l has *one dimension in length* or its dimension is [L], those in mass and time being *zero*; and the dimensions of g, the acceleration due to gravity are L^1T^{-2}, that in mass being *zero*.

Hence the dimensions of the term $2\pi\sqrt{\frac{l}{g}}=\sqrt{\frac{L^1}{L^1T^{-2}}}=\sqrt{T^2}=[T]$

i.e. it has only one dimension in time : the same as the term on the left hand side. The relation $t=2\pi\sqrt{\frac{l}{g}}$ is, therefore, a correct one,

We can only say that the relation $t=2\pi\sqrt{\frac{l}{g}}$ is dimensionally correct. By the above proof, we cannot say whether it is numerically correct or not. For instance the relation $t=4\pi\sqrt{\frac{l}{g}}$ is also dimensionally correct but numerically wrong.

(2) *To check the relation $S=V_0t+1/2\ at^2$, for the distance covered in t seconds by a body, having an initial velocity V_0 and an acceleration a.*

Here, the dimension of the term S on the left hand side is *one in length*, or L^1, and taking the terms on the right hand side we have

(*i*) dimensions of V_0 (Velocity)$=L^1T^{-1}$
(*ii*) dimensions of t (time)$=T^1$
(*iii*) dimensions of 1/2 (a number)$=$no dimensions
(*iv*) dimensions of a (acceleration)$=L^1T^{-2}$
(*v*) dimensions of t^2 (time)$^2=T^2$

dimensions of the term,

$$V_0t+1/2\ at^2=[L^1T^{-1}]\times[T^1]+[L^1T^{-2}]\times[T^2]=L^1+L^1$$

i.e. the dimensions of each term on the right hand side is the same as that of the term on the left hand side, hence the given relation is correct.

(b) Deriving a correct relationship between different Physical Quantities

The principle of homogeneity of dimensions also enables us to deduce a relationship between different physical quantities which are related, or at any rate, a preliminary form of such a relationship. For knowing the factors on which a physical quantity may possibly depend, an expression for it can be obtained in terms of these factors, so that the dimensions of the terms on the two sides of the expression are the same. A few examples will illustrate the point.

(*1*) *To deduce an expression for the time period of a simple pendulum.*

The factors on which the time period (t) may possibly depend are the following :—

(*i*) the mass of the bob (m),
(*ii*) the length of the Pendulum (l),
(*iii*) acceleration due to gravity (g) and
(*iv*) the angle of swing of the Pendulum (θ).

Let us further suppose that

$$t \propto m^a$$
$$\propto l^b$$
$$\propto g^c$$
$$\propto \theta^d$$

So that $t = K m^a \, l^b \, g^c \, \theta^d$, where K is a constant of proportionality.

Taking dimensions of the terms on either side of the sign of equality, we have

$T^1 = M^a \, L^b \, (L^1T^{-2})^c$ as K and θ have no dimensions
or $T^1 = M^a \, L^{b+c} T^{-2c}$

Since the dimensions of the terms on the two sides must be the *same*, we have, equating the indices of M, L and T.

$$\boxed{a=0}$$

$b+c=0$ and $-2c=1$

$$\therefore \quad \boxed{c=-\tfrac{1}{2}}$$

Thus, $b-1/2=0$, or $\boxed{b=1/2}$

Therefore, $t = K \, l^{1/2} \, g^{-1/2}$

or $t = K\sqrt{l/g}$

The value of K cannot be found by the method of dimensions since it is just a number. It can be found experimentally or by mathematical investigation which shows that $K = 2\pi$

Hence $t = 2\pi\sqrt{l/g}$

(2) *To deduce a relationship for the velocity of sound in a material medium, the temperature of the medium remaining constant.*

The velocity V may depend upon

(*i*) the elasticity of the medium (E)
and (*ii*) the density of the medium (ρ), so that

$V = K . E^a . \rho^b$, where K is a constant of proportionality

Again taking dimensions of the terms on both sides, we have

$$M^0L^1T^{-1} = (M^1 \, L^{-1} \, T^{-2})^a (M^1 \, L^{-3})^b$$
$$= M^{a+b} \, L^{-a-3b} \, T^{-2a}$$

Since the dimensions on the two sides must be the *some*, it is clear that

$$a+b=0\ ;$$

$$-a-3b=1$$

and $-2a=-1$, whence, $\boxed{a=1/2}$

and $b=-a=-1/2$ or $\boxed{b=-1/2}$

Hence $V=K\,E^{1/2}\rho^{-1/2}$

$$V=K\sqrt{E/\rho}$$

The value of constant of proportionality K is again determined by experiment, and is found to be 1 in this case; so that

or $V=\sqrt{E/\rho}$

(3) A liquid is flowing steadily through a pipe. Assume that the volume of the liquid issuing out per second depends on (i) the coefficient of viscosity of the liquid, η. (ii) the radius of the pipe, r and (iii) the pressure gradient along the pipe (pressure gradient is drop in pressure per unit length of the pipe and it is equal to p/l where p is the pressure difference between the ends of the pipe and l is the length of the pipe). Deduce by the method of dimensions the formula for the volue of the liquid flowing out per second (given the dimensions of the coefficient of viscosity η are $M^1L^{-1}T^{-1}$).

Let V = Volume of the liquid issuing out per second.

Then, V depends upon

(*i*) coefficient of Viscosity, η,

(*ii*) the radius of the pipe, *r*;

(*iii*) the pressure gradient along the pipe, *p*/*l*

Let $V \propto \eta^a\, r^b\, (p/l)^c$

so that $V=K\,\eta^a\, r^b \left(\frac{p}{l}\right)^c$ where K is constant of proportionality.

Writing the dimensions of the various terms on both sides in the above equation, we have

$$M^0L^3T^{-1}=[M^1L^{-1}T^{-1}]^a\ \ [L]^b \left[\frac{M^1L^{-1}T^{-2}}{L^1}\right]^c$$

or $M^0L^3T^{-1}=M^{a+c}\ \ L^{-a+b-2c}\ \ T^{-a-2c}$

Equating the powers of M, L and T on either side, we have

$a+c=0$...(*i*) $-a+b-2c=3$...(*ii*) and $-a-2c=-1$...(*iii*)

From (*i*), $a=-c$

Putting it in (*iii*), $c-2c=-1$ or

$$\boxed{c=1}$$

$$\boxed{a=-1}$$

Putting a and c in (*ii*), $1+b-2=3$ or $\boxed{b=4}$

Hence the formula for V becomes $V=K\,\eta^{-1}\,r^4\left(\frac{p}{l}\right)^1$

or $$\boxed{V=\frac{K\,r^4\,p}{\eta l}}$$

Limitations of Dimensional Analysis :

It will be readily seen from the examples given above, that the method of dimensional analysis is, after all, not quite so simple in its application. It has also its own limitations. Thus, for example :

(*i*) Its one obvious drawback is that it gives little or no information about *pure numerics and non-dimensional constants* (like constant of proportionality) involved in various physical relations and which, therefore, have to be determined by separate calculation or experiment.

(*ii*) Since at best only *three* equations can be obtained by equating the dimensions of M, L and T, this method is of no avail in deducing the exact form of a physical relation which happens to depend upon more than three quantities.

(*iii*) It is of no use when :

(*a*) trigonometric ratios are involved.

(*b*) complex relations like

$$M=\frac{M_0}{\sqrt{1-\frac{V^2}{c^2}}}$$ are to be derived

unless one of them is related to another and the relation is given.

(*iv*) It does not tell us whether the quantity is vector or scalar.

SUMMARY

1. Scientist deals with a wide range of measurements from the *infinite* to the *infinitesmal.*
2. Length is any interval of distance. We can measure many distances directly by laying objects representing the unit end to end. Often, however, we

need to measure very large distances or distances to inaccessible points *e.g.* heights of mountains, distance to heavenly objects. In such cases, we can use methods of *triangulation* and *parallax*. Distances to more remote stars are measured by methods involving their *brightness* or the *Doppler Shift*.

3. One of the important measuring units used by astronomers is the *astronomical unit* (AU). 1 AU = 149×10^6 km. A still large unit of distance is the *light year*. One light year is the distance travelled by light in one year, about 9.5×10^{12} km.

4. For measuring very small distances such as those within molecules and atoms, only *indirect methods* can be adopted.

5. A phenomenon that repeats itself can be used as a measure of time. Greater accuracy has been achieved with a *cesium clock*.

6. The three standard units, the *Metre*, *Kilogram*, and *Second* form what is called the Metre-Kilogram Second (MKS) system. In the scientific field SI system is now used. In SI system, there are *six* fundamental and *two* supplementary units. The six fundamental units are : *metre* (for length) ; *Kilogram* (for mass) ; *Second* (for time) ; *Kelvin* (for temperature) ; *Candela* (for luminous intensity) and *ampere* (for electric current). The two supplementary units are : *radian* (for plane angle) and *steradian* (for solid angle).

7. Physical quantities in mechanics are expressed in terms of only three fundamental quantities of *length*, *mass* and *time*.

8. The *dimensions* of any derived unit are the power to which the fundamental units of length, mass and time must be raised so as to completely represent the derived unit. The equations which represent the dimensions of any physical quantity in terms of the fundamental units are called *dimensional equations*.

QUESTIONS

1. What is a light year ? How will you estimate the distance of a distant star by Parallax method ?

2. What is a coherent system of units ? What are the fundamental and the supplementary units of measurement in the International system of units (SI system) ? Define these units.

3. What are the desirable characteristics of a physical standard ? What is the advantage in choosing the wave-length of light radiation as a standard of length ?

4. What is the average distance of the sun from the earth ? How does the triangulation method of distance measurement differ from the parallax method ?

5. Suggest an indirect method of measuring the height of a tree on a sunny day.

6. Mention some repetitive phenomenon occuring in nature which could serve as time standards. What will be your criterion in selecting the most appropriate one as a time standard ?

7. What is the basis for the measurement of sidereal time ? What is one advantage of the use of sidereal time over the use of solar time ?

8. Explain the terms : light year, galaxy and Hubble's principle.

9. What do you mean by fundamental and derived units ? Give examples.

10. What do you mean by dimensions, dimensional formulae and dimensional equations.

11. What is the advantage in expressing physical quantities in terms of dimensional equations ? Mention two applications of dimensions with examples.

12. Write the dimensions of the following physical quantities :

    (*i*) Angular Momentum (*ii*) Temperature (*iii*) Heat Energy (*iv*) Specific Heat (*v*) Force Constant (*vi*) Planck's Constant (*vii*) Boltzmann Constant (*viii*) Torque (*ix*) Elasticity (*x*) Impulse (*xi*) Angular Velocity (*xii*) Coefficient of Viscosity. [*A.I.S.S.C. 1986*]

13 How is it estimated that universe is expanding ?

14. Suggest a distance corresponding to each of the following orders of length—

    (*i*) $10^{26}m$ (*ii*) $10^{7}m$ (*iii*) $10^{4}m$ (*iv*) $10^{3}m$ (*v*) $10^{2}m$ (*vi*) $10^{-4}m$ (*vii*) $10^{-8}m$ (*viii*) $10^{-14}m$.

15. Derive dimensional formulae of the following physical quantities.

    (*a*) Velocity (*b*) Acceleration (*c*) Momentum (*d*) Force (*e*) Pressure (*f*) Work (*g*) Kinetic Energy (*h*) Potential Energy (*i*) Power (*j*) Stress (*k*) Strain (*l*) Surface Tension (*m*) Gravitational constant (*n*) Moment of Inertia.

16. Which is longer, a kilosecond or a microcentury ?

17. The three fundamental units used in mechanics are the metre (*m*) to measure length, the kilogram (kg) to measure mass, and the second (s) to measure time. What unit is employed in this system to specify each of the following quantities ? Use abbreviations, so that each answer involves one or more of the fundamental units *m*, **kg** and s.

    (*a*) A Volume

    (*b*) An acceleration

    (*c*) The rate at which oil is moved by a pipe line.

    (*d*) The rate at which a mower cuts a lawn.

    (*e*) The rate at which a machine gun fires bullets.

18 Write down a convenient unit to use when talking about

(*a*) the diameters of test tubes

(*b*) the wavelengths of light of various colours, in the range $4 \times 10^{-7} m$ to $8 \times 10^{-7} m$.

(*c*) the diameters of the planets, ranging from $5 \times 10^{6} m$ to $1 \times 10^{8} m$.

(*d*) the masses of grains of sand, of the order of 10^{-6} kg ;

(*e*) the times taken for radar echoes to return from objects a few **kilometres** away, of the order of 10^{-5} s.

19. **Mass,** Length, and time are commonly accepted as the fundamental quantities of mechanics, but there is no theoretical justification for choosing these three.

(*a*) If force, length and time were chosen as fundamental quantities, what would be the dimensions of mass ?

(*b*) Why in practice is it more convenient to use mass rather then force as a fundamental quantity ?

PROBLEMS

1. The moon subtends an angle of 57′ at the base line equal to the radius of the earth. Compute the distance (Radius of the earth = $6{\cdot}378 \times 10^{6} m$). [$3{\cdot}844 \times 10^{8} m$]

2. Suppose that a planetoid comes so near the earth that if it is observed from opposite sides of the earth, so that one observer sees it on the horizon in the east and the other sees it on the horizon in the west, the angle which the earth's diameter subtends at the planetoid is 0.05°. How far is the planetoid from the earth ? (Radius of earth = $6{\cdot}378 \times 10^{6} m$)

(15 million km)

3. An exam candidate has forgotten whether the period of swing T of *a* pendulum of length *l* is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \text{or} \qquad T = 2\pi\sqrt{\frac{g}{l}}$$

where *g* is the acceleration due to gravity.

(*a*) write down the dimensions of T, *l* and *g*.

(*b*) 2 is a number and has no dimensions. What is π ? Why does it have no dimensions ?

(*c*) By dimensional analysis, which of the two expressions is correct ?

4. The thrust (force) F developed by a rocket motor is given by

$$F = mV + A(p_1 - p_2)$$

where m=mass of gas ejected per unit time,

V = Velocity of this gas,

A = area of exhaust nozzle,

P_1 = pressure of the exhaust gas, and

P_2 = pressure of the surrounding atmosphere

Show that this equation is dimensionally correct.

5. The fundamental unit of time is the second, but several other units, such as the minute and the hour, are also in common use. These are not related to the second by simple powers of ten. Suppose an autocratic government decides to introduce decimal time. The second, minute and hour are abolished and the day becomes the fundemental unit. As at present the day is taken to start at midnight, when the time is 0·000 O'clock. Noon becomes 0·500 O'clock and the following midnight 1·000 O'clock.

(*a*) what time is 0·750 O'clock ? 0·333 O'clock ?

(*b*) what is the decimal equivalent of 10 p.m. ?

(*c*) An exam is to run from 0·600 to 0·725 O'clock. How long is this in decimal time ? in hours ?

(*d*) what would be the principal advantages and disadvantages of changing to this system ?

[(*a*) 6 p.m., 8 a.m (*b*) 0·972 O'clock
(*c*) 0·125 day ; 3 hours.]

6. The distance of some of the nearer stars has been measured

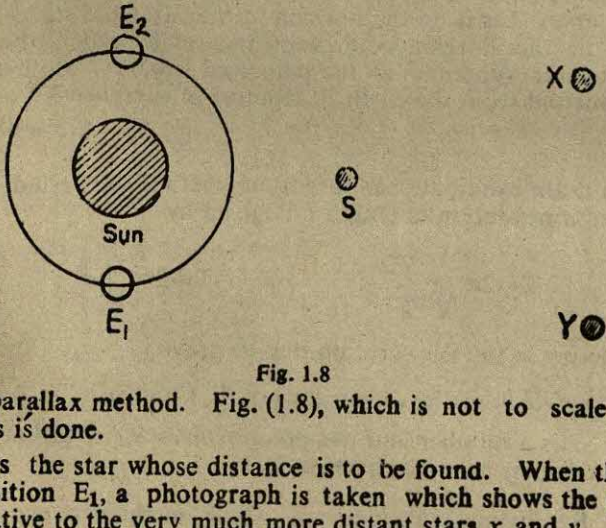


Fig. 1.8

by the parallax method. Fig. (1.8), which is not to scale, shows how this is done.

S is the star whose distance is to be found. When the earth is in position E_1, a photograph is taken which shows the position of S relative to the very much more distant stars x and y.

(*a*) Sketch what this photograph would look like assuming that the earth and the three stars all lie in one plane, *i.e.* the stars appear along a straight line in the photograph.

(*b*) Six months later the earth has moved to E_2, and another photograph is taken. Sketch how the three stars would look now.

(*c*) S will appear to have moved relative to X and Y, and if the smallest parallax angle E_1SE_2 that can be measured with accuracy is about 10^{-7} radian, how far away is the star ? The diameter of earth's orbit is $3 \times 10^{11} m$?

(*d*) Why must stars X and Y be much more distant than S ?

(*e*) Calculate how far away a 10 *p* coin 2·4 cm. in diameter would have to be to subtend an angle of 10^{-7} rad at your eye.

[(c) $3 \times 10^{18} m$ or 300 light years; (e) 240 km away)]

7.

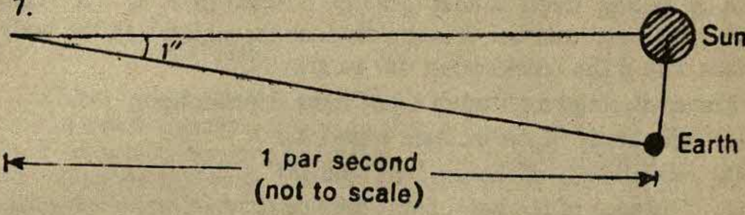


Fig. 1.9

The *parsec* is a unit of length used by astronomers for describing distances *outside the solar system*. It is defined to be the distance at which the radius of the earth's orbit subtends an angle of 1 second of arc, (Fig. 1.9)

(*a*) Taking one radian to be 57°, convert one second of arc to radians.

(*b*) If the distance from earth to sun is $1{\cdot}5 \times 10^{11} m$, how many meters are there in a parsec ?

(*c*) How does a parsec compare with a light year, which is $9{\cdot}5 \times 10^{15} m$?

[(*a*) $4{\cdot}9 \times 10^{-6}$ rad (*b*) $3{\cdot}1 \times 10^{16}$ *m* (*c*) 3·3 light years.]

8. Check the dimensional consistency in the case of the following equations :

(*i*) $S = V_0 t + 1/2 at^2$ (*ii*) $V = V_0 + at$ (*iii*) $FS = 1/2 mV^2 - 1/2 m V_0^2$ where S is the distance moved in time *t*, V_0 and V are the velocities initially and at time *t*, *a* is the acceleration, *m* is the mass and F is the force acting.

9. The acceleration '*a*' of a particle moving on a circumference of a circle depends upon the linear velocity 'V' and radius '*r*' of the circle. Prove making use of the knowledge of dimensions that $a = V^2/r$

10. The time of oscillation of a small drop of liquid under surface tension depends upon the density ρ, radius *r* and surface tension σ. From the knowledge of dimensions, prove that

$$\text{the period of oscillation } T \propto \sqrt{\frac{\rho r^3}{\sigma}}$$

11. Show that the equation for vertical distance travelled by a

freely falling body in time t, $S=1/2\, gt$ is dimensionally incorrect. Use the method of dimensions to obtain the correct form. [$S=1/2\, gt^2$]

12. Derive an expression for the frequency of a string stretched with a given tension. The frequency of vibration n of the string may depend upon :
(*a*) the mass per unit length of the string (m), (*b*) tension acting on the string (T), (*c*) Length of the string (1)
$$\left[n = \frac{K}{l} \sqrt{\frac{T}{m}} \right]$$

13. Use the method of dimensions to show that the velocity V of a body falling freely under gravity is equal to $K\sqrt{gh}$ where K is a pure number, h is the distance through which the body falls and g the acceleration due to gravity.

14. The scattering amplitude (A_s) of light depends upon
(*a*) amplitude of the incident wave (A_i)
(*b*) wavelength of the incident light (λ)
(*c*) Distance of the point from the Scatterer (r)
(*d*) Linear dimension of the scattering particle (l).
Using dimensional theory show that the scattering amplitude varies as $1/\lambda^2$.

15. Find the dimensions of ϵ_0 from the following relation
$F=1/4\pi\epsilon_0\, q_1 q_2/r^2$
where F is the force between two charges q_1 and q_2 separated by a distance r and charge = current × time. [$M^{-1} L^{-3} T^4 A^2$]

16. If a planet moves round the sun in a circular orbit and its time period t is found to depend upon, the radius of the orbit (a), the mass (m) of the sun and the Gravitational constant (G). Prove that the square of the time period of the planet is directly proportional to the cube of its distance.

17. Find dimensionally an expression for the critical velocity (V_c) if the critical velocity depends upon the coefficient of viscosity (η); density of the liquid (ρ) and the radius of the spherical body (r) $\left[V_c = K \frac{\eta}{\rho r} \right]$

18. Reynold number N_R (a dimensionless quantity) determines the condition of laminar flow of a viscous liquid through a pipe. N_R is a function of the density of the liquid 'ρ,' its average speed 'v', and the coefficient of viscosity of the liquid 'η'. Given that N_R is also directly proportional to 'd', (the diameter of the pipe), show from dimensional consideration that
$$N_R \propto \frac{d\rho v}{\eta}$$
[The unit of η in S.I. system is kg $m^{-1}s^{-1}$]
[*Delhi Senior Sec. 1980*]

UNIT 2

MOTION

2.1 INTRODUCTION

That objects in the real world appear to be in a continual state of relative motion is one of the primary data of experience. Traffic moves in various directions in the streets, pedestrians move relative to the buildings, the sun moves over the sky, autumn leaves fall to the ground etc. Like all really fundamental ideas, the idea of motion is difficult to describe in words, and attempts to do so inevitably reduce to circular statements like "Motion is when things move"—with a shrug to indicate the speaker's contempt for the futility of the question. Yet philosophers of all ages have pondered over the problems inherent in the definition of motion. The Greek thinker Zeno, and others following him came to the conclusion that the popular idea of motion is self-contradictory. Consider Zeno's paradox of Achilles and the tortoise. The assertion was that Achilles ran 10 times as fast as the tortoise, yet, if the tortoise had (say) a start of 100 *m*, it could never be overtaken. For when Achilles had gone the 100 *m*, the tortoise would still be 10 *m* in front of him ; by the time he had covered these 10 *m* it would still be in front of him : and so on for ever. Thus Achilles would get nearer and nearer to the tortoise but would never overtake it. The resolving of the paradox depends on the recognition that the sum of an infinite series of terms need not be infinite, but this explanation rests on the assumption, which Zeno would not have admitted, that space and time are infinitely divisible. Consideration of ancient paradoxes such as this makes clear the need for a new language suitable for describing the motion of objects, and the purpose of the present unit is to analyze the essential elements of the simplest kind of motion—that in a straight line—in the language of calculus.

2.2 DISPLACEMENT

Consider an automobile travelling along a straight stretch of road. At regular intervals of 100 *m* there are posted observers to record the times at which the automobile passes so that a complete record of the motion can be obtained. The automobile starts opposite one observer who may be called the reference observer and labelled 0. All time measurements are synchronized with him, and it is convenient to measure time from the instant the automobile starts. Thus for observer 0, $t=0$, and the times recorded by the other observers are the times that have elapsed since the automobile started. In the same way, the observer 0 may be taken as the reference point for the positions of the others. Thus successive observers P, Q, R, ... to the right of 0 are said to be at positions, $+100\ m$, $+200\ m$, $+300\ m$, etc. and observers N, M, L,... to the left of 0 are said to be at positions $-100\ m$, $-200\ m$, etc. These are the coordinates of the various observers along the line of the road, and may be given the symbol x. Thus $x_P=+100\ m$, $x_R=+300\ m$, $x_M=-200$, etc. The *displacement* of one observer relative to another is the coordinate of the first minus the coordinate of the second. Thus the displacement of observer P relative to observer M is $x_P-x_M=+100\ m-(-200\ m)=+300\ m$. This means that P is 300 m to the right of M. Conversely the displacement of M relative to P is $-300\ m$, since M is to the left of P. *The coordinate or position of any observer is just his displacement, relative to the reference observer.* If the reference observer is changed, then all the coordinates will change, but the displacement of one point relative to another will not change. If in any interval of time the car changes its position, the displacement of the car is defined as this change of position. Such a displacement may be positive or negative according as the movement is to the right or the left, and the amount of the displacement is independent of the choice of reference observer.

Thus in general it follows from the above discussion that **a change of position of a particle is called displacement.** Let a particle move from position A to position B (Fig. 2.1). Its

Fig. 2.1. A displacement vector

displacement is $\overrightarrow{AB}$, the arrow on the top signifying that the particle has moved from A to B. If the particle moves from B to A (Fig. 2.2), its displacement will be $\overrightarrow{BA}$. In either case, the magni-

Fig. 2.2. Displacement

tude of the displacement is the same but the direction in the second case is opposed to that in the first case.

The Nature of a Vector. *In a three-dimensional space there are many physical quantities that have direction as well as magnitude. Such quantities are* called **vectors.** For example displacement, velocity, acceleration, force, momentum etc.

A quantity that has magnitude but no direction is called a **scalar.** Examples of scalars are time, mass, speed, temperature, electric charge, volume, density, energy etc.

The magnitude of a vector, a scalar quantity, is called *modulus of the vector.* The modulus of the vector $\overrightarrow{AB}$ (Fig. 2.1) is denoted by $|\overrightarrow{AB}|$

Symbol for a Vector. In print, the symbol for a vector is set in bold face type, whereas the symbol for a scalar is set in italics. The vector representing a velocity would therefore appear as **V**, whereas the corresponding speed would be *V*. In handwriting, this distinction is not possible and it is convenient to indicate a vector quantity by placing an arrow above it ; for example $\vec{V}$, the magnitude of velocity is, however, represented by V or sometimes by $|\mathbf{V}|$.

2.3 GRAPHICAL REPRESENTATION OF VECTORS :

A vector is represented pictorially by a line that is drawn in the same direction as the vector and has a length proportional to the magnitude of the vector on some agreed scale. An arrow-head indicates one of the two possible directions along the line.

Suppose, a body possesses a velocity of 30 km / h^{-1} due east. If 1 cm is chosen to represent a velocity of 10 km / h^{-1} a line OA, 3 cm long and drawn towards east with arrow-head at A (Fig 2.3), will represent completely the velocity of the body. The end where the arrow-head is marked is called the *head* of the vector,

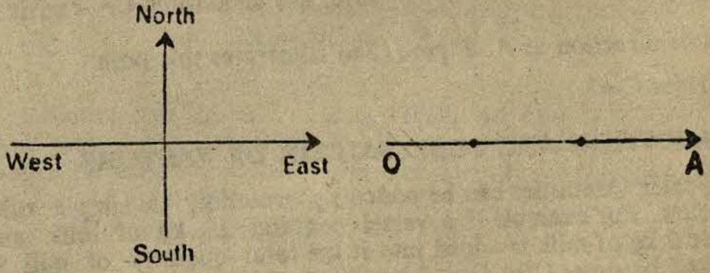


Fig. 2.3

whereas the other end is called the *tail* of the vector. Thus in Fig. (2.3), A is the head and O is the tail of the vector OA.

Similarly a velocity of 60 km per hour due north could be represented by a line with an arrow-head pointing due north and a length of 6 cm, if it had been agreed that 1 cm was to represent 10 km per hour.

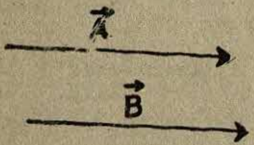


Fig. 2.4. Two equal Vectors

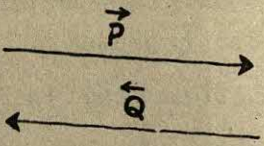


Fig 2.5. Negative of a Vector

Equal Vectors. Two vectors of the same physical quantity are equal if they have the same length and drawn in the same direction. Fig (2.4) shows two vectors $\vec{A}$ and $\vec{B}$ which have the same length and are drawn along the same direction. Hence they are equal, *wherever they may be drawn.*

The *negative* of a vector is defined as another vector having the same length but drawn in opposite direction. Thus in Fig. (2.5), the vector $\vec{Q}$ is equal in length but opposite in direction to vector $\vec{P}$.

$$\therefore \qquad \vec{P} = -\vec{Q}$$

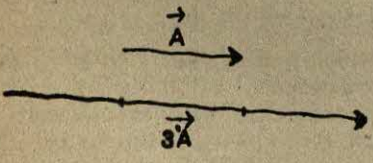


Fig (2.6)
Multiplication of a vector by a scalar

Multiplication of a vector by a scalar does not change the direction of the vector but multiplies its magnitude by the scalar in question. The vector $\vec{A}$ has magnitude A and vector $\overrightarrow{CA}$, therefore, has magnitude CA and is in the same direction as $\vec{A}$. Figure (2.6) illustrates this point.

Here C=3.

2.4 ADDITION AND SUBTRACTION OF VECTORS

Scalar Quantities can be added up according to simple rules of algebra. For example if a vessel contains 10 kg of milk and another 5 kg of milk is added into it the total quantity of milk in the vessel becomes (10+5)=15 kg. But it is not possible to add up vector quantities according to simple algebraical rule. Let us illustrate it by an example involving two displacements.

Figure 2.7 is a map of India. The displacement from Bombay to Bangalore is represented by the vector $\overrightarrow{PQ}$. The displacement from Bangalore to New Delhi is represented by the vector $\overrightarrow{QR}$. From the point of view of one's eventual location a trip from Bombay to Bangalore followed by a trip from Bangalore to New Delhi has exactly the same end result as a trip directly from Bombay to New Delhi. The vector $\overrightarrow{PR}$ is, therefore, exactly equivalent to the sum of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$. This is written

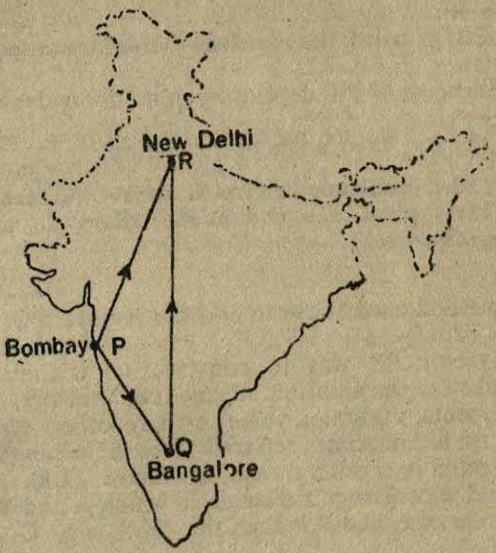


Fig. 2.7

$$\boxed{\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}} \qquad \dots (2\text{-}1)$$

Notice that this is a vector equation and must be clearly distinguished from an ordinary algebraic equation involving scalar quantities, with which the reader may already be familiar. It is *not* true for example, that

$$PR = PQ + QR \qquad \dots (2\text{-}2)$$

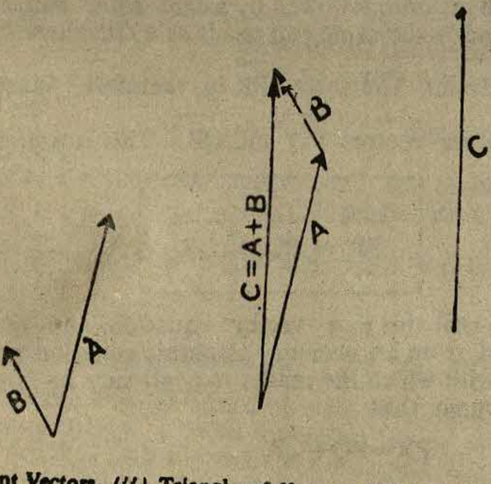
since one covers fewer kilometres in going directly from Bombay to New Delhi than if one went via Bangalore.

Since the displacement $\overrightarrow{PR}$ in the above example produces the same effect as the displacements $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ together do, therefore, $\overrightarrow{PR}$ is called the **resultant displacement**. The magnitude and direction of $\overrightarrow{PR}$ depend upon the individual displacements $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ and their angle of inclination.

Thus, **the resultant of two or more vectors is a single vector which produces the same effect as the individual vectors together do.**

The method used above to add the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ and obtain the vector $\overrightarrow{PR}$ was to construct a triangle PQR. This method works for the addition of any two vectors, whether they are displacements, velocities, forces or any other type of vector. The procedure is sometimes referred to as the **triangle of vectors** and is illustrated in a general case in figure (2.8). To add the vectors **A** and **B** construct a triangle in which **A** and **B** are adjacent sides. The sum of **A** and **B** is then the third side **C** and one can write

$$\boxed{\mathbf{C}=\mathbf{A}+\mathbf{B}} \qquad \dots (2\text{-}3)$$



(*i*) Component Vectors, (*ii*) Triangle of Vectors, (*iii*) Resultant Vector

Fig. 2.8

Notice that the arrow-heads on the vectors being added together, **A** and **B** both proceed in the same direction around the triangle [counter clockwise in Fig 2.8 (*ii*)], but the arrow-head on the resultant vector **C** points in the opposite direction [clockwise in figure 2.8 (*iii*)]. The effect of **C** is exactly the same as the combined effect of the vectors **A** and **B** . **C** is called the resultant of **A** and **B**, whereas **A** and **B** are the **components** of **C**. For all practical purposes, the vectors **A** and **B** can be replaced by the single vector **C**.

Thus according to **triangle law of vectors**, *if a particle is simultaneously acted upon by two vectors and they can be represented in magnitude and direction by the two adjacent sides of a triangle* **taken in order**, *then the third side of the triangle taken in the* **opposite order** *completely represents the resultant vector of the two component vectors.*

Again consider two vectors **A** and **B** acting upon a particle simultaneously inclined at angle θ [Fig 2.9 (*i*),], Move **B** parallel

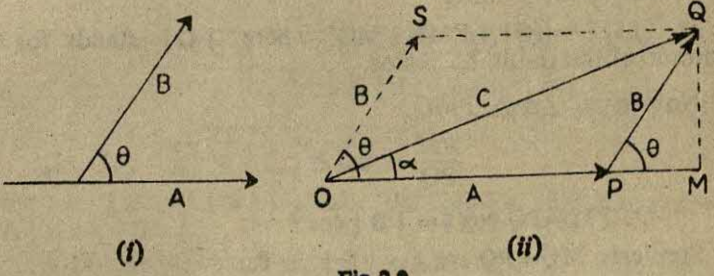


Fig 2.9

to itself till its tail touches the head of **A** as shown in Fig. [2.9 (*ii*)]

Let $\overrightarrow{OP}$ and $\overrightarrow{PQ}$ in Fig. [2.9 (*ii*)] represent in direction as well as in magnitude the two vectors **A** and **B** respectively. The vector $\overrightarrow{OQ}$ which connects the foot of $\overrightarrow{OP}$ and the head of $\overrightarrow{PQ}$ represent the resultant vector **C**.

Now complete the parallelogram OPQS. As mentioned above, $\overrightarrow{OQ}$ is the resultant of $\overrightarrow{OP}$ and $\overrightarrow{PQ}$;

or $$\overrightarrow{OP}+\overrightarrow{PQ}=\overrightarrow{OQ}$$

But $\overrightarrow{OS}=\overrightarrow{PQ}=\mathbf{B}$ (as both the vectors are equal and parallel)

$$\therefore \qquad \overrightarrow{OP}+\overrightarrow{OS}=\overrightarrow{OQ}$$

Now $\overrightarrow{OP}$ and $\overrightarrow{OS}$, which represent the two vectors **A** and **B** respectively, constitute the two sides of the parallelogram meeting at O and $\overrightarrow{OQ}$ which gives the resultant vector **C**, is the diagonal of the parallelogram passing through the same point O, This method of finding the resultant of two vectors is known as **Parallelogram Law of Vectors** and may be stated as below :—

If a particle is simultaneously acted upon by two vectors represented both in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, the resultant is represented both in magnitude and direction by the diagonal of the parallelogram passing through the same point.

To calculate the *magnitude* of the resultant vector **C**, draw from Q a normal QM on OP produced.

Now $\angle SOP = \angle QPM = \angle \theta$

In rt. ∠d. △ OMQ,

$$OQ^2 = OM^2 + MQ^2$$

or $|\mathbf{C}|^2 = (OP + PM)^2 + MQ^2$ where $|\mathbf{C}|$ stands for the magnitude of the resultant vector.

Now in rt. ∠d △ PMQ,

$$\frac{PM}{PQ} = \cos\theta$$

$\therefore\ PM = PQ\cos\theta = |\mathbf{B}|\cos\theta$

Similarly $MQ = PQ\sin\theta = |\mathbf{B}|\sin\theta$.

Thus,

$$\begin{aligned}|\mathbf{C}|^2 &= [|\mathbf{A}| + |\mathbf{B}|\cos\theta]^2 + |\mathbf{B}|^2\sin^2\theta\\ &= |\mathbf{A}|^2 + |\mathbf{B}|^2\cos^2\theta + 2|\mathbf{A}|\,|\mathbf{B}|\cos\theta\\ &\qquad + |\mathbf{B}|^2\sin^2\theta\\ &= |\mathbf{A}|^2 + |\mathbf{B}|^2(\sin^2\theta + \cos^2\theta) + 2|\mathbf{A}|\,|\mathbf{B}|\cos\theta\\ &= |\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}|\,|\mathbf{B}|\cos\theta\end{aligned}$$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$$\boxed{\text{or } |\mathbf{C}| = \sqrt{|\mathbf{A}|^2 + |\mathbf{B}|^2 + 2|\mathbf{A}|\,|\mathbf{B}|\cos\theta}} \qquad \text{...(2-4)}$$

Thus the equation (2-4) gives the magnitude of resultant of two vectors inclined at an angle.

To determine the *direction* of the resultant vector **C**, let α be the angle which it makes with the direction of **A**.

$$\text{Now } \boxed{\tan\alpha = \frac{QM}{OM} = \frac{|\mathbf{B}|\sin\theta}{|\mathbf{A}| + |\mathbf{B}|\cos\theta}} \qquad \text{...(2-5)}$$

Thus knowing, $|\mathbf{A}|$, $|\mathbf{B}|$ and θ, the value of α can be determined from the tables.

Special cases :—

(*i*) If the two vectors **A** and **B** are acting along the same direction $\theta=0°$. Therefore, the resultant is given by

$$|\mathbf{C}|=\sqrt{|\mathbf{A}|^2+|\mathbf{B}|^2+2|\mathbf{A}|\;|\mathbf{B}|\cos 0°}$$

$$=\sqrt{|\mathbf{A}|^2+|\mathbf{B}|^2+2|\mathbf{A}|\;|\mathbf{B}|}\quad(\because \cos 0=1)$$

$$=\sqrt{(|\mathbf{A}|+|\mathbf{B}|)^2}$$

$$\text{or}\quad |\mathbf{C}|=|\mathbf{A}|+|\mathbf{B}| \qquad \text{...(2-6)}$$

(*ii*) If the two vectors are acting at right angles to each other, $\theta=90°$

$$\therefore\quad |\mathbf{C}|=\sqrt{|\mathbf{A}|^2+|\mathbf{B}|^2+2|\mathbf{A}|\;|\mathbf{B}|\cos 90°}$$

$$\text{or}\quad |\mathbf{C}|=\sqrt{|\mathbf{A}|^2+|\mathbf{B}|^2}\qquad(\because \cos 90°=0)\qquad \text{...(2-7)}$$

(*iii*) If the two vectors are acting along opposite direction $\theta=180°$

$$\therefore\quad |\mathbf{C}|=\sqrt{|\mathbf{A}|^2+|\mathbf{B}|^2+2|\mathbf{A}|\;|\mathbf{B}|\cos 180°}$$

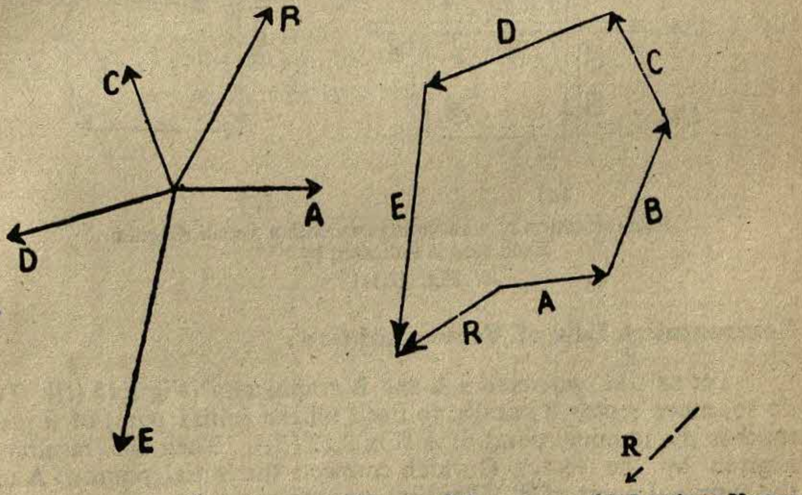
$$=\sqrt{|\mathbf{A}|^2+|\mathbf{B}|^2-2|\mathbf{A}|\;|\mathbf{B}|}\qquad(\because \cos 180°=-1)$$

$$=\sqrt{(|\mathbf{A}|\sim|\mathbf{B}|)^2}$$

$$\text{or}\quad |\mathbf{C}|=|\mathbf{A}|\sim|\mathbf{B}| \qquad \text{...(2-8)}$$

[*Note* :— $(|\mathbf{A}|\sim|\mathbf{B}|)$ means the positive difference between $|\mathbf{A}|$ and $|\mathbf{B}|$]

A similar procedure is used to add together more than two vectors as illustrated in Fig. (2.10). The vectors are joined



(a) Component Vectors (b) Polygon of Vectors (c) Resultant Vector

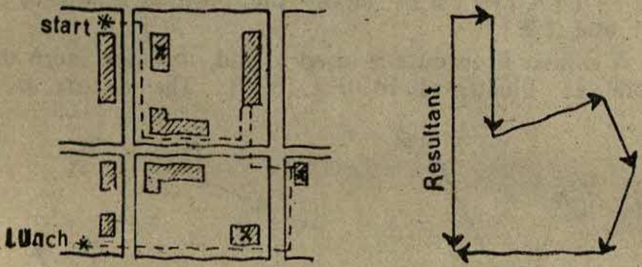
Fig. (2.10)

together, following one after the other with tail joined to head. *The resultant is then obtained by joining the tail of the first vector to the head of the last vector.* This diagram is sometimes called the **polygon of vectors**. Thus in Fig. (2.10) **A**, **B**, **C**, **D** and **E** represent a number of vectors, drawn along their given directions. The resultant, as explained above, is given by the vector **R** where

$$\boxed{\mathbf{R}=\mathbf{A}+\mathbf{B}+\mathbf{C}+\mathbf{D}+\mathbf{E}} \qquad \text{...(2-6)}$$

There is no reason why the vectors should all be in the same plane. It is easy to imagine a skew polygon with individual vectors jutting out in various directions in three dimensional space. Consider, for example, the case of a person on a shopping trip who travels from home to a variety of stores before stopping at a restaurant for lunch Fig. (2.11 *a*). If we wish to work out the overall displacement between home and the lunch stop, we represent each stage of the journey—between one store and the next—by a vector arrow and sum all the vectors by placing the tail of each arrow on the head of the preceding one [Fig. 2.11 (*b*)]

Notice that the arrow-heads on the component vectors are all pointing in the *same direction* around the polygon, whereas the arrow-head on the resultant vector points in the *opposite* direction.



(*a*) (*b*)

Representation of a shopping trip with a vector diagram
Each stop is indicated by "*"

Fig. (2.11)

Commutative Law of Vector Addition :

Let us add two vectors **A** and **B** graphically [Fig 2.12 (*i*)]. To do so, move vector **B** parallel to itself till the initial point of **B** just touches the terminal point of **A** [Fig 2.12 (*ii*)]. Then the resultant is given by the vector **C** which connects the initial point of **A** to the terminal point of **B**. Thus we may write

$$\mathbf{A}+\mathbf{B}=\mathbf{C}$$

It is further seen from Fig [2.12 (*ii*)] that the resultant vector is the same even if the orders in which the vectors are added is reversed

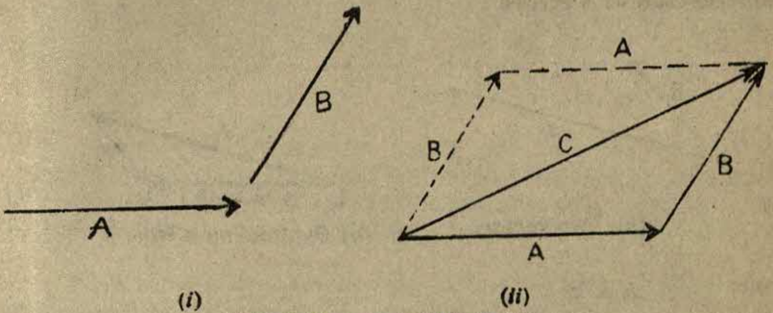


Fig. 2.12 Commutative Law

i.e., $\mathbf{B}+\mathbf{A}=\mathbf{C}$

$$\therefore \quad \boxed{\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}} \qquad \ldots(2\text{-}7)$$

Thus, in adding two vectors **A** and **B**, the order of addition makes no difference. This is called Commutative Law.

Associative Law of Vector Addition:

Let us add three vectors **A**, **B** and **C**. We can first add **A** and **B** to find their sum **D** *i.e.*, $\mathbf{D}=\mathbf{A}+\mathbf{B}$ which can then be added to **C** to find the resultant **R** [Fig 2.13]

$$\mathbf{R}=\mathbf{D}+\mathbf{C} \qquad \text{or} \qquad \mathbf{R}=(\mathbf{A}+\mathbf{B})+\mathbf{C}$$

It is further clear from the figure that the resultant is the same if **B** and **C** are first added to get **E** *i.e.*,

$\mathbf{E}=\mathbf{B}+\mathbf{C}$ which then is added to **A**.

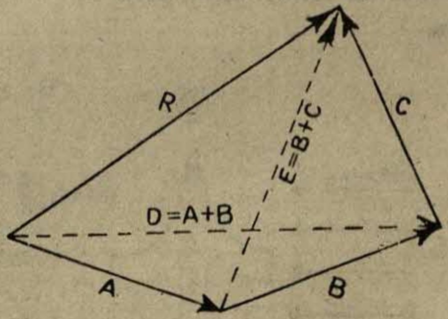
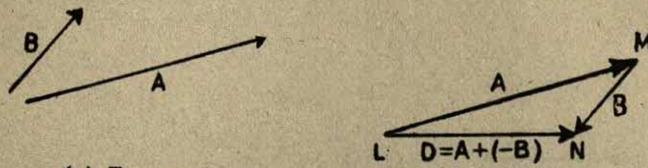


Fig. 2.13. Associative Law

$$\text{Thus,} \quad \boxed{(\mathbf{A}+\mathbf{B})+\mathbf{C}=\mathbf{A}+(\mathbf{B}+\mathbf{C})} \qquad \ldots(2\text{-}8)$$

Since the vectors can be grouped in any order, the vector addition is associative. This is called Associatve Law.

Subtraction of Vectors :



(a) Two vectors
A & B

(b) Subtracting B from A

Fig. 2.14 Subtraction of Vectors

Let **B** be a given vector which has to be subtracted from another vector **A** [Fig 2.14(*a*)]. The procedure is very simple. *Reverse* the direction of **B** and then *add* it to **A**, This is shown in Fig. [2.14 (*b*)] where LM represents **A** and NM represents—**B**. LN represents **D** the sum of **A** and **—B,** that is **A—B**. The reader can easily convince himself that, if

$$\boxed{\mathbf{A}-\mathbf{B}=\mathbf{D}} \qquad ...(2\text{-}9)$$

then

$$\boxed{\mathbf{A}=\mathbf{B}+\mathbf{D}} \qquad ...(2\text{-}10)$$

Example 2.1 : An aircraft is trying to fly due **north** at a speed of 100 m/sec but is subject to a cross-wind blowing from the West to the East at 50 m/sec. What is the actual velocity of the aircraft relative to the surface of the earth ?

Solution.

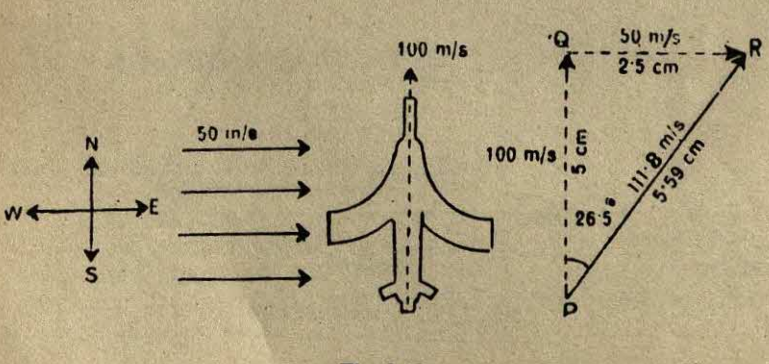


Fig. 2.15

The aircraft flies northward at 100 m/sec relative to the air, but the air is moving eastward at 50 m/sec, so the resultant velocity of the aircraft is the vector sum of there two velocities. In the vector diagram of Fig (2.15), let us assume that the scale is 1 cm= 20 m/sec. The northward velocity of the aircraft is therefore represented by a vector 5 cm long and the eastward velocity of the air by a vector 2.5 cm long. Drawing the triangle of vectors, PQR, the resultant $\overrightarrow{PR}$ is 5·59 cm long and the magnitude of the resultant velocity is therefore $5·59 \times 20 = 111·8$ m/sec. The angle QPR, which the resultant makes with the northerly direction. can be measured with a protactor and is found to be 26·5°.

Thus the actual velocity of the aircraft relative to the surface of the earth is 111·8 m/sec in a direction N 26° 30′ E.

Example 2·2 : A motor boat can move with a maximum speed of 10 m/sec, relative to the water. A river 400 m wide flowing at 5 m/sec must be crossed in the shortest possible time to reach a point on the other bank directly opposite the starting point. In which direction must the boat be pointed and how long will it take to cross ?

Solution.

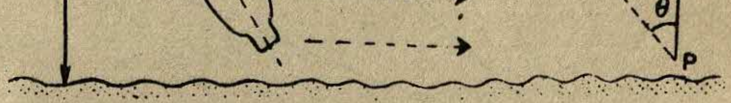


Fig. 2.16

If the boat were pointed directly at the opposite bank, then during the crossing it would drift downstream and it would not reach the other bank at the point directly opposite the starting point. It must, therefore, be pointed in a direction tilted in the *upstream* direction as shown in Fig (2.16). As illustrated in the vector diagram PQR, the result of adding the velocity of the boat relative to the water to the velocity of the water must be a resultant velocity **V** pointing directly toward the opposite bank. We cannot draw this triangle of vectors immediately because we do not know the angle θ between the direction of motion and the direction straight across the stream. However, inspecting the triangle PQR and remembering that in trigonometry the sine of the angle θ is defined as

$$\sin\theta = \frac{QR}{PQ} = \frac{5}{10} = 0{\cdot}5$$

We refer to tables of sines and find that the angle whose sine is 0·5 is 30°

The boat must, therefore, be pointed upstream at an angle of 30° from the direction perpendicular to the bank.

Applying Pythagoras' theorem to the triangle PQR

$$PQ^2 = QR^2 + PR^2$$

$$\text{or} \qquad PR^2 = PQ^2 - QR^2$$

$$\text{that is,} \qquad |\mathbf{V}|^2 = (10)^2 - (5)^2 = 75$$

$$\therefore \qquad |\mathbf{V}| = \sqrt{75} = 8.66 \ m/sec.$$

The boat, therefore, crosses the river at the speed of 8.66 m/sec. Since the distance across the river is 400 m, the time taken is $\frac{400}{8{\cdot}66}$ = *46·2 seconds.*

2.5 RESOLUTION OF A VECTOR

A single vector may be split up into two or more vectors along given directions which, when applied together, will produce

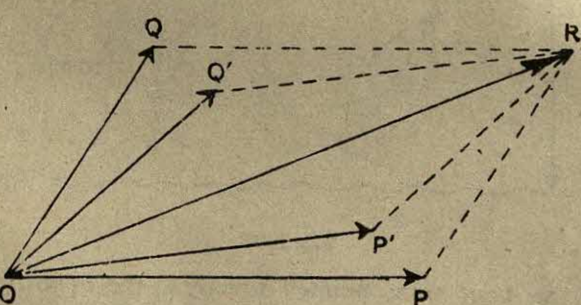


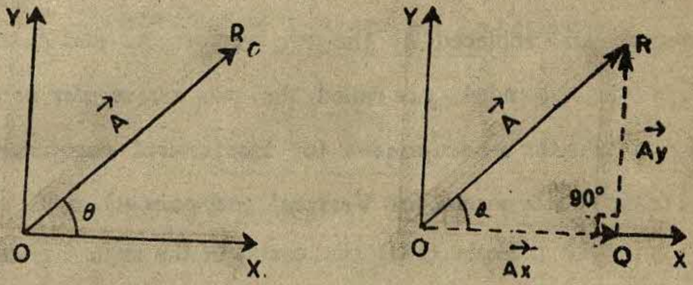
Fig. 2.17

the same effect as the single vector does. *This process of splitting a vector into two or more vectors is known as the* **resolution of a vector** *and the parts so obtained are known as the* **components of a vector**.

Whereas two or more vectors have got a single resultant, a given vector may be resolved into its components in an infinite number of ways, because the adjacent sides of any parallelogram, whose diagonal represents the given vector, give the possible components. Thus in figure (2.17), the given vector $\overrightarrow{OR}$ can be resolved into its components in several ways, such as $\overrightarrow{OP}$ and $\overrightarrow{PR}$. $\overrightarrow{OP'}$ and $\overrightarrow{P'R}$; $\overrightarrow{OQ}$ and $\overrightarrow{QR}$; $\overrightarrow{OQ'}$ and $\overrightarrow{Q'R}$ etc.

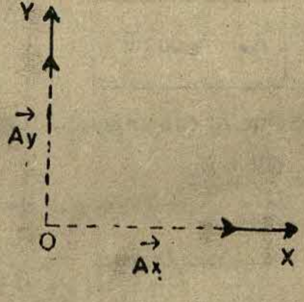
Rectangular components :

It is very convenient to resolve a given vector into two components at right angles to each other where one of the components makes a given angle with the direction of the given vector. Such components are called **rectangular components** or **resolved parts.**



(*a*)
The Original Vector

(*b*)
The Triangle of Vectors



(*c*)

The two rectangular components $\vec{A}_x$ and $\vec{A}_y$.
Fig. 2.18

Consider any vector $\vec{A}$, as in figure [2.18 (*a*)]. With the starting point of the vector as the origin O, draw any two axes OX

and OY at right angles to one another. Let the angle between OX and the direction of $\vec{A}$ be θ, which is determined by our initial choice of the direction OX. From the end R of the vector $\vec{A}$ drop **a perpendicular RQ to the axis OX (the angle RQO is then 90°). Now consider the triangle of vectors ORQ in figure [2.18 (*b*)].**

Clearly

$$\vec{OR}=\vec{OQ}+\vec{QR}$$

or

$$\boxed{\vec{A}=\vec{A_x}+\vec{A_y}} \qquad ..(2\text{-}11)$$

The original vector of figure [2.18 (*a*)] may therefore be removed and replaced by the two vectors $\vec{A_x}$ and $\vec{A_y}$ of figure [2.18 (*c*)]. $\vec{A_x}$ and $\vec{A_y}$ are called the two ***rectangular components*** of $\vec{A}$. $\vec{A_x}$ is the **x-component** (or **Horizontal component**) and $\vec{A_y}$ is the **y-component** (or **Vertical component**).

In the triangle ORQ the cosine of the angle θ is defined as

$$\cos\theta=\frac{OQ}{OR}=\frac{A_x}{A}$$

Therefore

$$\boxed{A_x=A\cos\theta} \qquad ...(2\text{-}12)$$

Similarly the sine of the angle θ is defined as

$$\sin\theta=\frac{QR}{OR}=\frac{A_y}{A}$$

Therefore,

$$\boxed{A_y=A\sin\theta} \qquad ...(2\text{-}13)$$

Given a magnitude A of the original vector A and the angle θ it makes with the axis OX, the rectangular components are therefore found by using the two formulas

$A_x=A\cos\theta$ and $A_y=A\sin\theta$

Since the component $\vec{A}_x$ making an angle θ with $\vec{A}$; $\vec{A}\,x$ is given by A cos θ, it follows that *the resolved part of a vector in a given direction is equal to the product of the given vector and cosine of the angle between the line of the vector and the given direction.*

Applying Pythagoras' theorem to triangle ORQ of figure [2.18 (*b*)],

$$OR^2=OQ^2+QR^2$$

or

$$A^2=A_x^2+A_y^2$$

$$\therefore \qquad \boxed{A=\sqrt{A_x^2+A_y^2}} \qquad \text{...(2-14)}$$

The definition of the tangent of the angle θ is

$$\boxed{\tan\theta=\frac{QR}{OQ}=\frac{A_y}{A_x}} \qquad \text{... (2-15)}$$

This may also be written

$$\boxed{\theta=\tan^{-1}\left(\frac{A_y}{A_x}\right)} \qquad \text{...(2-16)}$$

which means that θ is the angle whose tangent has the value $\frac{A_y}{A_x}$

Given the number $\frac{A_y}{A_x}$, tables are available to find the angle whose tangent is equal to this number.

If we are given two rectangular components $\vec{A}_x$ and $\vec{A}_y$ of the vector $\vec{A}$, the magnitude and direction of the vector itself may therefore be found using the two formulas

$$A=\sqrt{A_x^2+A_y^2}$$

and

$$\theta=\tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Example 2·3. An aircraft of Indian Airlines is climbing with a steady speed of 200 m/sec at an angle of 20° to the horizontal. What are the horizontal and vertical components of its velocity ?

Solution.

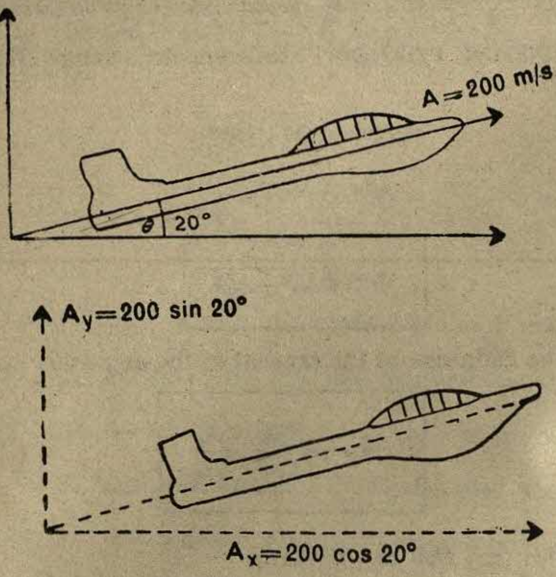


Fig. 2·19

Refer to Fig. 2·19

Horizontal component $= A_x = A \cos \theta = 200 \cos 20°$

Vertical component $= A_y = A \sin \theta = 200 \sin 20°$

Trigonometric tables tell us that

$\cos 20° = 0{\cdot}9397$ and $\sin 20° = 0{\cdot}3420$

$\therefore$ Horizontal component $= 200 \times 0{\cdot}9397$
$= 187{\cdot}94$ m/sec.

Vertical component $= 200 \times 0{\cdot}3420$
$= 68{\cdot}40$ m/sec

Notice that the sum of 187·94 and 68·40 is **not** *200, but you can check that* $(187{\cdot}94)^2 + (68{\cdot}4)^2 = (200)^2$. *This sort of extra check on the results of a calculation is a good policy ; since it helps to reveal any theoretical or arithmetical errors that might otherwise go undetected.*

Example 2.4. A boy can throw a ball horizontally with a speed of 20 m/sec. If he performs this feat in a car that is moving at 30 m/sec in a direction perpendicular to the direction in which he is throwing, what will be the actual speed and direction of motion of the ball ?

Solution.

If the resultant velocity is R m/sec at an angle θ to the direction in which the car is moving, then

$$R^2=(20)^2+(30)^2=1300$$

$$\text{or} \quad R=\sqrt{1300}=36{\cdot}06 \text{ m/sec.}$$

$$\text{Also, } \tan\theta=\frac{20}{30}=0{\cdot}666$$

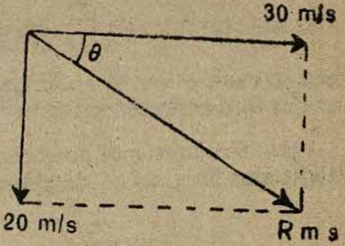


Fig. 2.20

From tables of tangents, θ=33·69°. Therefore, the ball has a speed of 36·06 m/sec in a direction at an angle of 33·69° to the direction in which the car is travelling.

2.6 SCALAR (DOT) PRODUCT OF TWO VECTORS :

The scalar or dot product of two vectors $\vec{A}$ and $\vec{B}$ is defined as the product of the magnitudes of the two vectors and the cosine of the angle between them. This can be written as

$$\boxed{\vec{A}\,.\,\vec{B}=|\mathbf{A}|\;|\mathbf{B}|\cos\theta} \qquad ...(2\text{-}17)$$

where $|\mathbf{A}|$ is the magnitude of $\vec{A}$,

$|\mathbf{B}|$ is the magnitude of $\vec{B}$,

and θ is the angle between two vectors measured from $\vec{A}$ to $\vec{B}$,

[**Note**—$\vec{A}.\vec{B}$ is read as $\vec{A}$ dot $\vec{B}$]

Since $|\mathbf{A}|$ and $|\mathbf{B}|$ are scalars and cos θ is a pure number, *the scalar product of two vectors is a scalar quantity.*

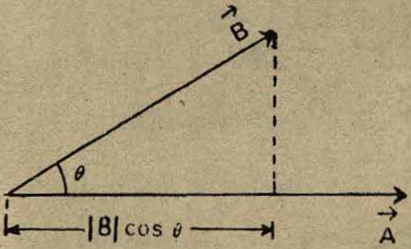


Fig 2.21

In Fig. (2·21), $|\mathbf{B}|\cos\theta$ is the component of $\vec{B}$ on $\vec{A}$. Hence scalar product of two vectors is also defined as the "*the product of the magnitude of one vector with the scalar component of the other vector in the direction of the first*"

As an example, let $\vec{F}$ represent force and $\vec{S}$ the displacement inclined to each other at an angle θ. Then $\vec{F}\ .\ \vec{S} = |\mathbf{F}|\ |\mathbf{S}| \cos\theta$ is a scalar and represents the *work done* by the force.

Again *Gravitational potential energy* (U) is the scalar product of weight and height *i.e.,* displacement.

$$U = \vec{W}\ \ \vec{h} = \vec{mg}\ .\ \vec{h}$$

Properties. The following properties easily follow from the above discussion on scalar product.

(*i*) When $\theta = 90°$, the dot product of twc vectors become zero.

If $\vec{A}$ and $\vec{B}$ are two vectors perpendicular to each other we have

$$\vec{A}\ \ \vec{B} = |\mathbf{A}|\ |\mathbf{B}| \cos 90° = 0$$

(*ii*) When $\theta = 0°$, the dot product is the same as the product of the magnitudes of the vectors.

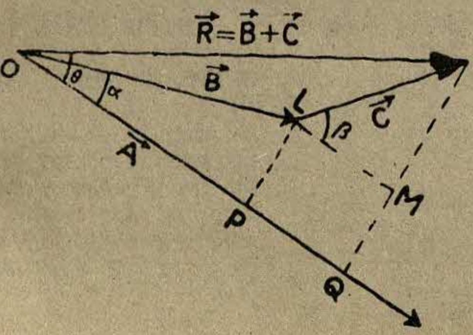
$$\vec{A}\ .\ \vec{A} = |\mathbf{A}|\ |\mathbf{A}| \cos 0° = |\mathbf{A}|^2$$
$$[\because \cos 0° = 1]$$

(*iii*) From the definition of dot product, it is obvious that, $\vec{A}\ .\ \vec{B} = \vec{B}\ .\ \vec{A}$, since $\cos(-\theta) = \cos\theta$ in both cases. The scalar product is thus **Commutative.**

(*iv*) Scalar product is **distributive** *i.e.,*

$$\vec{A}\ \ (\vec{B} + \vec{C}) = \vec{A}\ .\ \vec{B} + \vec{A}\ .\ \vec{C}.$$

To prove it let us refer to figure 2·22 involving three vectors



$\vec{A}$, $\vec{B}$ and $\vec{C}$. Let the angle between $\vec{A}$ and $\vec{B}$ be α and between $\vec{A}$ and $\vec{C}$ be β. Let $\vec{R}$, the resultant of $\vec{B}$ and $\vec{C}$, make an angle θ with $\vec{A}$.

$$\text{Now } \vec{A} \cdot (\vec{B}+\vec{C}) = \vec{A} \cdot \vec{R}$$
$$= |\mathbf{A}|\ |\mathbf{R}| \cos\theta$$
$$= |\mathbf{A}|\,(OQ)$$

where OQ is the component of $\vec{R}$ along the vector $\vec{A}$.

$$\text{Further, } \vec{A}\ \vec{B} = |\mathbf{A}|\ |\mathbf{B}| \cos\alpha$$
$$= |\mathbf{A}|\,(OP)$$

and
$$\vec{A} \cdot \vec{C} = |\mathbf{A}|\ |\mathbf{C}| \cos\beta$$
$$= |\mathbf{A}|\,(PQ) \qquad (\because\ PQ = LM)$$

Since $|\mathbf{A}|\,(OQ) = |\mathbf{A}|\,(OP) + |\mathbf{A}|\,(PQ)$, it follows that

$$\vec{A}\ (\vec{B}+\vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}.$$

2·7 VECTOR (CROSS) PRODUCT OF TWO VECTORS

The vector or cross product of two vectors $\vec{A}$ and $\vec{B}$ is a single vector C whose magnitude is equal to the product of the magnitudes of the given vectors multiplied by the sine of the smaller angle θ between them.

Thus the vector product of two vectors $\vec{A}$ and $\vec{B}$ is given by

$$\boxed{\vec{C} = \vec{A} \times \vec{B} = |\mathbf{A}|\ |\mathbf{B}| \sin\theta} \qquad \ldots(2\text{-}18)$$

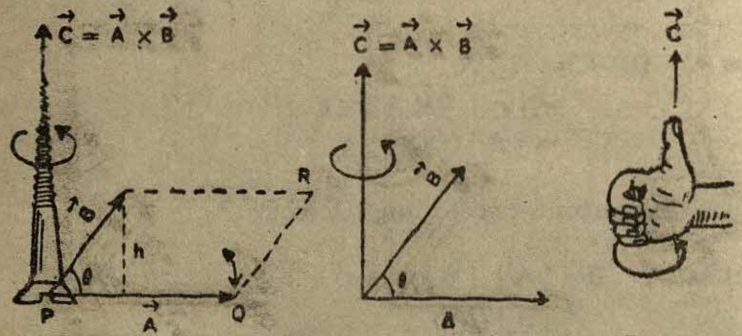
where $|\mathbf{A}|$ is the magnitude of $\vec{A}$

and $|\mathbf{B}|$ is the magnitude of $\vec{B}$.

The magnitude of the resulting vector $\vec{C}$ is given by

$$\boxed{|\mathbf{C}| = |\mathbf{A}|\ |\mathbf{B}| \sin\theta} \qquad \ldots(2\text{-}19)$$

The *direction* of $\vec{C}$ is perpendicular to the plane, containing $\vec{A}$ and $\vec{B}$ and is given by **right-hand Screw Rule.**



$\vec{C} = \vec{A} \times \vec{B}$
R
h
θ
$\vec{B}$
P
$\vec{A}$
Q
$\vec{C} = \vec{A} \times \vec{B}$
$\vec{B}$
θ
$\vec{A}$
$\vec{C}$

(*i*) Right hand screw rule (*ii*) (*iii*) Right hand rule

Fig. 2·23 Cross Product of two vectors.

Let us imagine the rotation of a right-handed screw whose axis is perpendicular to the plane containing the vectors $\vec{A}$ and $\vec{B}$ Fig. [2·23 (*i*)]. If we turn the screw from $\vec{A}$ to $\vec{B}$ *through the smaller angle θ between them*, then the direction along which the screw advances gives the direction of the resultant vector $\vec{C}$.

Another rule of fixing the direction of $\vec{C}$ is **right-hand rule.** *Let the right hand be held as shown in figure [2·23 (iii)] with the thumb erect and fingers folded round. Let the direction of thumb be perpendicular to the plane containing the vectors $\vec{A}$ and $\vec{B}$. If the direction of rotation of vector from $\vec{A}$ to $\vec{B}$ is the same as the direction of folding of fingers, then the thumb points towards the direction of the resultant vector $\vec{C}$ of vector product.*

As an example, **Torque** $\vec{\tau}$ is the vector product of position vector $\vec{r}$ and force vector $\vec{F}$.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Again, **Angular Momentum** $\vec{L}$ is the vector product of position vector $\vec{r}$ and linear momentum vector $\vec{p}$.

$$\vec{L} = \vec{r} \times \vec{p}$$

Also, **Linear Velocity** $\vec{v}$ is a vector product of angular velocity vector $\vec{w}$ and position vector $\vec{r}$.

$$\vec{v} = \vec{w} \times \vec{r}$$

Properties. The following properties easily follow :

(*i*) The vector product of two vectors is a vector.

(*ii*) If the two vectors are *at right angles*, their vector product is equal to the product of their magnitudes and in a direction perpendicular to both the vectors.

Thus if $\theta = 90°$

$$\vec{C} = |\mathbf{A}|\ |\mathbf{B}| \text{ as } \sin 90° = 1.$$

(*iii*) The vector product is **anti-commutative** *i.e.*,

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

But $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

The reason is that when right hand screw is rotated from $\vec{A}$ to $\vec{B}$, it advances in a direction opposite to that when it is rotated from $\vec{B}$ to $\vec{A}$ (Fig 2·24). The vector product changes the sign when the order of vectors in the product is reversed.

(*iv*) If the two vectors are *parallel*, their vector product vanishes

$$\vec{C} = \vec{A} \times \vec{B} = |\mathbf{A}|\,|\mathbf{B}| \text{ Sin } 0° = 0$$

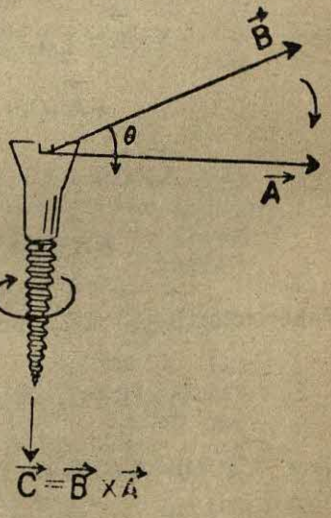


Fig. 2·24

(*v*) The vector product is **distributive** *i e.,*

$\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$ where $\vec{B}$ and $\vec{C}$

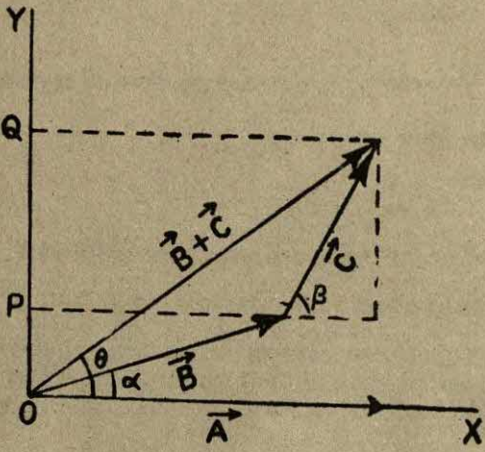


Fig. 2·25

are three vectors. ~~Let these~~ three vectors lie in the same plane. Let $\vec{B}$ and $\vec{C}$ be inclined to $\vec{A}$ at angles α and β respectively [Fig. 2·25].

Now $\vec{A}\times\vec{B}=|\mathbf{A}|\ |\mathbf{B}|\sin\alpha$

$=\vec{A}\,(OP)$

and $\vec{A}\times\vec{C}=|\mathbf{A}|\ |\mathbf{C}|\sin\beta$

$=\vec{A}\,(PQ)$

If the vector$(\vec{B}+\vec{C})$ makes an angle θ with $\vec{A}$, we have

$$\vec{A}\times(\vec{B}+\vec{C})=|\mathbf{A}|\ |(\mathbf{B}+\mathbf{C})|\sin\theta=\vec{A}\,(OQ)$$

Since $\vec{A}\,(OQ)=\vec{A}\,(OP)+\vec{A}\,(PQ)$, it follows that

$$\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$

(*vi*) We know that

$$|C|=|A|\ |B|\ \sin\theta$$

But $|B| \sin\theta = h$, the height of the parallelogram formed by the two vectors $\vec{A}$ and $\vec{B}$ as sides [Fig 2·23 (*i*)].

$$\therefore \quad |C| = |A|\, h = \text{area of the parallelogram.}$$

Thus the magnitude of a vector product is equal to the magnitude of the area of the ||gram formed by the two vectors as sides.

2·8 KINEMATICS

Kinematics is the study of motions considered abstractly without any reference to force or mass. In the present chapter motions in nature will be treated purely analytically and geometrically, no account being taken of their physical cause or of their relations with the forces producing them. In every day life, we generally come across *three* types of motion.

(*i*) Linear Motion (*ii*) Rotational Motion and (*iii*) Vibratory Motion.

For example a rolling wheel is an example of a body possessing both linear motion and rotational motion. A pendulum of a clock has a vibratory motion.

In this chapter, we shall consider the simplest case of a linear motion of a body, treating it as a particle.

2·9 SPEED AND VELOCITY

To understand the basics of motion or kinematics better, we must understand the concepts that relate displacement and distance to time in studies of motion. These concepts, one a scalar, the other a vector, are *speed* and *velocity*.

SPEED

The simpler of the two, because it is a scalar, is speed. **Speed is the ratio of the distance travelled by any object, irrespective of its direction, to the time it takes to travel that distance**. Speed involves both distance and time. Therefore its unit in SI system is *metre per second* (m/sec or ms^{-1})

Ordinarily, the speed of a body does not remain uniform over a certain time interval we are considering. For instance, a bus that carry passengers between Delhi and Agra, with a half dozen intermediate stops, gains speed when it starts from a station and loses speed when it is approaching a station. Again it slows down in motion while passing over the bridges etc. It also changes its direction quite often while proceeding along its journey. Thus the speed of the bus is not uniform but *variable*. When the speed of a body varies, then we should use the term *average speed* since we are determining the average value of the speed over the time interval we are considering. Thus the **average speed** *of a body* *between two points is measured by the total distance covered by the body between those points divided by the total time taken by the body to travel that distance.* Mathematically, this is expressed as

Average speed$=\frac{x}{t}$ where x is the distance covered by the body in time t.

At various points on the journey, however, the speedometer in the bus driver's cab may register a speed of anything say between zero and 60 km h^{-1}. The average speed say 40 km h^{-1}, is the speed of the bus over the whole journey, taking into account all delays and speed ups on the way, while the reading on the speedometer is the **instantaneous** speed at the time the driver glances at the instrument's dial. The relation between average and instantaneous speed is best depicted diagrammatically in Fig. [2·26]

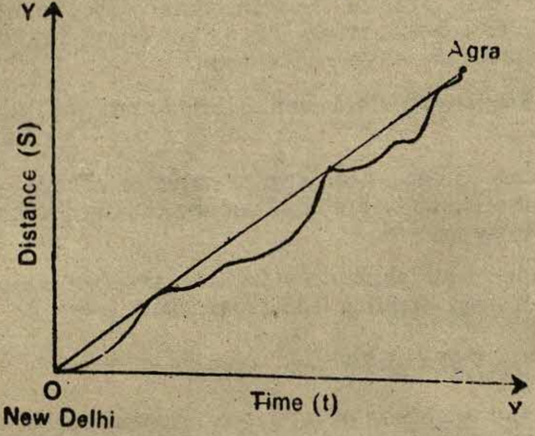


Fig. 2·26

VELOCITY

Velocity, the vector, is related to speed in the same way that displacement is related to distance. It is the rate of displacement *i.e.*, **it is the ratio of the displacement—which takes account of direction as well distance—to time interval.** Thus, while a bus driver looking at his speedometer obtains only a measure of his speed alone, an airplane navigator referring to his instrument calculates his vehicle's actual velocity—a necessity because winds can alter the direction in which planes travel.

The velocity of a body like speed is also expressed in **metres per second (ms^{-1})** or **kilometre per sec** or **kilometre per hour (km h^{-1})**. Its dimensions are $M^0L^1T^{-1}$.

When the velocity of a body is constant, the velocity is very simply defined as the distance travelled divided by time taken; but when the velocity changes with time (*i.e.*, either its speed changes or direction of motion changes or both change), then a more careful definition is required.

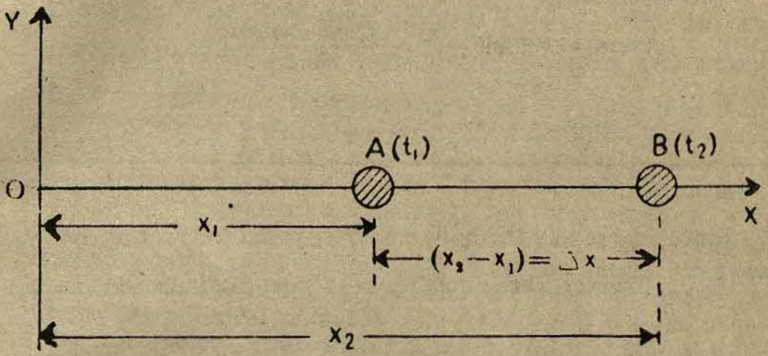


Fig. 2 27

Let us consider a particle to the moving along positive x-direction as shown in Fig. (2.27). Under this circumstance the instantaneous position of the particle is uniquely determined by its distance from the origin o, that is, its x coordinate. At a time t_1 let the particle be at a point A and at a distance x_1 from the origin. Let the particle reach point B at position x_2 at the time t_2. Here we see that the distance travelled by the particle depends on the time. Mathematically we say that the distance travelled by the particle is a function of time interval.

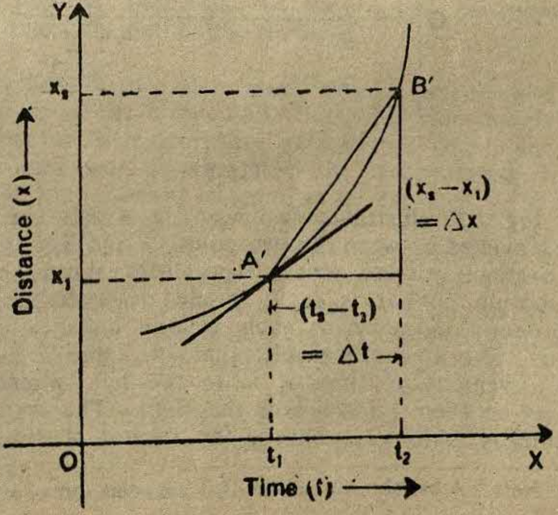


Fig 2.28

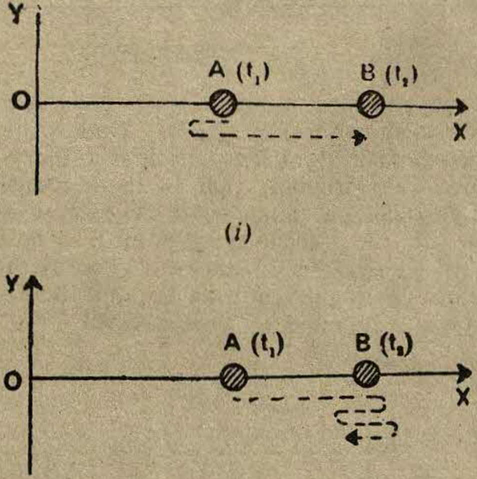
If the distance travelled by the particle in the above case is plotted on a graph as a function of time we get the graph as shown in Fig. (2.28) points A′ and B′ correspond to points A and B. From the graph, it is clear that, however, complicated the motion during this time interval, the *net* distance travelled is (x_2-x_1) and the time taken is (t_2-t_1). The ratio of the displacement of the particle (x_2-x_1) to the time interval (t_2-t_1) gives the **average velocity** between A and B.

$$\text{Average velocity} = \bar{V} = \frac{\text{Displacement}}{\text{Time}}$$

$$= \frac{(x_2-x_1)}{(t_2-t_1)}$$

or $\bar{V} = \frac{\Delta x}{\Delta t}$ where Δx is a very small distance covered by the body in a very small interval of time Δt.

However, the simplicity of this definition can be deceptive.



(*i*)

(*ii*)

Fig. 2.29

Fig. 2·29 illustrates two ways in which the particle might have travelled between the two points A and B, and in both cases the average velocity reveals very little about the true nature of the motion. What is really needed is a definition of the **instantaneous velocity** at each definite instant of time during the motion. Notice for example, that in figure [2·29 (*i*)] the instantaneous velocity at time t_1 is to the left, whereas the average velocity between t_1 and t_2 is to the right. The way to avoid this difficulty is obvious *i.e.*, choose the instant of time t_2 sufficiently

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Note. A bar above V implies that it is some sort of average.

close to t_1 so that the motion has not had time to reverse itself during the interval (t_2-t_1), and then, at least, the body will always be travelling in the same direction. Even so the magnitude of the velocity might change so much between t_1 & t_2 that the average velocity would bear no direct relationship to the instantaneous velocity at time t_1. Suppose, for example, that the particle starts out very slowly and covers only $\frac{1}{100}$ of the distance (x_2-x_1) during the first half of time interval (t_2-t_1), but then suddenly accelerates and covers the remaining $\frac{99}{100}$ during the second half of the time interval. After thinking about such examples, it becomes clear that the only way to avoid such difficulties is to make the time interval $(t_2-t_1)=\Delta t$ *so short* that the velocity does not have time to change between t_1 and t_2. Since we cannot. in general, know how rapidly the velocity is changing, the only thing to do is to make $(t_2-t_1)=\Delta t$ very small indeed. How small ? The mathematical answer is that although Δt must be larger than zero, it should be smaller than any number you choose to quote, *however small.* The practical answer is that Δt must be as small as possible, subject to the conditions that the small distance $(x_2-x_1)=\Delta x$ and the small time interval Δt can be measured with sufficient accuracy for the purpose in hand, bearing in mind that as quantity becomes very small, it becomes increasing more difficult to measure it.

This complicated discussion has been given to emphasize *two* points. Theoretically, the instantaneous velocity of a body is found by studying its motion over a very small time interval—the smaller the better. In practice, however, the motion has to be measured with instruments of limited accuracy and this automatically places a limit on how small the time interval can be. We must fully realize that our knowledge of the universe is limited by the crudity of our methods of observation, and, as these methods are improved and their accuracy increases, we should not be surprised to find that new and finer details are revealed.

The thoughtful student might object that in fact there is a '*true*' velocity of the body and that we can *imagine* a perfect measurement with instruments refined to the point of perfect accuracy, and we can then use the mathematical approach of making the time interval as small as is necessary to reveal all the fine details. This eminently reasonable argument is refuted by modern physics. One of the fundamental features of quantum mechanics is that it will not allow us to decrease the size of the time interval without limit. If we attempt to do this, however perfect our measuring instruments, there comes a point where an accurate measurement of velocity becomes impossible. Fortunately, in the case of an every day moving object such as a cricket ball this point is not reached until the time interval has the incredibly small value of say 10^{-35} seconds. So we are not aware of this in every day experience.

At this stage let us be satisfied with the well behaved nature of cricket balls and the definition of instantaneous velocity is as follows :

The instantaneous velocity of a body is the actual value of the velocity of the body at any point of its path or at any instant of time. It is obtained by dividing the distance it travels by the time taken, with the time interval as short as it possibly can be without exceeding the limits of accuracy of the measuring equipment or runnning into difficulties raised by Quantum mechanics.

Let us find the instantaneous velocity of the particle (Fig. 2·27 and 2·28) at the point A. The average velocity between A and B is associated with the entire displacement $\triangle x$ and with the entire time interval $\triangle t$. Move the point B (Fig 2·28) closer and closer to the point A' and calculate the average velocity for shorter and shorter time intervals. When the time $\triangle t$ is very small such that $\triangle t \to 0$, then points A' and B' are very close to each other and the line A'B' nearly coincides with the tangent line at the point A' (Fig 2·28). Then the velocity calculated by the ratio $\frac{\triangle x}{\triangle t}$ in the case when $\triangle t \to 0$ gives the instantaneous velocity at the given instant and is written as $\frac{dx}{dt}$. Here $\frac{dx}{dt}$ is called the derivative of x with respect to t.

Mathematically we can express this statement as

$$\boxed{\text{Instantaneous Velocity} = V = \underset{\triangle t \to 0}{\text{Lt.}} \left(\frac{\triangle x}{\triangle t} \right) = \frac{dx}{dt}} \quad \ldots \text{(2-20)}$$

In the (Figure 2·27) as the point B approaches point A, point B' approaches point A' in (Figure 2·28). In the case when $\triangle t \to 0$ (which is also known as limit of $\triangle t$) the slope of chord A'B' equals the slope of the tangent to the curve at point A'. *The instantaneous velocity at any point of $x-t$ graph is equal to the slope of the tangent to the graph at that point.*

The velocity of a particle is said to be **uniform** *if the displacement of the particle in equal intervals of time are equal,* **however small** *these intervals may be.*

Suppose, a particle is moving with a uniform velocity of 30 km. h^{-1} in a particular direction, say, towards South. It implies that the direction of motion of the particle is the same throughout, namely, towards the South and that in every half-hour, it moves 15 km ; in every one minute, half km ; in every one second, $\frac{1}{120}$ th of a km and in every one hundredth of a second, $\frac{1}{1,20,000}$ of a km and so on. Even if we consider *very short possible* equal intervals of time, the distances travelled in those intervals will be equal. This is what we mean by uniform velocity and this is the significance of the words "however small the intervals may be."

Example 2·5. On February 20, 1962, American astronaut John Glenn orbited the earth three times, a distance of approximately 120 000 kilometers in 4 hours and 56 minutes. Compute the average speed and the average velocity of his mercury capsule in km/h.

Solution. The average speed V is given by the total distance covered divided by the time required for the journey.

$$\overline{V} = \frac{x}{t} = \frac{120\,000}{4\cdot93} = 24\cdot3\times10^3 \text{ km/hr}$$

Note that while Glenn's *velocity* was changing each instant during his trip, since his direction of motion was changing continuously, his *speed* was essentially constant.

Now the *average velocity* over the three complete trips was actually *zero*, since Glenn's total displacement was zero. He went around in a circle returning to his starting point

2·10 ACCELERATION

Acceleration is a familar word to almost everyone, and in physics it is an extremely important quantity. We must, therefore, define it carefully in an exact manner. When we "take off" in our car from a stop light at the instant the light turns green, we say that we *accelerate*. We have changed our speed in this case in a positive manner ; increasing it with time. When we put on our brakes in a car to stop the machine we "*decelerate*", a special type of acceleration referring to a decrease in speed with time. **This rate of change of velocity with time is known as acceleration**. Whereas velocity is the ratio of the displacement to the time taken for the journey, **acceleration is the ratio of the change in velocity to the time over which this change occurs.**

If $\triangle V$ is the change in velocity of a particle in a short time interval $\triangle t$, then the acceleration 'a' is given by

$$a = \frac{\triangle V}{\triangle t} \quad \text{where } a \text{ is a vector since } \triangle V \text{ is a vector.}$$

Acceleration is generally measured in **metre per second**^{2} (**ms**$^{-2}$). It can be both *positive* in magnitude, representing an increase in speed, or *negative* in magnitude, representing a decrease in the speed, commonly called **deceleration** or **retardation**. The dimensions of acceleration are $M^0L^1T^{-2}$.

Because velocity has both magnitude and direction, any vehicle or other object can accelerate in either of two ways. The magnitude of the velocity, the speed, can change, or the direction in which the object or vehicle is moving can change, or both can change simultaneously. Clearly, an object that speeds up or slows down even if it still maintains its original direction is undergoing acceleration. However a model electric train moving around a circular track at constant speed is also accelerating because the direction in which it moves is constantly changing.

For a more general motion, such as that of a car on the Grand Trunk Road or a train on its tracks, both speed and the direction of motion are changing constantly. To simplify our discussion in this section, we shall limit our study only to motion in one dimension *i.e.* along a straight line (rectilinear motion). This will enable us to consider accelerations as changes only in speed.

Like speed and velocity, acceleration can be measured in both *average* and *instantaneous* terms.

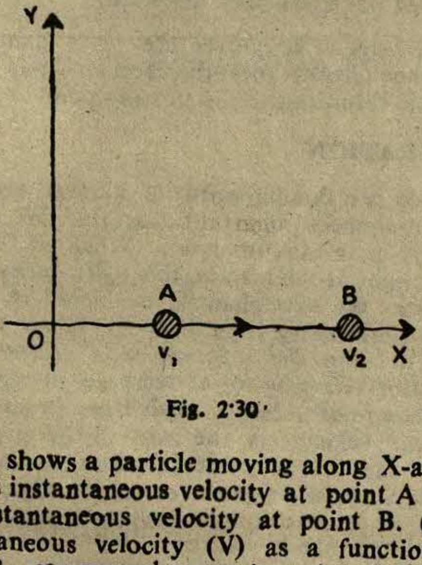


Fig. 2·30

(Fig. 2·30) shows a particle moving along X-axis. The vector V_1 represents its instantaneous velocity at point A and vector V_2 represents its instantaneous velocity at point B. (Fig. 2·31) is a graph of instantaneous velocity (V) as a function of time (*t*). Points A′ and B corresponds to points A and B in (Figure 2·30). The **average acceleration** *of the particle as it moves from A to B is defined as the ratio of the change in instantaneous velocity to the elapsed time.*

$$\text{Average acceleration} = \frac{\text{Change in instantaneous velocity}}{\text{Time taken}}$$

or

$$a = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta V}{\Delta t}$$

The **instantaneous acceleration** of a body *i.e.* the acceleration at a given instant of time, is defined in the same way as instantaneous velocity. The instantaneous acceleration at the point A is defined as the limiting value of the average acceleration when point B is taken very close to A. This means if Δt is taken exceedingly small, ΔV also becomes proportionately small. If $\Delta t \to 0$ as a limit, the average acceleration $\frac{\Delta V}{\Delta t}$ becomes the instantaneous acceleration at the given instant. Thus

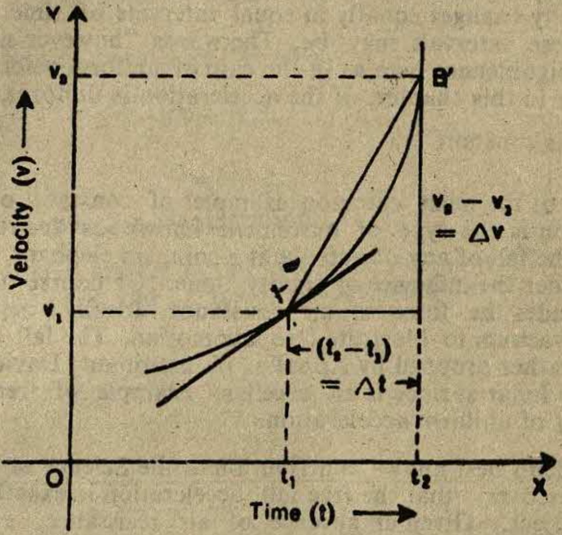


Fig. 2·31

$$\boxed{\text{Instantaneous acceleration } (a) = \underset{\Delta t \to 0}{\text{Lt.}} \left(\frac{\Delta V}{\Delta t} \right) = \frac{dV}{dt}} \quad \text{...(2-21)}$$

$$\text{Also } a = \frac{d}{dt}\left(\frac{dx}{dt} \right) = \frac{d^2x}{dt^2} \qquad \left(\because \ V = \frac{dx}{dt} \right)$$

or

$$\boxed{a = \frac{d^2x}{dt^2}} \quad \text{... (2-22)}$$

Here $\frac{d^2x}{dt^2}$ is called second derivative of x with respect to t.

Also the instantaneous acceleration at any point on V$-t$ graph is equal to the slope of the tangent to the graph at that point.

A body is said to be moving with **uniform acceleration** if its velocity changes equally in equal intervals of time, "however small" these intervals may be. The words "however small" have the same significance here as in the case of uniform velocity explained earlier in this chapter. If the acceleration is uniform, it means that $\frac{dV}{dt}$ is constant.

One of the most common examples of constant or uniform acceleration is the type of movement known as **free fall**. This refers to the fall of any object—be it a coin, an elephant, or an airplane—under the influence of gravity alone. Of course this condition precludes the force of air resistance. Ideally, therefore, we require a vacuum to illustrate free fall motion. The fall of hammer and the feather dropped by Appollo 15 astronaut David Scott on the airless lunar surface is an excellent example of free fall and hence that of uniform acceleration.

Galileo's best known contribution to the Science of Kinematics was his discovery that the free fall acceleration is exactly the same for any object. Given an absence of air resistance, each object will fall to earth with exactly the same acceleration. Modern measurements yield a value for this free fall acceleration of $9{\cdot}8\ ms^{-2}$. The universal free fall acceleration, that is, acceleration caused by the earth's gravity, is identified by the symbol 'g'.

2.11 EQUATIONS OF MOTION

When a body is having rectilinear motion, the relations between its velocity, acceleration, distance covered and the time are represented by simple equations known as equations of Motion. These equations can be derived as follows :—

(1) Distance covered by a body moving with uniform velocity

As explained earlier in this chapter, if a body moves with a uniform velocity V for a time t, the distance covered (S) by the body is given by

$$\boxed{S = Vt} \qquad (2\text{-}23)$$

(2) Velocity of a uniformly accelerated body after a certain time

Let a body be in linear motion with constant or uniform acceleration a. As the acceleration (a) of a body at any instant is given by the derivative of velocity (V) with respect to time (t), we have

$$a=\frac{dV}{dt} \text{ or } dV=adt.$$

Integrating both sides, we have

$$\int dV=\int adt.$$

or $$V=at+c$$

where c is a constant of integration.

If u and V are the initial and final velocities of the body at the beginning and end of time interval t, we have

$$V=u, \text{ when } t=0$$

∴ $$u=c$$

Hence $$\boxed{V=u+at} \qquad ...(2\text{-}24)$$

(3) **Distance covered by a uniformly accelerated body in a given time**

Let a body moving with an initial velocity u and uniform acceleration a covers a distance S in time t. Since the instantaneous velocity V of a body is given by the derivative of displacement with respect to time, therefore,

$$V=\frac{dS}{dt}$$

or $$dS=Vdt=(u+at)\,dt$$

or $$dS=udt+at\,dt$$

Integrating both sides, we get

$$\int dS=\int udt+\int at\,dt$$

$$S=ut+\tfrac{1}{2}at^2+C$$

where C is constant of integration.

Now, when $t=0$; $S=0$

∴ $$C=0$$

Hence $$\boxed{S=ut+\tfrac{1}{2}at^2} \qquad ...(2\text{-}25)$$

(4) Velocity of a uniformly accelerated body after covering a certain distance

Let a body move with an initial velocity u and attain a final velocity v after time t. Let it cover a distance S with uniform acceleration a.

$$\because \quad a=\frac{dV}{dt} \text{ (Instantaneous acceleration).}$$

$$\therefore \quad a=\frac{dV}{dS}\;\frac{dS}{dt}=\frac{dV}{dS}.V \qquad \left(\because \; V=\frac{dS}{dt}\right)$$

or
$$dS=\frac{dV.V}{a}=\frac{VdV}{a}$$

Integrating both sides, we have

$$\int ds=\int\frac{VdV}{a}$$

$$S=\frac{V^2}{2a}+C$$ where C is constant of integration.

Now, when $S=0$; $V=u$ and when $S=S$, $V=v$

$$\therefore \quad C=-\frac{u^2}{2a}$$

Hence
$$S=\frac{v^2}{2a}-\frac{u^2}{2a}=\frac{v^2-u^2}{2a}$$

or
$$\boxed{2aS=v^2-u^2} \qquad \ldots(2\text{-}26)$$

(5) Distance covered by the body in the *nth* second

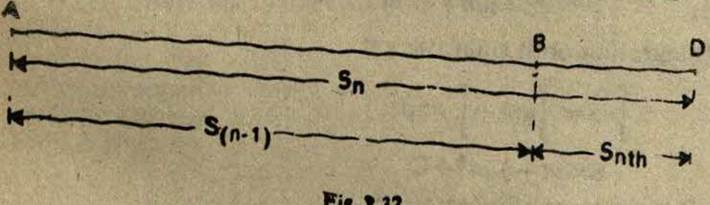


Fig 2.32

Let a body move with an initial velocity u and uniform acceleration a. Let it cover AD ($=S_n$) distance [Fig. 2.32] in n seconds and AB ($=S_{(n-1)}$) distance in $(n-1)$ seconds respectively. Then the distance BD ($=S_{nth}$) is called the distance covered by the body in the *nth* second of its motion.

From the relation $S=ut+\frac{1}{2}at^2$, the distances covered in n and $(n-1)$ seconds are given by

$$S_n=un+\tfrac{1}{2}\,an^2 \qquad [\because t=n]$$

and

$$S_{(n-1)}=u\,(n-1)+\tfrac{1}{2}a\,(n-1)^2 \qquad [\because t=(n-1)]$$

Since $S_{nth}=S_n-S_{(n-1)}$

$$\therefore \quad S_{nth}=[un+\tfrac{1}{2}an^2]-[u(n-1)+\tfrac{1}{2}a(n-1)^2]$$

$$=(un+\tfrac{1}{2}an^2]-[un-u+\tfrac{1}{2}an^2-an+\tfrac{1}{2}a]$$

$$=u+an-\tfrac{1}{2}a$$

or

$$\boxed{S_{nth}=u+\tfrac{1}{2}a(2n-1)} \qquad ...(2\text{-}27)$$

If a body has a *free fall* or moves with an acceleration due to gravity then $\boxed{a=g}$ and hence the important equations of motions can now be presented as-

$$\boxed{V=u+gt} \qquad ...(2\text{-}28)$$

$$\boxed{S=ut+\tfrac{1}{2}gt^2} \qquad ...(2\text{-}29)$$

$$\boxed{2gs=v^2-u^2} \qquad ...(2\text{-}30)$$

$$\boxed{S_{nth}=u+\tfrac{1}{2}g(2n-1)} \qquad ...(2\text{-}31)$$

Remember that these equations are meaningless unless the exact significance of each symbol is understood. Also they are applicable only if the acceleration is constant.

In any particular problem, the student will have to decide which of these equations to use. To do this he should notice that each equation contains only four different kinds of symbol. Three quantities will be given in the problem and a fourth quantity will have to be calculated. He should therefore choose the equation that involves the three known quantities plus the quantity to be calculated, and nothing else.

Example 2·6. A man standing on the roof of a building 30m high throws a ball vertically downward with an initial velocity of 500 cm $\sec^{-1}$ as it leaves his hand. The acceleration due to gravity is 9·8 ms^{-2}. (*i*) What is the velocity of the ball after it has been falling for 0·5 sec ? (*ii*) Where is the ball after 1·5 sec. ? (*iii*) What is the velocity of the ball as it strikes the ground ?

Solution

Place the origin at the top of the building. Then S=0 when $t=0$. Let the positive direction of S be downward. The initial velocity is downward and therefore positive, so

$$u=500 \text{ cm sec}^{-1}=5 \text{ ms}^{-1}$$

The acceleration is downward and therefore positive, so

$$a=+9{\cdot}8 \text{ ms}^{-2}$$

(*i*) In this part of the problem one is given *u*, *a* and *t*, and must deduce *v*. Therefore,

$$v=u+at$$

$$=5+(9{\cdot}8)\times(0{\cdot}5)=+9{\cdot}9 \text{ ms}^{-1}$$

After 0·5 sec the velocity is 9·9 ms^{-1} downward.

(*ii*) In this part of the problem one is given *u*, *a* and *t* and must calculate S. Therefore,

$$S=ut+\tfrac{1}{2}at^2$$

$$=5\times1{\cdot}5+\tfrac{1}{2}\times9{\cdot}8\times(1{\cdot}5)^2=18{\cdot}525 \text{ m}$$

After 1·5 sec the ball is 18·525 m below the roof or 11·475 m above the ground.

(*iii*) When the ball strikes the ground $x=+30$m. So one is given *u*, *a* and S, and asked to calculate *v*. Therefore,

$$v^2-u^2=2aS$$

or

$$v^2=u^2+2aS$$

$$=(5)^2+2\times9{\cdot}8\times30=613$$

$$\therefore \quad v=\sqrt{613}=24{\cdot}76 \text{ ms}^{-1}$$

When it strikes the ground, the ball has a velocity of 24·76 ms^{-1}.

Example 2·7. Repeat the previous problem for the case when the ball is thrown *upward* with a velocity of 5 ms^{-1}. Also (*iv*) What is the maximum height reached by the ball ?

Solution

The only difference is that the initial velocity is in the *negative* direction and so $u=-5 \text{ ms}^{-1}$. Otherwise the arguments for parts (*i*), (*ii*) and (*iii*) are identical.

(*i*) $v=u+at$

$=-5+9{\cdot}8\times0{\cdot}5=-0{\cdot}1\ \text{ms}^{-1}$

After 0·5 sec the body is moving *upward* with a velocity of 0.1 ms^{-1}.

(*ii*) $S=ut+\frac{1}{2}\,at^2$

$=(-5)\times1{\cdot}5+\frac{1}{2}\times9{\cdot}8\times(1{\cdot}5)^2=+3{\cdot}525\ \text{m}$

After 1·5 sec the ball is 3·525 m *below* the roof.

(*iii*) $v^2=u^2+2aS$

$=(-5)^2+2\times9{\cdot}8\times30=613$

or $v=\sqrt{613}=24{\cdot}76\ \text{ms}^{-1}$

When it strikes the ground, the ball has a velocity of 24·76 ms^{-1}, exactly the same as in the previous problem.

(*iv*) The ball starts to move upward but the downward acceleration steadily reduces the magnitude of its velocity. When the ball reaches its highest point, the velocity has been reduced to zero and subsequently the velocity is reversed and the ball begins to move downward. In the highest position, therefore, it is known that $v=0$. The known quantities are therefore u, a and v and the desired quantity is S. Thus

$v^2-u^2=2aS$

or $v^2=u^2+2aS$

or $0=(-5)^2+2\times9{\cdot}8\times S$

or $19.6S=-25$

$S=-1{\cdot}277\ \text{m}$

The negative sign indicates that the highest point is *above* the roof where the S coordinates are negative. The maximum height reached by the ball is therefore 1·277 m above the roof or 31·277 m above the ground.

Note. We can also solve this problem with the origin still at the roof, but the positive direction of S *upward*. Then the initial velocity is upward and positive ($u=+5\ \text{ms}^{-1}$) but the acceleration is downward and therefore negative ($a=-9{\cdot}8\ \text{ms}^{-2}$).

Example 2·8. A motorist rushing at a speed of 20 ms^{-1} finds a child on the road 201 m ahead of him. He immediately applies brakes so as to stop within 1 m of the child (supposed stationary). Find the deceleration produced and also the time taken to stop the car.

Solution

Here $u=20\ \text{ms}^{-1}$ $S=(201-1)=200\ \text{m}$

$v=0$ $a=?$ $t=?$

Since $v^2-u^2=2aS$

$\therefore$ Retardation $=a=\dfrac{v^2-u^2}{2S}=\dfrac{0-(20)^2}{2\times 200}=-1\ \mathbf{ms^{-2}}$

Again $v=u+at$

$\therefore$ Time $=t=\dfrac{v-u}{a}$

$$=\frac{0-20}{-1}=\mathbf{20\ sec.}$$

Example 2·9. A body travels 2 m in the first 2 seconds and 2·20 m in the next 4 seconds. What will be the velocity at the end of the 7th second from the start ?

Solution

Let u be the initial velocity of a body and a be its uniform acceleration.

$$S=ut+\tfrac{1}{2}at^2$$

Distance travelled in the first 2 seconds is 2 m.

$\therefore$ $2=u\times 2+\frac{1}{2}a\times(2)^2=2u+2a$

or $1=u+a$ ...(*i*)

Distance travelled in the next 4 seconds is 2·20 m. Therefore distance travelled in the first 6 seconds is 4·20 m

$\therefore$ $4{\cdot}20=u\times 6+\frac{1}{2}a(6)^2$

or $4{\cdot}20=6u+18a$

or $0{\cdot}7=u+3a$ ...(*ii*)

Subtracting (*ii*), from (*i*)

$$0{\cdot}3=-2a$$

$$a=\frac{-0{\cdot}3}{2}=-0{\cdot}15\ \text{ms}^{-2} \quad \text{...(iii)}$$

Putting (*iii*) in (*i*), we have

$$u=1{\cdot}15\ \text{ms}^{-1}$$

Since $v=u+at$

$\therefore$ Velocity at the end of 7th second is

$$v=1{\cdot}15+(-0{\cdot}15)\times 7$$

$$=1{\cdot}15-1{\cdot}05=\mathbf{0{\cdot}10\ ms^{-1}}$$

2·12 NEWTON'S FIRST LAW OF MOTION

Sir Isaac Newton (1642-1727) studied and developed Galileo's (1564-1642) ideas about motion and subsequently stated the three laws which now bear his name. He established the subject of dynamics. His laws are a set of statements and definitions that help to explain very important principles of Physics. Observations and experiments furnish evidence of their truth as applied to the motion of both terrestrial and celestial bodies. We believe these laws to be true because the results they predict are found to be in very exact agreement with experiment over a wide range of conditions. We do not regard them as absolutely true and more exact laws are required for certain extreme cases.

Newton's First law. *Every body continues in its state of rest or of uniform motion in a straight line unless and until it is compelled by some external force to change that state of rest or of uniform motion.*

The first law of motion thus enunciated by Newton consists of two parts. The first part, which states that every body continues in its state of *rest* unless and until it is compelled by an external force to change this state of rest, is self evident and presents no difficulty whatsoever. For example, a book kept on the table continues to be at the same place until and unless some one removes it.

The second part, which states that every body continues in *its state of uniform motion in a straight line* until and unless it is compelled by an external force to change that state does not admit of easy verification by experiment in our daily life as it is *impossible* to imagine a body moving with uniform velocity in a straight line perfectly free from *external forces like resistance of air, friction etc.* Thus, even a very smooth ball rolled on the flat and smooth ground comes to rest owing to the resistance of the air and friction of the ground. The nearest approach to *free motion* (motion free from external force) is the motion of a glass or ivory ball on smooth ice in vacuum. The evidence in support of the second part of the First Law of Motion is contained in the astronomical predictions regarding the movement of the various planets in all most perfect vacuum. The positions of the Moon, Sun and the Planets, Mercury, Venus, Earth Mars, Jupiter, Saturn Uranus, Neptune and Pluto which do not encounter any air resistance are fore-told to a wonderful degree of accuracy. The time and duration of Solar and Lunar eclipses are likewise correctly predicted. All these predictions have come out true and demonstrate the truth contained in the second part of Newton's First Law. It seems that the question to be asked about a moving body is not 'what keeps it moving' but 'what changes or stops its motion'. Frequently it is friction and if a body does move under near frictionless conditions, its velocity is infact almost uniform. This is shown in figure (2·33) which is a photograph of a puck, illuminated at equal time intervals by a flashing Xenon lamp, moving on a cushion of carbon dioxide gas across a clean, level glass plate.

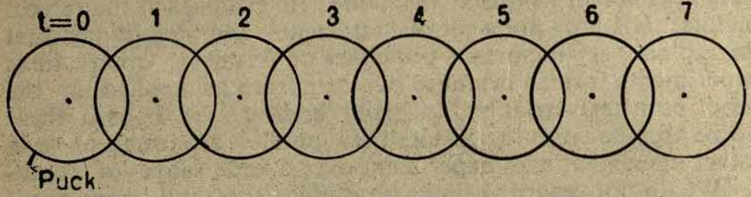


Fig 2·33 Motion of ice puck on glass plate—no friction results in uniform velocity

Again, by means of the *linear air track* (Fig. 2·34) an almost frictionless track can be obtained. It consists of long hollow tube of aluminium. A number of small holes are drilled on the top of the tube. Air under pressure is forced into one end of the tube. The air comes out through the small holes and forms a cushion of air under a cart (A) placed on the top of the track. A

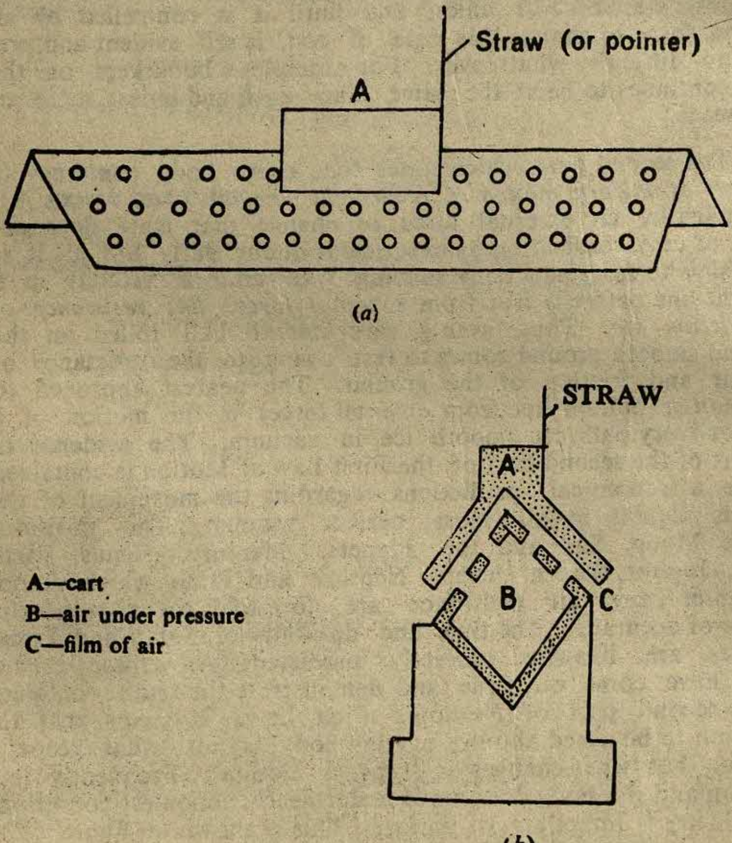


Fig 2·34 Linear-Air track Apparatus

straw (or pointer) is attached to the cart as shown. Given a push, the cart (A) moves almost floating on the cushion of air. As the cart moves, flash photographs are taken at equal short intervals of time. From the pictures obtained (Fig. 2·34*c*) the distances covered in these equal short intervals are calculated. They will be found to be equal throughout the motion showing that the cart moved with almost constant velocity.

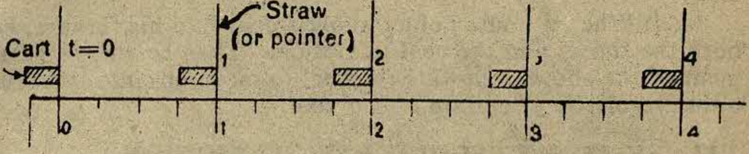


Fig 2·34 (*c*)

Law of Inertia. Newton's First Law of Motion is also called Law of Inertia. *By inertia we mean the inertness i.e. built in reluctance of matter* and *hence its inability to change its position of rest or of uniform motion in a straight line without the application of an external force.* (Galileo was the first to recognize this property.) In other words, the first law of motion gives us the *definition of force.* The force is the cause of change of position of rest or of uniform motion in a straight line. The force may be applied in the form of a push or a pull. It may be defined as follows.

Force is a push or a pull which produces or tends to produce, destroys or tends to destroy motion in a body.

It should be very clearly understood that the force must act on a body or a system of bodies from *outside.* An *internal* force cannot cause any change of state of a body. It is impossible for a man to lift himself by putting his feet in two buckets and pulling up their handles. A boat cannot be moved by leaning against its stern and pushing forward on its thwarts with the feet.

The *mass* of a body is a measure of its inertia. A large mass requires a large force to produce a certain acceleration because the greater the mass of the body the greater is its tendency to oppose any change in its state of rest (*inertia of rest*) or of uniform motion (*inertia motion*).

Illustrations of Inertia

(*a*) A bullet fired against a window glass pane makes a clean hole in it and the glass is not cracked. The glass surface surrounding the place where the bullet strikes *cannot* share the quick motion of the bullet and remains at rest

(*b*) When a train *suddenly* starts a passenger standing in it falls backwards. This is due to the fact that the lower part of his body being in contact with the floor of the train shares its motion whereas the upper part tends to remain in the *former state of rest in space.*

(*c*) When the bus goes round a turn, the passenger will continue to move in a straight line rather than, turn with the bus owing to his or her inertia *until* he or she grabs some thing attached firmly to the bus that will pull him or her along with the turning bus.

(*d*) When a house holder shovels coal into his furnace, he sets both the shovel and the coal in motion. Then he stops the movement of the shovel, allowing the coal to move on into the furnace owing to inertia of motion of the coal.

2·13 NEWTON'S SECOND LAW OF MOTION

The enunciation of the Second Law of Motion contains a physical term called **momentum** which needs some elucidation before we proceed further. **Momentum of a body is the sum total of motion contained in it.** The motion contained in a body depends upon its *mass* and *velocity* as will be clear from the following example.

Let us imagine that a tennis ball and a cricket ball are falling from the same height. Common experience tells us that it is easier to catch the tennis ball than the cricket ball. Why ? The balls are falling from the same height and as such have the *same* velocity on reaching the hand. But the cricket ball is heavier than the tennis ball. *Thus, the quantity of motion contained in a body depends upon its mass.*

Again consider two cricket balls, one falling from a height of 1000 m and the other from 500 m. On reaching the hand the velocity of the first will be

$$\sqrt{2gs}=\sqrt{2\times 9{\cdot}8\times 1000}=140 \text{ ms}^{-1}$$

and that of the second ball will be

$$\sqrt{2\times 9{\cdot}8\times 500}=98{\cdot}99 \text{ ms}^{-1}$$

It will be difficult to catch the former as it has greater velocity than the latter.

Thus, the motion contained in a body depends upon its mass and velocity. Hence we may define momentum of a body as follows.

"Momentum of a body is the sum total of motion contained in it and is measured by the product of its mass and velocity."

If m is the mass of the body and $\vec{V}$ its velocity then its linear momentum $\vec{p}$ is defined as

$$\boxed{\vec{p}=m\vec{V}} \qquad ...(2\text{-}32)$$

since velocity $\vec{V}$ is a vector, the linear momentum $\vec{p}$ is also a *vector* as momentum of a body depends upon its velocity.

The unit of momentum is the momentum possessed by a body of unit mass moving with a unit velocity. The SI unit of momentum is the momentum of a mass of 1 kg moving with a velocity of 1 ms^{-1}. *i.e.* **kg ms** $^{-1}$.

The dimensions of momentum are $M^1L^1T^{-1}$.

Illustrations of Momentum

(*a*) The great havoc sometimes done by a cyclone is due to the great momentum of the mass of air. The mass of air may be small, but its velocity is very great, and so the momentum (*i.e.* mass × velocity) is large.

(*b*) By taking the hammer at a distance before striking a nail in order to drive it into a piece of wood, a greater velocity of the hammer is acquired and consequently a greater momentum is obtained.

Newton's Second Law of Motion. Whereas Newton's First Law of Motion gives the definition of a force, Second Law of Motion gives the measurement of a force. In other words it defines the effects of a force. The Second Law of Motion states that

"The rate of change of momentum of a body is proportional to the resultant force impressed and takes place in the direction of the impressed force."

Let a body of mass m move with a velocity $\vec{V}$ at any instant.

The momentum $\vec{p}$ of the body is defined as

$$\vec{p}=m\vec{V}$$

The rate of change of momentum can be obtained by differentiating $\vec{p}$ with respect to time t. Thus

$$\frac{d\vec{p}}{dt}=\frac{d}{dt}(m\vec{V})=m\frac{d\vec{V}}{dt} \text{ taking } m \text{ as a constant.}$$

$$=m\vec{a}$$

where $\frac{d\vec{V}}{dt}=\vec{a}$ is the acceleration produced.

According to Newton's Second Law of Motion, stated above, the rate of change of momentum $\frac{d\vec{p}}{dt}$ is proportional to the resultant force $(\vec{F})$; but since $\frac{d\vec{p}}{dt}=m\vec{a}$, we have

$$\vec{F} \propto m\vec{a}$$

or $\vec{F}=k\,m\vec{a}$ where k is a constant of propartionality.

Now let us choose a unit force as that force, which produces in a unit mass a unit acceleration. Thus putting

$$F=1\ ;\quad m=1 \text{ and } a=1, \text{ we get}$$

$$k=1$$

Hence $$\boxed{\vec{\mathbf{F}}=\mathbf{m}\vec{\mathbf{a}}} \qquad \text{...(2-33)}$$

Since $\vec{a}=\frac{d^2x}{dt^2}$

$$\therefore \quad \boxed{\vec{F}=m\frac{d^2x}{dt^2}}$$

or **Force = Mass × Acceleration**

Units of Force

The unit for mass together with the units for acceleration can be employed in the Second Law equation (2·33) to define the basic unit of force.

In the *SI system*, the unit of force is known as **the newton** and **is defined as the force required to give a 1-kilogram mass an acceleration of 1 metre per second per second**.

$$1 \text{ newton } (N)=1 \text{ kg}\times 1 \text{ms}^{-2}=1 \text{ kg-ms}^{-2}$$

According to the smaller scale units (cgs system), the basic unit of force is the *dyne*.

"One dyne is the amount of force necessary to give a mass of 1 gram an acceleration of 1 centimetre per second per second."

Since $1 \text{ kg} = 10^3 \text{ g}$ and $1 \text{ m} = 10^2 \text{ cm}$

We have

$$1 \text{ newton (N)} = 1 \text{ kg} \times 1 \text{ ms}^{-2}$$

$$= 10^3 \text{ g} \times 10^2 \text{ cms}^{-2} = 10^5 \text{ g cms}^{-2}$$

$$= 10^5 \text{ dyne} \quad (\because 1 \text{ dyne} = 1 \text{ g} \times 1 \text{ cms}^{-2})$$

imensions of force are $M^1 L^1 T^{-2}$

2.14 PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The law of conservation of linear momentum is a fundamental law and is of great importance in physics. It may be stated as follows :

"When the resultant external force acting on a system of several bodies is zero, the total linear momentum of the system is conserved (constant). The total linear momentum is the vector sum of the linear momentum of each particle in the system."

Suppose there is only one object in an isolated system. If the object is moving with a velocity $\vec{V}$, according to Newton's first Law it will continue to move with the same velocity. Hence its linear momentum $m\vec{v}$ remains constant where m is the mass of the object.

Let there be two objects in an isolated system and let them be interecting with each other (*e.g. colliding*). If we consider any one object, its velocity will be changing due to interaction. hence its linear momentum will be changing. The same will be the case with second body also. Since there are no external forces acting on the system, the vector sum of linear momenta of two objects would remain the same. The same thing applies even if there are more than two objects interacting among themselves in an isolated system.

Proof. Let us consider an **isolated* system consisting of n particles of masses $m_1, m_2, m_3, \ldots\ldots\ldots\ldots, m_n$ moving with velocities $\vec{V}_1, \vec{V}_2, \vec{V}_3, \ldots\ldots, \vec{V}_n$ respectively mutually interacting amongst themselves. The total linear momentum $\vec{p}$ of this system is given by the vector sum of the linear momenta of all the individual particles in the system. Thus

---

*An isolated system is one which is free from the influence of any external force.

$$\vec{p} = m_1 \vec{V}_1 + m_2\vec{V}_2 + m_3\vec{V}_3 + \ldots\ldots + m_n\vec{V}_n$$

$$\text{But } m_1\vec{V}_1 + m_2\vec{V}_2 + m_3\vec{V}_3 + \ldots\ldots + m_n\vec{V}_n = M\vec{V}_{c.m.}$$

where M= total mass of all the particles in the system and $\vec{V}_{c.m.}$= Velocity of the **Centre of mass* of the system, thus it follows that

$$\vec{p} = M\vec{V}_{c.m.}$$

Differentiating the above equation with respect to time, we have

$\frac{d\vec{p}}{dt} = \frac{d}{dt}(M\vec{V}_{c.m.}) = M\vec{a}_{c.m}$ where $\vec{a}_{c.m}$ is the acceleration of the centre of mass of the system.

According to Newton's Second Law of Motion, we have

$$\frac{d\vec{p}}{dt} = \text{Resultant external force on the system} = \vec{F}_{ext}$$

---

*The centre of mass of a system of particles is that single point within the system which moves in the same way in which a single particle having the total mass of the system and subject to the same external force would move. The centre of mass of a system of particles depends on the mass of the particles and positions of the particles relative to one another.

If there are '*n*' particles of mass $m_1, m_2, m_3, \ldots, m_n$ distributed in space then

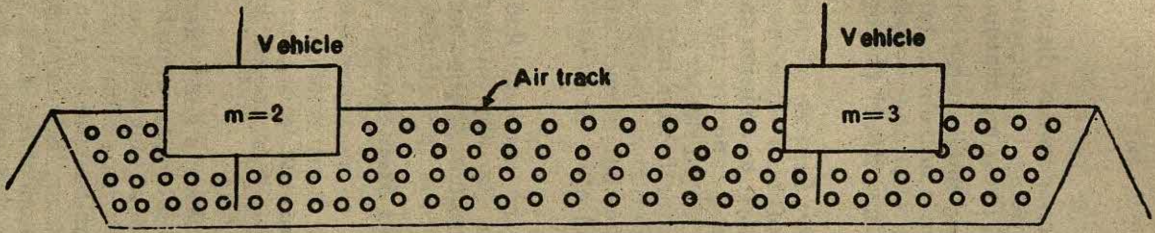
$$X_{c.m.} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots + m_nx_n}{m_1 + m_2 + m_3 + \ldots + m_n} = \frac{\Sigma mx}{M}$$

and
$$Y_{c.m.} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \ldots + m_ny_n}{m_1 + m_2 + m_3 + \ldots + m_n} = \frac{\Sigma my}{M}$$

Similarly
$$Z_{c.m.} = \frac{\Sigma mz}{M}$$

where $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), \ldots, (x_n, y_n, z_n)$ are coordinates of the particles with respect to three coordinate axes x, y, z respectively and $(X_{c.m.}, Y_{c.m.}, Z_{c.m.})$ are the coordinates of centre of mass of the system.

Experimental Test



Air Track Fig. (2.35) (a)

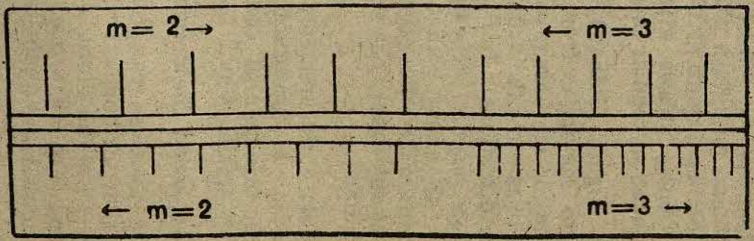


Fig. 2.35(b)
(Collision between two vehicles)

When the resultant external force on the system is zero

i.e $\vec{F}_{ext.}=0$, then,

$$\frac{d\vec{p}}{dt}=0 \text{ or } \vec{p} = \text{constant.}$$

That is, the total linear momentum (*Vector sum*) of the system is constant, if the resultant external force acting on the system is zero. This is the principle of conservation of linear momentum.

The principal can be investigated experimentally using a linear air track and multiflash photography of electric stop clocks to measure velocities. Figure [2·35 (a)] shows an air track and two vehicles with drinking straws attached so that a multiflash photograph can be taken at regular intervals.

In Figure [2·35 (b)] a collision is shown between a vehicle of mass 'two' moving in from the left and one of mass 'three' from the right, the top markers give the velocities before the collision and the bottom ones after the vehicles.have 'rebounded' and are moving in opposite directions. [In Figure[2·35 (b)] the hinged shutter is shown in the up position revealing to the camera the bottom half of one straw. At the exact instant of collision the shutter is rotated from one position to the other so that only one half of each straw is visible at any time]. The velocities before and after the collision are thus obtained.

Make measurements on Fig. (2.35 b) and you would find that momentum is conserved in this collision. In any attempt to verify the principle of conservation of momentum, friction must be negligibly small. Why ?

Illustrations of the conservation of Linear Momentum

(1) When a man jumps from a boat to the shore, it is well known that the boat experiences a backward thrust which displaces it away from the shore. It is due to the impulsive force exerted by the man. The change of momentum of the boat caused by the force is equal and opposite to that of the man.

(2) **Motion of a shot and Gun.** When a gun is fired the powder is almost instantaneously converted into a gas at high pressure, which by expansive action forces the shot out of the muzzle. The force on the shot at any instant, before it leaves the muzzle, is equal and opposite to that exerted on the gun backwards. The time for which both these forces act being the same, their impulses* are equal but opposite. So the change of momentum of the shot is equal and opposite to that of the gun. But both the shot and the gun being initially at rest, the momentum produced in the shot is equal and opposite to that in the gun.

Suppose m and M are the masses of the shot and the gun respectively, $\vec{v}$ the velocity with which the shot emerges from the muzzle and $\vec{V}$, the *recoil* velocity of the gun, supposing it to be free to move. Thus $m(\vec{v}-o)= M(\vec{V}-o)$

---

*Impulse means change in momentum produced in a body

$$or\ m\vec{v} = M\vec{V}$$

So the Vector sum of the momentum of the system is conserved.

(3) Rocket and Jet Propulsion

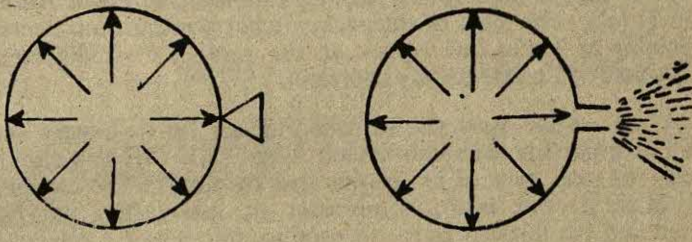
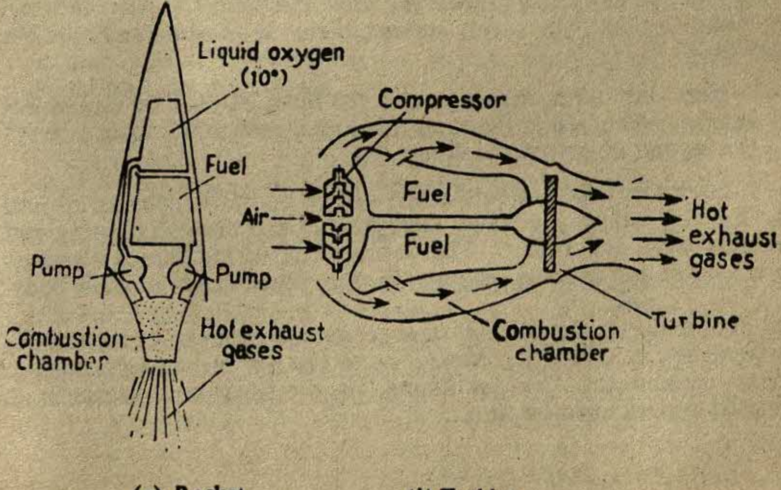

(*a*) (*b*)

Fig 2·36

The principle of both rocket and jet propulsion is illustrated by the behaviour of an inflated balloon when released with its neck open. If the neck is closed there is a state of balance inside the balloon with equal pressure at all points [Fig. 2.36 (*a*)]. When the neck is opened the pressure on the surface opposite the neck is now



(*a*) Rocket (*b*) Turbine

Fig. 23·7

unbalanced and the balloon is forced to move in the opposite direction to that of the escaping air [Fig. 2.36 (*b*)]. According to the principle of conservation of linear momentum the air and the balloon have equal but opposite amounts of momentum, that is

$$m_{air} \times \vec{v}_{air} = m_{balloon} \times \vec{v}_{balloon}$$

In a rocket and a jet engine a stream of gas is produced at a very high temperature and pressure and then escapes at high velocity through an exhaust nozzle. The thrust arises from the large increase in momentum of the exhaust gases. A rocket carries its own supplies of oxygen (*liquid*) and fuel (*e.g., Kerosene or liquid hydrogen*) Fig. [2.37 (*a*)]. The mass of the rocket is not constant but decreases appreciably as it uses fuel (*often at the rate of over 3000 kg* s^{-1}). The acceleration consequently increases.

A jet engine uses the surrounding air for its oxygen supply and so is unsuitable for space travel. Fig. [2.37 (*b*)] is a simplified drawing of one type of jet engine (gas turbine). The compressor draws in air at the front, compresses it, fuel (often paraffin) is injected and the mixture burns to produce hot exhaust gases which escape at high speed from the rear of the engine. These cause forward propulsion and drive the turbine which in turn rotates the compressor. The backward momentum produced in the escaping gases is equal and opposite to the forward momentum imparted to the rocket and the jet.

2·15 NEWTON'S THIRD LAW OF MOTION

Newton's Third Law of Motion states that "**when any object exerts a force on another object, the second object also exerts an equal and opposite force on the first.**" Alternatively, we might say that "*for every action, there is an equal and opposite reaction*".

Here the term *action* means the force exerted by any object on another object while the term, *reaction*, means the force exerted by the second object on the first.

Let an isolated system consist of two bodies of masses m_1 and m_2 moving along the same straight line and mutually acting and reacting with each other. The mutual interaction causes a change in velocities and hence a change in the momentum of each body.

Let $\Delta\vec{p}_1$ and $\Delta\vec{p}_2$ denote the change in momentum of m_1 and m_2 respectively, in time Δt. According to the law of conservation of linear momentum, the net change in the linear momentum in an isolated system must be zero.

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$$

or

$$\Delta\vec{p}_2 = -\Delta\vec{p}_1$$

Dividing by the time Δt and taking the limiting case,

$$\underset{\Delta t\to 0}{\mathrm{Lt}}\left(\frac{\Delta\vec{p_2}}{\Delta t}\right)=-\underset{\Delta t\to 0}{\mathrm{Lt}}\left(\frac{\Delta\vec{p_1}}{\Delta t}\right)$$

or
$$\frac{d\vec{p_2}}{dt}=-\frac{d\vec{p_1}}{dt}$$

i.e., Rate of change of momentum of m_2

= −Rate of change of momentum of m_1.

or Force on m_2 = −Force on m_1

or $\boxed{\text{Action}=-\text{Reaction}}$ which is Newton's third Law of Motion. The third law of motion is also called the *Law of Reaction* or *Law of stress.*

Illustrations :

When a carpenter hits the head of a nail with a hammer, the hammer exerts a force on the nail that drives it into a piece of wood. At the same time, the nail exerts an equal force on the hammer that stops the hammer's motion in midswing.

In the engines of a jet airplane, expanding hot gases produced by the explosion of a mixture of alcohol and oxygen whistle out of small nozzles facing the rear of the plane at very high speeds. In response, the plane itself speeds forward through the air.

The basic truth illustrated by these two common place examples is that **forces always occur in pairs.** *A single force cannot exist.* To every force in nature, there must correspond an equal and opposite force.

Everyday life is full of other examples of this particular law. These include the recoil of a gun when it fires a bullet, the crazy zigzag movement of a balloon that has been blown up and then released, and the movement of a boat through the water as the Oarsman pushes backward on the water with his oars.

When we step forward from rest our foot pushes backwards on the earth and the earth exerts an equal and opposite force forward on us. Two bodies and two forces are involved. The comparatively small force we exert on the large mass of the earth produces no noticeable acceleration of the earth, but the equal force it exerts on our very much smaller mass causes us to accelerate. However, if we could somehow get to a very small asteroid and step on that, the asteroid would undoubtedly move a few centimetres downward and send us a few metres into the air in the process.

If you pull a string attached to a block with a force P to the right [Fig. 2·38], the string pulls you with an equal force P to the left. Generally we can assume the string transmits the force unchanged and so there is another pair of equal and opposite forces at the block. The string exerts a pull P to the right *on the block* and the block exerts an equal pull to the left on the *string*—one force acts on the block and the other on the string. The string is pulled outwards at both ends and is in a state of tension.

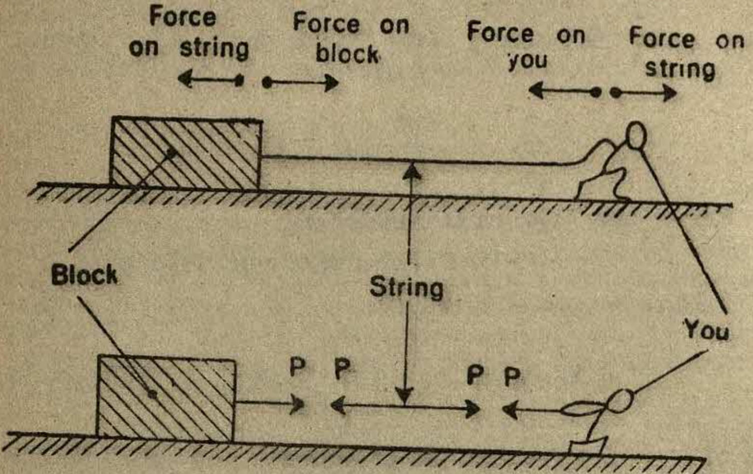


Fig. 2·38

Two important points must be borne in mind when we apply Newton's Third Law. **The forces of action and reaction act on different objects**; and *it is the* **vector sum** *of all the forces acting* **on** *a particular object that determines whether or not the object accelerates.* These points are illustrated by the following paradox.

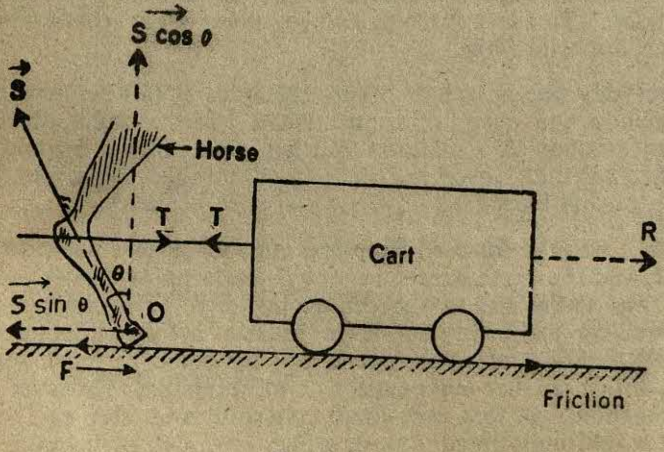


Fig. 2·39

A horse is pulling a heavily loaded cart. The cart pulls the horse backward. The question is: why does the system move forward? Let us answer this question.

In the process of pulling the cart forward, the horse exerts pressure on the ground which reacts back along OS Fig. (2·39). The reaction, $\vec{S}$, of the earth may be resolved into two rectangular components. The component $\vec{S}\cos\theta$ supports the weight of the horse where as the component $\vec{S}\sin\theta$ (which is greater than the force of friction F) is responsible for the forward motion of the system as a whole. It may be noticed here that this external force has been provided by the earth which does not form part of the system. So far as the cart is concerned, the weight is balanced by the reaction due to the earth. The forward pull of the horse on the cart and the backward pull of the cart on the horse, each being equal to T, cancel each other.

Example 2·10. A Saturn V rocket develops an initial thrust of $3{\cdot}3\times10^7$ N and has a lift-off mass of $2{\cdot}8\times10^6$ kg. Find the initial acceleration of the rocket at lift-off. (Take $g=10\ \text{ms}^{-2}$)

Solution.

Let T be the initial thrust on the rocket, m its mass and W its weight, then

$$W=mg=2{\cdot}8\times10^6\ \text{kg}\times10\ \text{ms}^{-2}$$
$$=2{\cdot}8\times10^7\ \text{N}$$

The *resultant* upward force on the rocket is (T−W) and if a is the initial vertical acceleration, then from

$$F=ma \text{ we have}$$
$$(T-W=ma$$
$$\therefore \quad a=\frac{(T-W)}{m}$$
$$=\frac{3{\cdot}3\times10^7-2{\cdot}8\times10^7}{2{\cdot}8\times10^6}=1{\cdot}8\ \text{ms}^{-2}$$

Note. We can apply $\vec{F}=m\vec{a}$ here since the rocket is instantaneouly at rest. In general this is not possible because the mass of the rocket changes.

Example 2·11. Two blocks A and B are connected as in Fig. (2·40) on a horizontal frictionless floor and pulled to the right

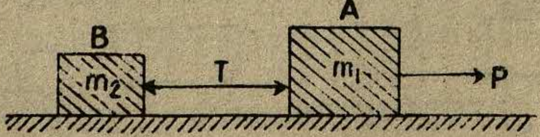


Fig. 2·40

with an accelaration of $2{\cdot}0\ \text{ms}^{-2}$ by a force P. If $m_1=50$ kg and $m_2=10$ kg, what are the values of T and P ?

Solution.

The forces acting on the blocks are shown in Fig. (2·40). Apply $\vec{F}=m\vec{a}$ to each

For Block B

$$\text{Resultant force}=T=m_2a=10\times 2=20\ \text{N}$$

For Block A

$$\text{Net or resultant force }=P-T=m_1a=50\times 2=100\ \text{N}$$

$$\therefore \qquad P=100+20=120\ \text{N}$$

Example 2·12. A jet of water emerges from a hose pipe of cross section area $5{\cdot}0\times 10^{-3}\ \text{m}^2$ with a velocity of $3{\cdot}0\ \text{ms}^{-1}$ and strikes a wall at right angles. Calculate the force on the wall assuming the water is brought to rest and does not rebound.

(Density of water$=1{\cdot}0\times 10^3\ \text{kg m}^{-3}$).

Solution.

If the water arrives with a velocity of $3{\cdot}0\ \text{ms}^{-1}$, $3{\cdot}0\ \text{m}^3$ hits every square metre of wall per second.

Hence Volume of water striking wall per second

$$=3{\cdot}0\times 5{\cdot}0\times 10^{-3}$$
$$=1{\cdot}5\times 10^{-2}\ \text{m}^3\ \text{s}^{-1}$$

Therefore, mass of water striking wall per second

$$=1{\cdot}5\times 10^{-2}\times 1{\cdot}0\times 10^3\ \text{kgs}^{-1}$$
$$=15\ \text{kg s}^{-1}$$

Velocity change of water on striking wall

$$=3{\cdot}0-0=3{\cdot}0\ \text{ms}^{-1}$$

Therefore, momentum change per second of water on striking wall

$$=15\times 3{\cdot}0$$
$$=45\ \text{kg ms}^{-2}$$

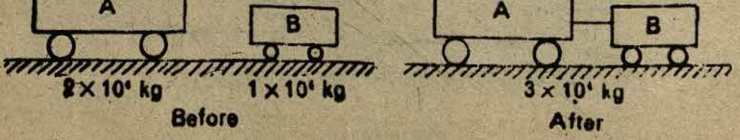


Fig 2·41

But Force=change in momentum per second=45 N

In practice the horizontal momentum of the water is seldom completely destroyed and so the answer is only approximate.

Example 2·13. A railway truck A of mass 2×10^4 kg travelling at $0·5$ ms^{-1} collides with another truck B of half its mass moving in the opposite direction with a velocity of $0·4$ ms^{-1}. If the trucks couple automatically on collision, find the common velocity with which they move.

Solution.

Total momentum *to the right* of A and B before collision (Fig. 2.41).

$$=2\times10^4\times0·5-1\times10^4\times0·4$$
$$=0·6\times10^4 \text{ kg ms}^{-1}$$

(If the momentum of A is taken as positive, that of B must be negative).

Total momentum to the right of A and B after collision

$=3\times10^4\times v$ kg ms^{-1} where v is the common velocity

By the principle of conservation of linear momentum,

$$3\times10^4\times v=0·6\times10^4$$
$$\therefore \quad v=0·2 \text{ ms}^{-1}$$

Example 2·14. A jet engine on a test bed takes in 20·0 kg of air per second at a velocity of 100 ms^{-1} and burns 0·80 kg of fuel per second. After compression and heating, the exhaust gases are ejected at 500 m s^{-1} relative to the aircraft. Calculate the thrust of the engine.

Solution.

Velocity change of 20 kg of air=(500−100)=400 ms^{-1}

∴ Momentum change per second of 20 kg of air

$$=20\times400 \text{ kg ms}^{-2}$$

The initial velocity of the fuel is *zero* and so its velocity change is 500 ms^{-1}.

∴ Momentum change per second of 0·80 kg of fuel

$$=0·80\times500 \text{ kg. ms}^{-2}$$

∴ Total momentum change per second of air and fuel

$$=(20\times400+0·80\times500) \text{ kg ms}^{-2}$$
$$=8·4\times10^3 \text{ kg ms}^{-2}$$

But force (thrust)=total change of momentum per second

∴ Thrust of engine=$8·40\times10^3$ N (∵ 1 kg ms^{-2}=1N)

2·16 INERTIAL MASS

Newton's first law led us to the concept of Inertia, a property of every body. Newton's second law proceeded a step further and provided us with a quantitative measure of Inertia namely mass. **Mass as defined by Newton's second law relates the net force applied to an object to the resulting acceleration**. If $\vec{F}$ is the net force applied on a body and $\vec{a}$ is the acceleration produced in it, then, according to Newton's second law of motion, the mass of the body is given by

$$m = \frac{\vec{F}}{\vec{a}}.$$

The greater the mass of an object, the greater its resistance to a change in motion—the greater its inertia and in terms of the second law of motion, we see that the greater the mass of an object for a given applied force, the smaller the change in motion—the smaller its acceleration. **Mass, then, is a measure of inertia and is related to the quantity of matter in a body.** We call such a mass as **inertial mass**. *It is given by the ratio of the resultant force applied upon a body to the acceleration produced in it along a smooth horizontal surface.*

If the same external force $\vec{F}$ acting on two bodies produces in them acceleration $\vec{a_1}$ and $\vec{a_2}$, then

$$\vec{F} = m_1\vec{a_1} = m_2\vec{a_2}$$

where m_1 and m_2 refer to the *inertial mass* of the bodies.

$$\therefore \quad \frac{m_1}{m_2} = \frac{|\vec{a_2}|}{|\vec{a_1}|}$$

If one of the bodies is taken as a standard, the above equation can be used to find the inertial mass of the other body in terms of the standard.

2·17 WEIGHT AND GRAVITATIONAL MASS

At this point, we must study one of the more subtle, but most important, distinctions in physics. This is the difference between mass and weight.

Let a body of mass *m fall freely* under the influence of gravity towards the centre of the earth. Its acceleration is g (9·81 ms^{-2}) called the acceleration due to gravity. It is *same* for all bodies at the *same place.*

The force producing this acceleration is provided by the pull with which the body is attracted by the earth towards its centre. This downward **gravitational** force is called the **weight** of the body. It may be defined as follows :

"Weight of a body is the pull with which it is attracted by the earth towards its centre."

By the second law of motion, force is equal to the product of mass and acceleration. Hence,

Weight of a body = its mass × acceleration due to gravity

$$\vec{W} = m\vec{g}$$

Thus the weight of a body is equal to the product of its mass, and acceleration due to gravity.

Since acceleration due to gravity is different at different points on the surface of earth. the weight of a body is *not* a constant quantity. The value of g is minimum at a place on the equator and maximum at poles. The weight of the body will, therefore, increase as it is moved from the equator towards the pole. Further as the body travels into space, the gravity force acting on the body and the resulting gravitational acceleration both change. However, the mass of the body remains constant as it is the sum total of matter contained in it. Mass is a *scalar* quantity, fundamental to the body under consideration. Your mass will not change if you take a trip to the Moon, however, the attractive force holding you to the moon's surface, your weight on the moon, is reduced to about one sixth of your weight on earth. Of course your free fall acceleration on the moon would be one-sixth that on the earth's surface. The high mobility of the astronauts on the moon arises from the reduction of their 'weight' on the moon. The inertia of the body does not play any role here.

The mass of a body which determines the gravitational pull due to the earth acting upon it is called its gravitational mass.

The weight of a body is measured by a springbalance because the extension produced in it depends upon the force pulling it downwards whereas the gravitational mass is measured by comparing its mass with another standard mass in an ordinary pair of scale pans.

Let m_1 and m_2 be the masses of two bodies and W_1 and W_2 be their respective weights at the same place. Then,

$$\vec{W_1} = m_1\vec{g}$$

and
$$\vec{W_2} = m_2\vec{g}$$

$$\therefore \qquad \frac{\vec{W_1}}{\vec{W_2}} = \frac{m_1}{m_2}$$

Thus the weight of a body is proportional to its mass.

Newton was the first to devise experiments to find if there was any difference between the inertial mass and the gravitational mass of a body. His experiments failed to show any difference. Since then several other experiments of much greater accuracy and sensitiveness have been performed and all these experiments show that these two masses are *equal* for a given body.

2·18 COLLISION

The law of conservation of linear momentum and law of conservation of energy are indispensable in dealing with collisions between particles. In such cases no external forces act on the participants, and therefore their total momentum before they collide equals their total momentum afterwards. The essential effect of the collision is to *redistribute* the total momentum of the particles.

A collision is said to have occurred if two particles or bodies physically collide against each other or even the path of a motion of a particle or a system is affected by the influence of a neighbouring system. Such influence may be due to the result of *long range* gravitational, magnetic or electric forces or *short range* nuclear forces.

There are *three types* of collisions as given below :

(*a*) **Perfectly Elastic Collisions**. *In perfectly elastic collisions, no mechanical energy is lost and both momentum and kinetic energy by the system are conserved.* Collisions between atoms, molecules, and fundamental particles are perfectly elastic. In practical life, collisions between ivory and glass balls are almost perfectly elastic.

(*b*) **Inelastic Collisions**. *If the kinetic energy of the system after the collision does not remain conserved, the collision is said to be inelastic.* During these types of collisions, some kinetic energy is invariably wasted and converted into heat energy and sound energy. In an inelastic collision, the kinetic energy after the collision is less than that before the collision. So in inelastic collisions, momentum is conserved, but kinetic energy is not **conserved.** In practical life, most of the impacts are of this type.

(c) **Perfectly Inelastic Collisions.** *After this type of collision, the two bodies stick together and move as one body.* A collision between a bullet and a block of wood when bullet remains embedded in the block after the collision, is a perfectly inelastic collision. The loss of kinetic energy is greatest in perfectly inelastic collisions. However momentum is conserved.

Perfectly Elastic Collision :

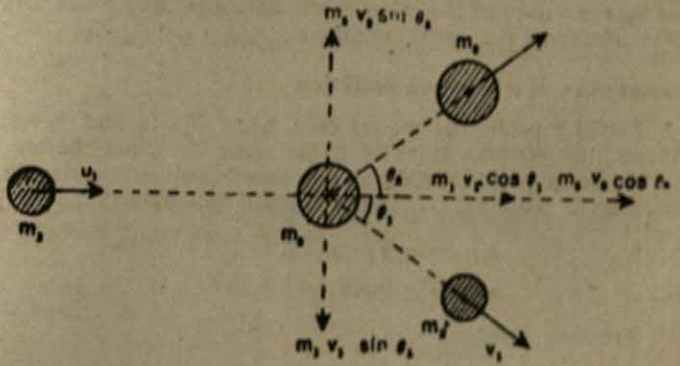


Fig. 2·42 Collision between one particle ahd another particle which is at rest

Let a particle of mass m_1 moving with a velocity $\vec{u}_1$, collide with another particle of mass m_2 *at rest.* (Fig 2·42). After the collision, let them move with velocities $\vec{V}_1$ and $\vec{V}_2$ along directions inclined at θ_1 and θ_2 respectively, with the initial direction of motion of m_1. *Let us assume that the collision is elastic.*

Taking the scalar components of velocities along the *x*- and *y*-directions and applying the principle of conservation of linear momentum, we have, for the *x*-component of motion

$$\boxed{m_1u_1 = m_1V_1 \cos\theta_1 + m_2V_2 \cos\theta_2} \quad ..(2\text{-}34)$$

and for the *y-component of motion,*

$$\boxed{0 = m_1V_1 \sin\theta_1 - m_2V_2 \sin\theta_2} \quad ..(2\text{-}35)$$

Since the collision is elastic, the kinetic energy is conserved.

Therefore,

$$\boxed{\tfrac{1}{2}m_1 u_1^2=\tfrac{1}{2}m_1 V_1^2+\tfrac{1}{2}m_2V_2^2} \quad \text{...(2-36)}$$

Suppose, we know only the values of m_1, m_2, and u_1 only, we cannot determine the motion after the collision and since there are three equations and four unknown quantities, *viz.*, V_1, V_2, θ_1 and θ_2. The value of one more quantity should be known to fix the motion after the collision. By giving different values for θ_1 or θ_2, the values of the other three unknown quantities can be determined from the above relations.

Special case of a head-on collision

Let us consider the special case when $\theta_1=0$ and $\theta_2=0$. In this case, the particles move along the same direction before and after the collision and it reduces to a case of collision in one dimension. The equations (2-34) and (2-36) will now reduce to

$$m_1u_1=m_1V_1+m_2V_2 \quad \text{...}(i)$$

and

$$\tfrac{1}{2} m_1u_1^2=\tfrac{1}{2}m_1V_1^2+\tfrac{1}{2}m_2V_2^2 \quad \text{...}(ii)$$

From equation (*i*)

$$m_2V_2=m_1u_1-m_1V_1 \quad \text{...}(iii)$$

and from equation (*ii*), we have

$$\tfrac{1}{2}m_2V_2^2=\tfrac{1}{2}m_1u_1^2-\tfrac{1}{2}m_1V_1^2$$

or

$$m_2V_2^2=m_1u_1^2-m_1V_1^2 \quad \text{...}(iv)$$

Dividing (*iv*) by (*iii*)

$$V_2=\frac{(u_1^2-V_1^2)}{(u_1-V_1)}=u_1+V_1 \quad \text{...}(v)$$

Putting (*v*) in (*iii*)

$$m_2(u_1+V_1)=m_1u_1-m_1V_1$$

or

$$m_2u_1+m_2V_1=m_1u_1-m_1V_1$$

or

$$V_1(m_1+m_2)=u_1(m_1-m_2)$$

$$\therefore \quad \boxed{V_1=u_1\,\frac{(m_1-m_2)}{m_1+m_2}} \quad \text{..(2-37)}$$

Now putting the equation (2-37) in (*iii*) we have

$$m_2V_2=m_1u_1-m_1V_1$$

or $$m_2V_2=m_1u_1-m_1\left[\frac{m_1-m_2}{m_1+m_2}\right]u_1$$

or $$m_2V_2=\frac{2m_1m_2u_1}{(m_1+m_2)}$$

$$\therefore \quad \boxed{V_2=\frac{2m_1u_1}{(m_1+m_2)}} \qquad \text{...(2·38)}$$

Further, if $m_1 < m_2$, then from equations (2-37) and (2-38) we have

$$\boxed{V_1=-u_1} \quad \text{and} \quad \boxed{V_2\approx 0}$$

this means that the *lighter* particle rebounds from the heavier particle with the *same* velocity.

Again if $m_1=m_2$ then from equations (2-37) and (2-38) we have

$$\boxed{V_1=0} \quad \text{and} \quad \boxed{V_2=u_1}$$

i.e., the two particles of *equal masses* suffering one dimensional elastic collision merely *exchange their velocities* after the collision. The moving particle transfers whole of its kinetic energy to the stationary one. As neutrons and protons have nearly equal masses, so kinetic energy of a fast neutron can be quickly reduced by making it pass through a substance like water which contains many protons. The neutron will lose energy due to perfectly elastic collisions with protons.

Example 2·15. A ball moving with a speed of 9 ms^{-1} strikes an identical stationary ball such that after the collision the direction of each ball makes an angle of 30° with the original line of motion. Find the speeds of the two balls after collision. Is the kinetic energy conserved in this collision process ?

Solution.

Let a ball A of mass m be moving with velocity $u_1=9\ ms^{-1}$ and another ball B of the *same* mass be stationary. Apply principle of conservation of linear momentum in the horizontal and vertical directions respectively, we get

$$9m=mV_1\cos 30^\circ+mV_2\cos 30^\circ \qquad \text{...}(i)$$

and $$0 = mV_1 \sin 30^\circ - mV_2 \sin 30^\circ \quad \text{...(2)}$$

From (2), we have

$$\boxed{V_1 = V_2}$$

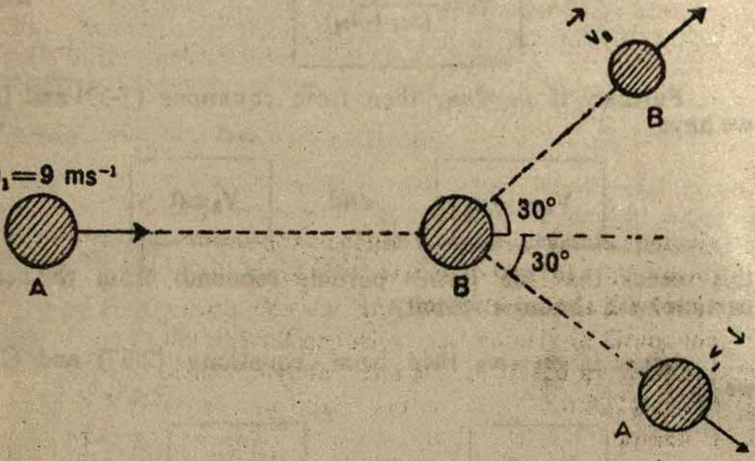


Fig. 2·43

and from (1)

$$9 = V_1 \times 2 \cos 30^\circ$$
$$= V_1 \times 2 \times \frac{\sqrt{3}}{2}$$

$$\therefore \quad \boxed{V_1 = 5{\cdot}2 \text{ ms}^{-1}}$$

$$\text{Initial K.E.} = \tfrac{1}{2} m u_1^2 = \tfrac{1}{2} m (9)^2 = \frac{81}{2} \text{ m}$$

$$\text{Final K.E.} = \tfrac{1}{2} m (V_1)^2 + \tfrac{1}{2} m (V_2)^2$$
$$= \tfrac{1}{2} m (5{\cdot}2)^2 + \tfrac{1}{2} m (5{\cdot}2)^2$$
$$= 27 \text{ m}$$

Since initial and final kinetic energies of the system before and after the collision are not equal, K.E. does not remain constant in this collision process.

2·19 IMPULSE

The effectiveness of a force in producing motion in a body depends upon its *magnitude* and the *time* for which it acts. Sometimes a large force acts for a very short duration and practically it is not possible to measure either its magnitude or time for which the force acts. In such cases the total effect of a force is measured by the impulse of force. **The total effect of a force is called impulse.** *It is measured by the change of momentum produced in a body.*

Let a moving body X collide with another body Y moving along the same straight line. Let the collision last for a time t during which the momentum of Y changes from $\vec{p}_1$ to $\vec{p}_2$. Assume that the bodies X and Y continue to move along the same straight line after the collision, though with different velocities.

Since the rate of change of momentum of a body is a measure of the force acting on the body, therefore, the force $\vec{F}$ acting on the body Y at any instant is given by

$$\vec{F}=\frac{d\vec{p}}{dt}$$ where $\vec{p}$ is the momentum at any instant.

or $$d\vec{p}=\vec{F}\,dt$$

Integrating the above equation between the limits, we have

$$\int_{p_1}^{p_2} d\vec{p}=\int_0^t \vec{F}dt$$

or $$\vec{p}_2-\vec{p}_1=\int_0^t \vec{F}dt.$$

We thus find that $\int_0^t \vec{F}\,dt$ is equal to the change in momentum $(\vec{p}_2-\vec{p}_1)$ of the body and hence is a measure of the *impulse* of a force. Impulse is denoted by $\vec{J}$.

$$\therefore \quad \boxed{\vec{J}=\int_0^t \vec{F}dt=\vec{p}_2-\vec{p}_1}$$

Now the magnitude of the force $\vec{F}$ generally varies during the time of a collision and if $\vec{\bar{F}}$ measures the *average force* during collision, we have

$$\boxed{\vec{J}=\vec{\bar{F}}t=\vec{p}_2-\vec{p}_1} \qquad \text{.. (2-39)}$$

Thus *impulse is also defined as the product of average force and the time for which the force acts and is equal to the total change in momentum.*

If the impulse of a force is positive, the momentum of the body on which it acts, increases algebraically. If the impulse is negative, the monemtum decreases. If the impulse is zero, there is no change in momentum. Impulse is a **vector** quantity and its unit in the SI system is **newton** × **Sec.**

If a *force-time* graph be drawn Fig (2·44), the area of the curve gives a measure of the blow or impulse. Thus if curves 1 and 2 in Fig (2·44) have *equal* areas, they represent equal impulse but it is evident from the figure that the force in the case of impulse No. 1 is much greater than that in the case of impulse No. 2.

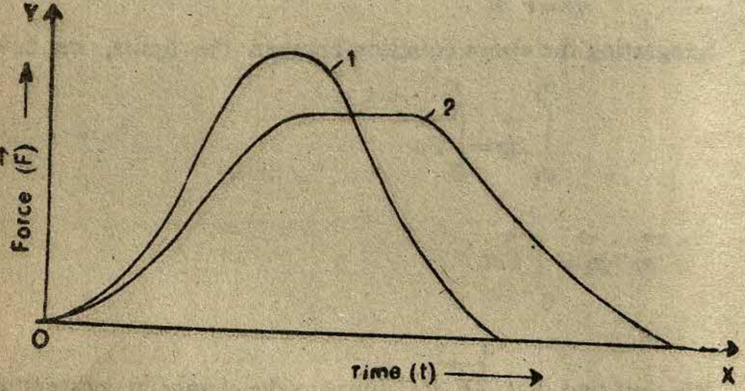


Fig 2·44

Illustrations.

(*i*) When a hammer drives a nail into a block of wood, the force $\vec{\bar{F}}$ acts for a very short time t and impulse$=\vec{\bar{F}}\times t$. It is this *impulse* that drives the nail into wood.

(*ii*) While catching a ball, a cricket player lowers his hands so that interval of time increases and from equation (2-39) for the same impulse, force will decrease and player will apply less force (reaction).

Suppose a cricket ball of mass 0·2 kg is falling from a height of 10 m. Its velocity will be equal to

$$\sqrt{2gs}=\sqrt{2\times 9{\cdot}8\times 10}=14 \text{ ms}^{-1}.$$

If a player brings it to *rest* in $\frac{1}{20}$th of a second, the average force $\vec{F}$ exerted by him is given by

$$\vec{F}\times t=\text{change in momentum}$$

$$\vec{F}\times\frac{1}{20}=0{\cdot}2\times 14$$

$$\therefore \qquad \vec{F}=20\times 0{\cdot}2\times 14=56 \text{ N}$$

If the player stops the ball in $\frac{1}{5}$th of a second, the average force exerted by him would be

$$\vec{F}\times\tfrac{1}{5}=0{\cdot}2\times 14$$

$$\therefore \qquad \vec{F}=5\times 0{\cdot}2\times 14=14 \text{ N}$$

The above example shows that if the time be made *four* times, the average force exerted is *reduced* to one-fourth.

3. For a similar reason, a man falling from a certain height on a cemented floor receives far greater injury that he would receive if he falls from the same height on a heap of sand.

4. In order to absorb the shocks or blows received by a tonga while moving over an uneven road, it is fitted with steel springs.

Example 2 16. A cricket ball having mass 500 g, initially at rest, is struck with a bat and acquires a velocity of 4 ms^{-1}. What is the magnitude of the change in momentum of the ball? What impulse is associated with the force exerted by the ball? If the bat is in contact with the ball for 0·1 sec, what is the magnitude of the average force exerted on the ball?

Solution.

Mass of the ball $=m=0{\cdot}5$ kg
Initial velocity $=u=0$
Final velocity $=v=4$ ms^{-1}

$$\text{Change in momentum} = 0{\cdot}5\,[4-0] = 2\ \text{kg ms}^{-1}$$

$$\text{Impulse } \vec{J} = \vec{F}t = \text{change in momentum} = 2\ \text{newton-sec}$$

$$\text{Time interval } (t) = 0{\cdot}1\ \text{sec}$$

$$\therefore\ \text{Average force } \vec{F} = \frac{\text{Impulse}}{\text{Time}} = \frac{2}{0{\cdot}1} = 20\ \text{N}$$

2·20. MOTION IN TWO DIMENSIONS—PROJECTILE MOTION.

Consider, for example, a ball thrown over the edge of a cliff with an initial velocity u_0 entirely in the horizontal direction Fig (2·45). The motion of the ball is also being affected by the downward pull of gravity so that the velocity of the ball at any instant has two components (*i*) the horizontal component which remains the same and(*ii*) the vertically downward component which increases by 9·8 ms^{-2} after every one second. The acceleration produced by gravity '*g*' is entirely in the vertical direction and cannot change the horizontal component of the velocity. This implies that a fielder starting from the foot of a cliff at the instant the ball is thrown and running outward with a constant horizontal velocity u_0 always remains directly underneath the ball.

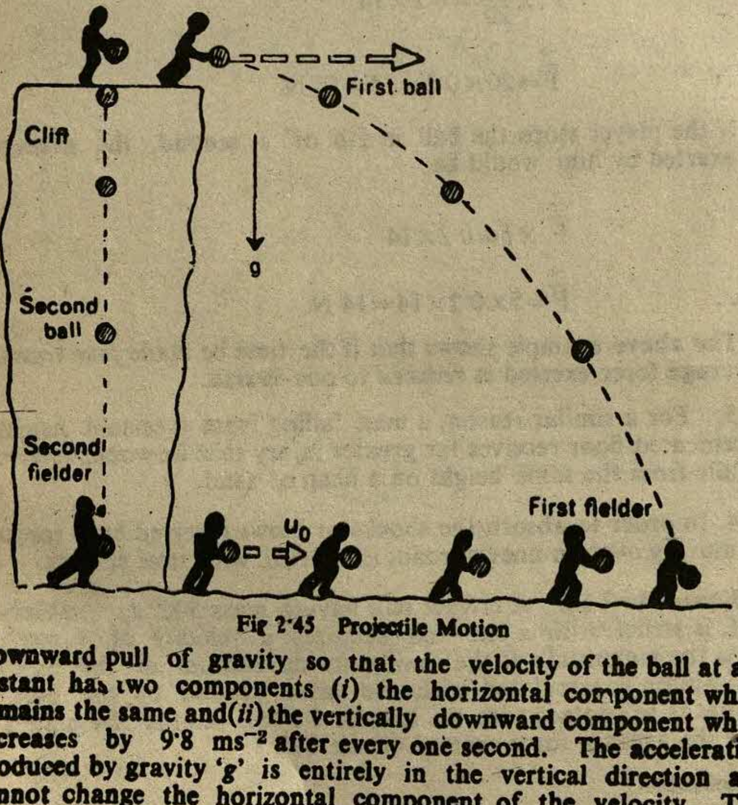
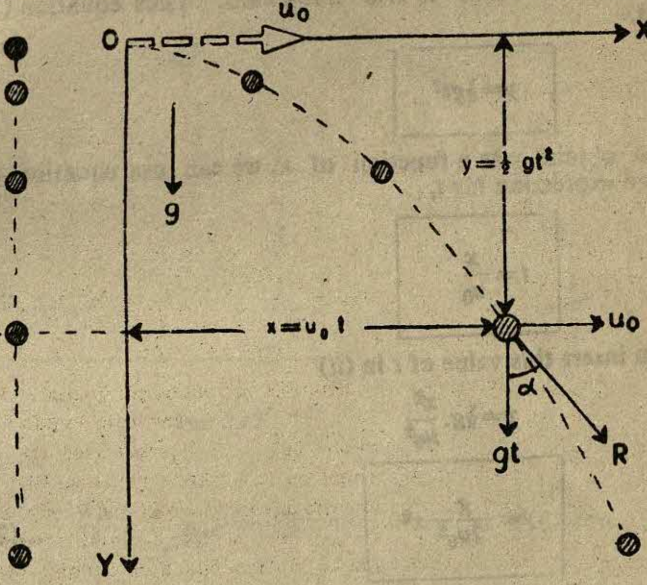


Fig 2·45 Projectile Motion

Meanwhile, the ball continues to fall in the vertical direction in exactly the same way as if it had no horizontal velocity.



(The use of cartesian coordinates to discuss Projectile Motion)
Fig 2·46

Imagine a second ball dropped from rest at the top of the cliff at exactly the same instant that the first ball is thrown horizontally outward. The two balls fall vertically in exactly the same way and are always at the same height. A second fielder standing stationary at the foot of the cliff catches the second ball at exactly the same instant that the running fielder catches the first ball.

To tackle the problem mathematically, use cartesian axes with the origin at the top of the cliff. Let the *X*-axis be horizontal with its *positive direction outward* from the cliff, let the *Y*-axis be vertical with its *positive direction downward* from the top of the cliff (Figure 2·46). After a time *t*, the horizontal distance covered by the first ball is the same as the horizontal distance covered by the running fielder and is

$$\boxed{x=u_0t} \quad ...(i)$$

Meanwhile, the vertical distance (*y*) through which either ball has fallen in the same time (*t*) is obtained by using equation of motion (2-25).

We must remember, though, that we are now considering motion in the *Y*-direction rather than in the *X*-direction, that the initial velocity u in the vertical direction is *zero*, and that the acceleration a is $+g$, which is positive because it is downward and the positive Y-direction is also downward. Thus equation (2-25) becomes,

$$\boxed{y=\frac{1}{2}gt^2} \qquad \text{...}(ii)$$

To obtain y as a function of x, we can use equation (i) to obtain an expression for t,

$$\boxed{t=\frac{x}{u_0}} \qquad \text{...}(iii)$$

and then insert this value of t in (ii)

$$y=\frac{1}{2}g.\frac{x^2}{u_0^2}$$

or

$$\boxed{y=\frac{g}{2u_0^2}x^2} \qquad \text{...(2-40)}$$

This is actually the equation of a *parabola* with its vertex at the top of the cliff. Thus the path followed by a body projected horizontally with a uniform velocity is a parabola.

A body thrown in the manner given above is called a **projectile** *and its path a* **trajectory**.

A **projectile** *may be defined as a body which after having been given an initial velocity is allowed to move under the action of gravity alone i.e.*, it is no longer being propelled by any fuel, *e.g.*, A javelin thrown by an athlete, a bomb released from an aeroplane, a bullet shot from a rifle etc.

The resultant velocity (R) at any instant (t) has two components, u_0 in the horizontal direction and gt in the vertical direction and thus the resultant velocity $R=\sqrt{u_0^2+g^2t^2}$ and its *direction* is along the tangent to the path at that point and makes an angle α with the vertical given by

$$\tan\alpha=\frac{u_0}{gt}$$

or

$$\alpha=\tan^{-1}\left(\frac{u_0}{gt}\right)$$

As t increases, $\tan\alpha$ and therefore, α decreases and approaches zero as t becomes large, and the ball begins to fall almost vertically downwards;

A body projected in any direction :

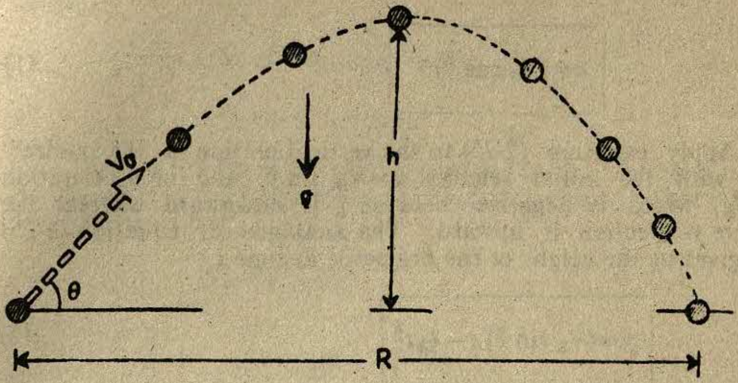


Fig. 2·47

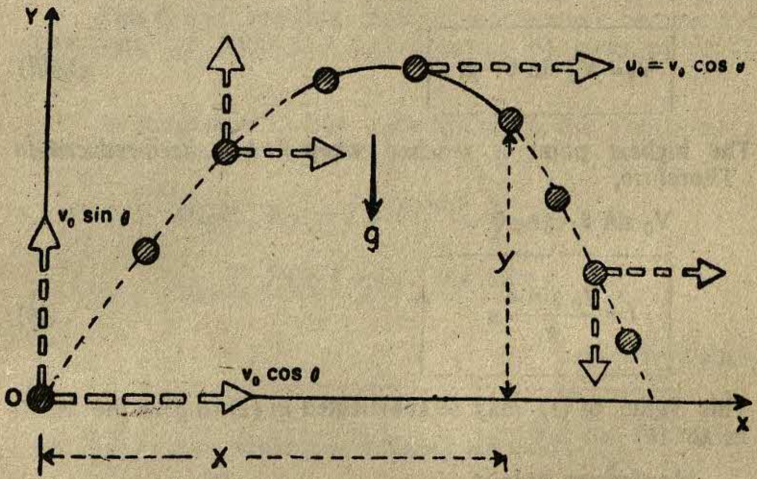


Fig 2·48

Now imagine a body projected upward with an initial speed V_0 at an angle θ to the horizontal as in Figure (2·47). As we shall shortly prove, its path is again a parabola. Let us calculate, h, **the maximum height** to which it rises, and R, its **horizontal range.**

As shown in Figure (2·48), take the origin at the point of projection, the *x*-axis horizontal, and the *y*-axis vertical with the *positive y*-direction now *upward*. The initial velocity V_0 may be replaced by a horizontal *x*-component $V_0 \cos\theta$ and a vertical *y*-component $V_0 \sin\theta$ upward. The horizontal component $V_0 \cos\theta$ is unaffected by the acceleration due to gravity and remains constant. After a time t, the displacement in the horizontal x-direction is

$$\boxed{x=(V_0 \cos \theta)\, t} \qquad \text{...(1)}$$

Apply equation (2-25) to the vertical motion in the y-direction, with the initial velocity $u=V_0 \sin \theta$, and the acceleration $a=-g$, which is negative because g is downward whereas the positive y-direction is upward. The analogue of equation (2-25) then gives us the height of the projectile at time t :

$$\boxed{y=(V_0 \sin \theta)\, t-\tfrac{1}{2}gt^2} \qquad \text{...(2)}$$

Similarly, equation (2-24) gives us the verticle component of the velocity (V_y) at any instant.

$$\boxed{V_y=V_0 \sin \theta-gt} \qquad \text{...(3)}$$

The highest point is reached when *V_y has been reduced to zero.* Therefore,

$$V_0 \sin \theta-gt=0$$

or

$$\boxed{t=\frac{V_0 \sin \theta}{g}} \qquad \text{...(4)}$$

This value of (t) may be substituted in (2) to give the maximum height (h)

∴ **Maximum height**

$$=h=(V_0 \sin \theta)\left(\frac{V_0 \sin \theta}{g}\right)-\tfrac{1}{2}g\left(\frac{V_0 \sin \theta}{g}\right)^2$$

or

$$\boxed{h=\frac{V_0^2 \sin^2 \theta}{2g}} \qquad \text{...(2-41)}$$

When the body is at its highest position, the vertical component of its velocity is zero and its resultant velocity is $V_0 \cos \theta$ horizontaly. This is identical with the situation discussed above if u_0 is put equal to $V_0 \cos \theta$. The previous proof that the *path is parabolic* can therefore be carried over to the present situation.

To obtain the range R, use the fact that when a body strikes the ground again $y=0$. At this instant of time, t_s, equation (2) becomes

$$(V_0 \sin \theta)t_s - \tfrac{1}{2}gt_s^2 = 0$$

or
$$(V_0 \sin \theta)\, t_s = \tfrac{1}{2}gt_s^2$$

This equation is satisfied by $t_s=0$, but this obviously corresponds to the instant of projection, when it is also true that $y=0$. If t_s is not zero, then

$$V_0 \sin \theta = \tfrac{1}{2}gt_s$$

or
$$\boxed{t_s = \frac{2V_0 \sin \theta}{g}} \qquad \text{...(2-42)}$$

This is an interesting piece of information because it tells us how long the flight last *i.e.*, equation (6) gives us the **time of flight.**

At the instant t_s, the x-coordinate is the horizontal range R and so equation (1) implies that

$$R = (V_0 \cos \theta)\left(\frac{2V_0 \sin \theta}{g}\right)$$

or
$$R = \frac{2V_0^2 \sin \theta \cos \theta}{g} = \frac{V_0^2 \sin 2\theta}{g}$$

$$(\because \quad \sin 2\theta = 2 \sin \theta \cos \theta)$$

or
$$\boxed{\textbf{Horizontal Range} = R = \frac{V_0^2 \sin 2\theta}{g}} \qquad \text{...(2-43)}$$

If the speed of projection V_0 is fixed, the horizontal range is maximum when $\sin 2\theta$ has a maximum value of 1 or when $2\theta=90°$ or $\theta=45°$. The maximum range is, therefore, realized when the angle of projection is 45° and its value is

$$\boxed{R_{max.} = \frac{V_0^2}{g}} \qquad \text{...(2-44)}$$

The proportionate ranges of projectiles fired with the same initial speed but at different angles of elevation are shown in Fig. (2-49). As seen, the horizontal range for the velocity of projection V_0 and the angle of projection θ is $\frac{V_0^2 \sin 2\theta}{g}$

Also when the angle of projection is $(90-\theta)$, the horizontal range $=\frac{V_0^2}{g} \sin 2(90-\theta)=\frac{V_0^2}{g} \sin(180-2\theta)=\frac{V_0^2 \sin 2\theta}{g}$

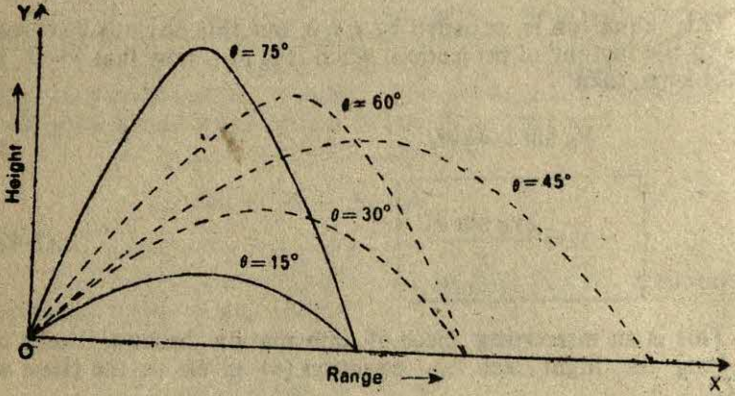


Fig. 2·49

Thus the horizontal range is the same for the angles of projection θ and $(90-\theta)$. Hence, with a given velocity of projection and for a given horizontal range, there are, in general, two directions of projection which are equally inclined to the horizontal and the vertical directions. Thus, for a given velocity, horizontal range is the same for $\theta=15°$ and $(90-15)=75°$; $\theta=30°$ and $60°$ and so on.

Example 2·17. A bomb is dropped from an aeroplane when it is directly above a target at a height of 1500 m. The aeroplane is going horizontally with a speed of 500 km h^{-1}. By how much distance will the bomb miss the target ? *[A.I.S.S.C. 1986]*

Solution

Here

Initial downward velocity $=u=0$

$S=1500$ m

and $a=g=9.8$ ms^{-2}

Thus using equation

$S=ut+\frac{1}{2}at^2$, we have

$1500=0+\frac{1}{2}\times 9.8\times t^2$

$\therefore \quad t=\frac{\sqrt{1500}}{4.0}$

Horizontal velocity of the aeroplane $=500$ km h^{-1}.

$$=\frac{500\times1000}{60\times60}=\frac{1250}{9}\text{ ms}^{-1}$$

∴ Horizontal Range = Horizontal velocity × Time

$$=\frac{1250}{9}\times\sqrt{\frac{1500}{4\cdot9}}$$

$$=2430\text{ m}$$

Example 2·18 : From the top of a building 19·6 m high, a ball is projected horizontally. After how long does it strike the ground. If the line joining the point of projection to the point where it hits the ground makes an angle of 45° with the horizontal, what is the initial velocity of the ball ?

Solution :

The time taken to reach the ground is independent of any horizontal velocity given to the ball. Now initial downward velocity $u=0$

$S=19\cdot6$ m
$a=9\cdot8\text{ ms}^{-2}$
$t=?$

Since $S=ut+\frac{1}{2}at^2$, we have

$$19\cdot6=0+\tfrac{1}{2}(9\cdot8)t^2$$

$$\therefore\quad t=2\text{ s}$$

Let Q be the point when the ball hits the ground [Fig 2·50]. Since ∠OQP=45°
PQ=PO=19·6 m

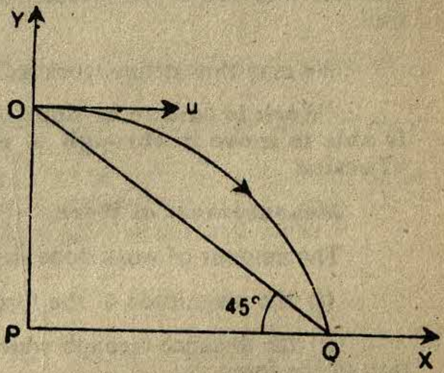


Fig. 2·50

Since PQ is the horizontal range covered by the ball in 2 seconds with the velocity (u) of horizontal projection, we have

$$u\times2=19\cdot6$$

$$\therefore\quad u=9\cdot8\text{ ms}^{-1}$$

2·21 WORK, ENERGY AND POWER

Meaning of Work

In science the term work has a definite meaning which differs from its every day one. For instance a student sitting in his chair on a holiday and forming a plan for spending his holiday and a coolie who stands still and holds a heavy weight on his head are doing *no* work in the sense in which we have to use the term here. Thus mental or physical work or exertion in which the working body *does not move* are excluded from the domain of work as understood in physics. *Hence, motion is an essential feature of work.* Morever, this type of work is the result of some action of one body upon the other. Such an action we have already defined as *force* which may be a push or pull due to muscular effort or otherwise.

There is another point about the meaning of work which must be noted here ; that is, *the direction of the motion of the body with respect to that of the force.* Let us consider the case of a passenger sitting in a rickshaw which is drawn by another man. The former is doing no work and the latter is doing a definite amount of work, although both of them are applying a force. This is explained by the fact that the passenger is pressing the rickshaw downwards with his weight but there is no motion in that direction. The man who is drawing the rickshaw exerts a force on the rickshaw in the same direction in which it is moving. He is doing no work against the force of gravity as this force is perpendicular to the direction of motion. Similarly, a cricket player does a definite amount of work in stopping a moving ball. Again a mason who simply fixes the bricks on the wall does very little work as compared to the labourer who carries the bricks from the ground to the top of the wall.

We may thus define work as follows :

"Work is said to be done if a force acting upon a body is able to move it through a certain distance along its line of action."

Measurement of Work

The amount of work done depends upon two factors :

(*i*) the magnitude of the force, and

(*ii*) the distance through which the body moves in the direction of the force.

The work done is measured by the product of the force and the distance through which the body moves along the direction of the force.

Thus if $\vec{F}$ is the force and $\vec{S}$ is the displacement along the direction of the force, the work done, W, is given by

$$\boxed{W = |\vec{F}|\ |\vec{S}|} \qquad \ldots(2\text{-}45)$$

Sometimes the displacement of the body is not along the same direction as the force applied. Consider a body which is acted on by a force $\vec{F}$ as in (Fig. 2·51). Suppose that, during a short interval of time, the body is displaced through a vectorial distance

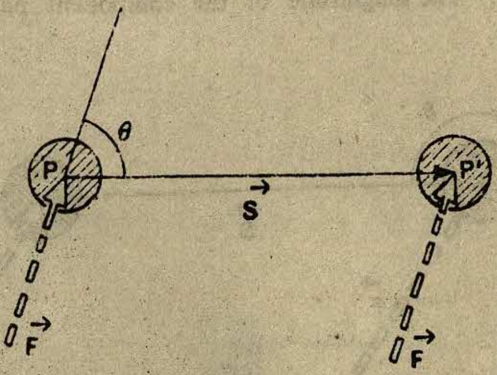


Fig. 2·51

$\vec{S}$ from P to P' If the time interval is short enough, the motion of a body is essentially a straight line and the force $\vec{F}$ does not have time to change appreciably. Here $\vec{F}$ and $\vec{S}$ are not in the same direction and let the angle between them be θ. The work done (W) by the force during this small displacement is given as

$$\boxed{W=|\vec{F}|\ |\vec{S}|\cos\theta} \qquad \ldots(2\text{-}46)$$

If $\vec{F}$ and $\vec{S}$ are at right angles, $\theta=90°$; $\cos\theta=0$ and hence work done is *zero.* A force *does no work* if the body on which it acts moves in a direction perpendicular to the force. It is for this reason that a coolie, carrying a load on his head and walking along a horizontal railway platform is not doing any work.

If $\vec{F}$ and $\vec{S}$ are in the same direction $\theta=0$ and hence

$$W=|\vec{F}|\ |\vec{S}|, \text{as } \cos 0°=1$$

The work done on a body can be *negative* if θ lies between 90° and 270° as shown in Fig. 2·52 because in this case cos θ is negative.

An alternative approach is to replace the Force $\vec{F}$ by its rectangular components parallel and perpendicular to the displacement $\vec{S}$ Fig. 2·53. The magnitude of the component parallel to $\vec{S}$ is

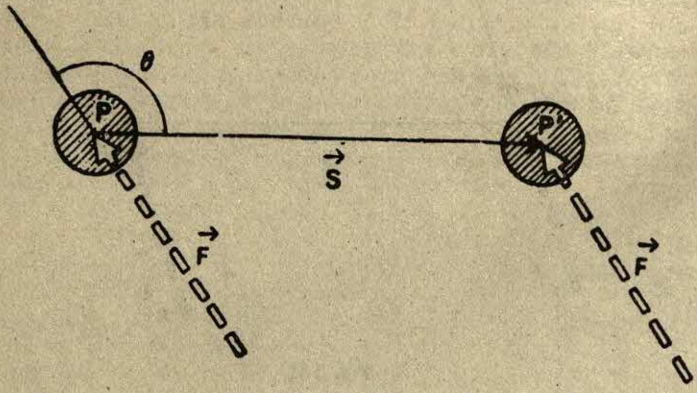


Fig. 2·52

$|\vec{F}|\cos\theta$ and the work done is the magnitude of this component multiplied by the distance S through which the body is displaced.

Work = Component of force parallel to the displacement × Displacement ...(2-47)

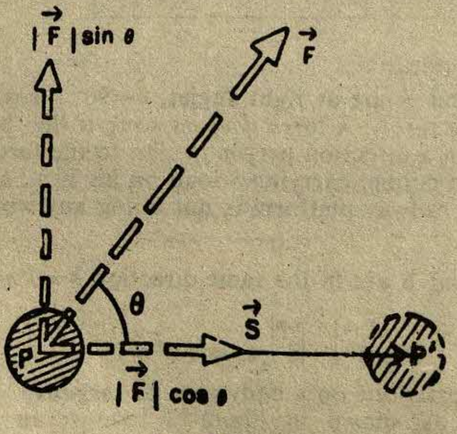


Fig. 2·53

If the $\cos\theta$ component of the force points in the same direction as displacement $\vec{S}$, the work done is positive (Fig. 2.54). If the component points in the opposite direction to the displacement $\vec{S}$, the work done is *negative* (Fig. 2·55). If the resultant force $\vec{F}$ is perpendicular to the displacement $\vec{S}$, $\theta=90°$, $\cos\theta=0$, there is *no* component of $\vec{F}$ in the direction of $\vec{S}$ and the work done is *zero* (Fig. 2·56.)

Positive work performed on an object increases the object's energy, while negative work on an object reduces its energy.

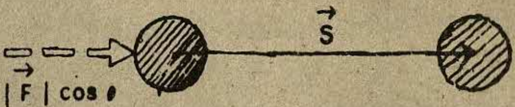


Fig. 2·54. Work done is positive.

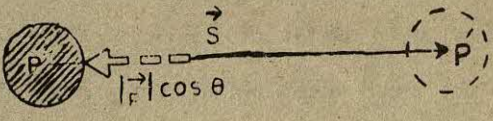


Fig. 2·55. Work done is negative.

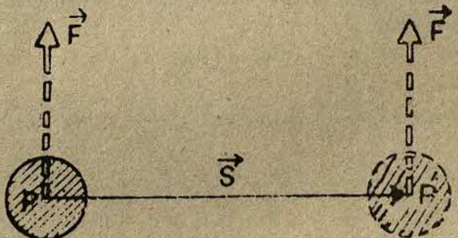


Fig. 2·56. Work done is zero.

Since work done $(W)=|\vec{F}|\,|\vec{S}|\cos\theta$, it means work done (W) is given by a *scalar or dot product* of $\vec{F}$ and $\vec{S}$ *i.e.*

$W=|\vec{F}|\,|\vec{S}|\cos\theta=\vec{F}\cdot\vec{S}$ where θ is the angle between $\vec{F}$ and $\vec{S}$. Further, since both force $\vec{F}$ and displacement $\vec{S}$

are vectors, their scalar product W is a scalar. Hence work is a *scalar quantity*.

Units of Work

In the SI system, if a force of *one newton* displaces a body through a distance of *one metre* parallel to the force, the work done is *one joule*.

In the C.G.S. system, if a force of *one dyne* displaces a body through a distance of *1 cm* parallel to itself, the work done is *one erg*.

$$1 \text{ joule} = 1 \text{ N} \times 1 \text{ m}$$

$$= 10^5 \text{ dynes} \times 10^2 \text{ cm} = 10^7 (\text{dynes} \times \text{cm}) = 10^7 \text{ ergs.}$$

$$\boxed{\therefore\ \mathbf{1\ joule = 10^7\ ergs}}$$

2·22 WORK DONE BY A CONSTANT FORCE

Let a constant force $\vec{F}$ act on a body and displace it through $\vec{S}$ in the direction of the force. Imagine that the body is displaced through a very small displacement $\vec{ds}$ and the corresponding work done dW is given by

$$dW = F ds \qquad \ldots(i)$$

The total work done by the force in displacing the body through the distance S in the direction of the force is given by integrating equation (*i*)

$$\int dW = \int F ds$$

$$\text{But} \quad F = ma = m\frac{dV}{dt} = m \cdot \frac{dV}{ds} \cdot \frac{ds}{dt} = m \cdot V\frac{dV}{ds} \quad \text{as} \quad \frac{ds}{dt} = V$$

$$\therefore \quad \int dW = \int mV\frac{dv}{ds} \cdot ds = \int mV dV$$

If u is the initial velocity of the body before the force was applied and 'v' the final velocity after the force acted through the distance S, then

$$\int dW = \int_u^v mV dV$$

$$\text{or} \qquad W = m\left|\frac{V^2}{2}\right|_u^v = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

But $W=FS$

$$\therefore \quad \boxed{FS=\tfrac{1}{2}mv^2-\tfrac{1}{2}mu^2} \qquad \text{...(2-48)}$$

If the body is initially at rest, $u=0$, then the work done by the force will be

$$\boxed{FS=\tfrac{1}{2}mv^2} \qquad \text{...(2-49)}$$

This expression (2-49) is a function of velocity and is a measure of the *kinetic energy* of the body *i.e.*, Energy of motion.

The right hand side expression of equation (2-48) gives the change in the Kinetic Energy of the body. Thus it follows that the work done by a force in displacing a body measures the change in its kinetic energy.

2·23 WORK DONE AGAINST A VARIABLE FORCE

In actual situations, the magnitude of the force acting on a body may not always remain constant. For example, the gravitational force acting on a freely falling body increases with the square of the distance. The restoring force per unit area developed in an elastic string increases with extension, within elastic limits. If the force varies, the work done can be obtained from a *force displacement graph* in which the component of the force in the direction of the displacement is plotted Fig. 2·57.

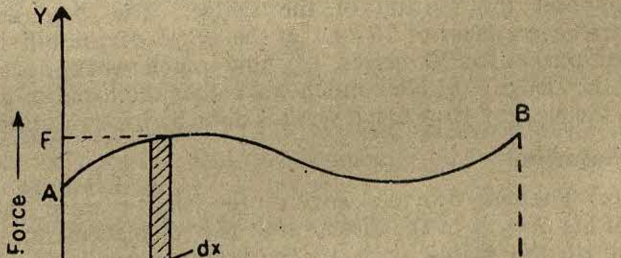


Fig. 2·57

Suppose the force is F when the displacement is x, then the work done during a further, very small displacement dx (which is so small that F can be considered constant during it) is Fdx, *i.e.* the shaded area of the thin shaped rectangle. By dividing the whole area under the curve into narrow strips, we see that the total work done during displacement S is represented by the area OABC. Thus

$$\text{Total work done} = W = \int_{x=OD}^{x=OC} F\,dx \qquad \text{...(2-50)}$$

Example 2·19. In 10 sec a body of 6 kg mass is dragged 8 metres with uniform velocity across a floor by a steady force of 20 N. Calculate

(*i*) How much work is done ?

(*ii*) What is the kinetic energy of the body ?

Solution.

(*i*) Work done $= FS = 20 \times 8$ joules $= 160$ joules

$$(ii)\ \because \quad S = ut + \tfrac{1}{2}at^2$$

$$\therefore \quad 8 = u \times 10 + \tfrac{1}{2} \times 0 \times (10)^2$$

$$\text{or} \quad 8 = 10u$$

$$\therefore \quad u = \tfrac{4}{5}\ \text{ms}^{-1}$$

$$\therefore \quad \text{Kinetic Energy} = \tfrac{1}{2}mu^2 = \tfrac{1}{2} \times 6 \times (\tfrac{4}{5})^2 \text{ joules}$$

$$= 1{\cdot}92 \text{ joules}$$

Example 2·20. A man pushes a boy up a hill on a sledge. He applies a horizontal force of 450 N, while a frictional force of 5 N opposes the motion of the sledge. The boy and sledge together have a mass of 36 kg. If the angle of the hill is 30° and the man pushes for 20 metres, (*a*) how much work does the man do on the sledge ? (*b*) How much work does the force of gravitation do on the sledge ? How much work is done by friction on the sledge ?

Solution.

(*a*) The man provides work on the sledge since the component of his push is in the direction of the displacement. The component of the man's push up the incline actually does the work. This is given by :

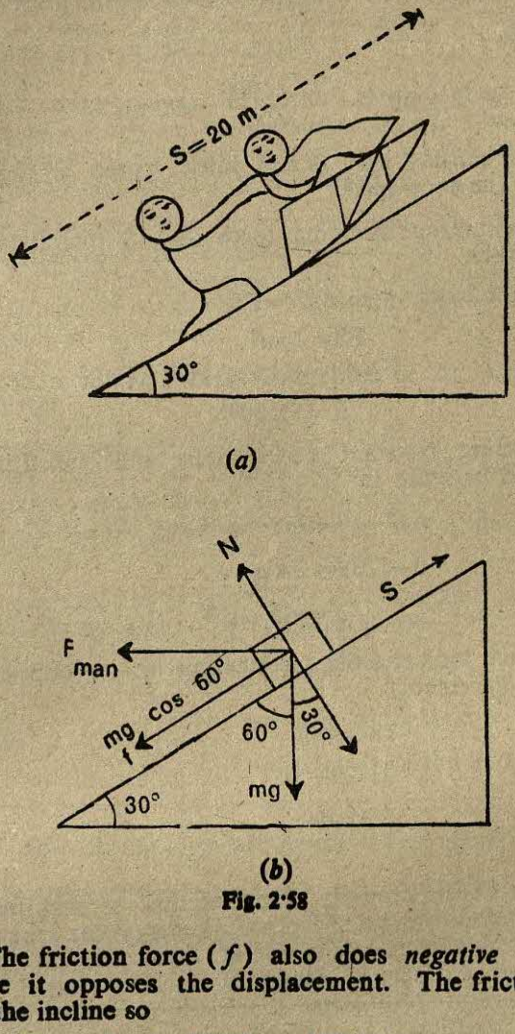
$$W = F \cos \theta\ S = +(450\text{N})\ (\cos 30)\ (20\text{ m}) = +7794 \text{ joules.}$$

(*b*) Gravity opposes the displacement ; hence, the gravity or weight force, of the boy and sledge does *negative* work on the sledge. The weight force makes an angle of 60° with the plane, we obtain the gravitational work

$$W = (F \cos \theta)S = -(mg \cos 60°) \times 20 \text{ m}$$

$$= -(36 \times 9{\cdot}8 \times \tfrac{1}{2})20 = -3528 \text{ joules.}$$

(We could also have computed the work by taking the product of the component of the weight along the plane ($-mg \sin 30°$) and the displacement (20 m) and product is the same).



(a)

(b)

Fig. 2·58

(c) The friction force (f) also does *negative* work on the sledge since it opposes the displacement. The friction force (f) lies along the incline so

$$W = -fs = (-5)(20) = -100 \text{ joules.}$$

This is equivalent to computing $W = fs \cos \theta$ where $\theta = 180°$ or $\cos \theta = -1$.

The total work done$=7794-3528-100=4166$ joules.

The fact that we have associated plus and minus signs with work does not mean that work is a vector. It is not! Whether or not the work is negative or positive is related to whether or not the force responsible for the work opposes the motion.

Example 2·21. A car of mass $1{\cdot}0\times10^3$ kg., travelling at 72 kmh^{-1} on a horizontal road is brought to rest in a distance of 40 *m* by the action of brakes and frictional forces. Find (*a*) the average stopping force, (*b*) the time taken to stop the car.

Solution.

$$\text{A speed of } 72\ \text{kmh}^{-1}=\frac{72\times1000}{3600}=20\ \text{ms}^{-1}$$

(*a*) If the car has mass *m* and initial speed *u*, then kinetic energy lost by the car $=\frac{1}{2}mu^2$

If F is the average stopping force and S the distance over which it acts, then

Work done by car against F=FS

$$\text{But} \qquad FS=\tfrac{1}{2}mu^2$$

$$\therefore \qquad F\times40=\tfrac{1}{2}(1{\cdot}0\times1000)\times(20)^2$$

$$\therefore \qquad F=5\times10^3\text{N}$$

(*b*) Assuming constant acceleration and substituting $v=0$, $u=20\ \text{ms}^{-1}$ and $S=40$ m in

$$v^2-u^2=2aS, \text{ we have}$$

$$0-(20)^2=2\times a\times40$$

$$\therefore \qquad a=-5\ \text{ms}^{-2}$$

(the negative sign indicates the acceleration in the opposite direction to the displacement)

$$\text{Using} \qquad v=u+at$$

$$0=20-5t$$

$$\therefore \qquad t=4\ \text{sec.}$$

2·24 ENERGY

Energy is a basic property of matter that is manifested in a bewildering variety of forms. Mechanical energy, for example, is a form possessed by objects as a result of their position, distortion or motion. Heat is a form of energy usually associated with the increased temperature of objects. Electricity is a form of energy, and a chemical reaction is a source of energy. All forms of radiation, such as sound, light, radar, radio waves, X-rays, and γ-rays, are types of energy. And the ultimate form of energy, as we know from Einstein's special theory of Relativity, is mass itself.

Energy is such a critical concept in physics and in its applications to engineering, chemistry, biology, geology, etc., that we shall now explore at some length its many facets. The most important characteristic of energy is the fact that it is a conserved quantity. To gain a preliminary understanding of this "principle of conservation of energy", we shall restrict-ourselves to one small part of the energy picture—*mechanical energy*

The capacity of a body for doing mechanical work is known as its mechanical energy. *It is measured by the total work the body can do under the circumstances. (position, configuration or motion) in which it is placed.*

Since energy means the amount of work that a body can do, **the units of energy are the same as those of work.** Therefore, *joule and erg* which are units of work in the SI system and *c.g.s.* system respectively are also units of energy.

The falling water at Niagara or Bhakra-Nangal Dam is used to drive dynamos which generate electricity. Hence the elevated water of the falls has got energy. A wound spring moves the hands of a watch, and so it has energy. Wind has energy, for work is done by it when it drives a boat.

Mechanical energy may have either of the two forms, *Kinetic and Potential.*

Kinetic energy. (*Energy of movement*) : A body in motion has energy due to its motion alone. Such energy is known as *kinetic energy* and is measured by the amount of work the body can perform against external impressed forces before its motion is stopped.

As explained in article (2·22) earlier in this unit, the kinetic energy of a body is given by equation (2-49)

i.e., Kinetic Energy$=\frac{1}{2}mv^2$

where $m=$mass of the body.

and $v=$Velocity of the body.

or $$\boxed{\textbf{Kinetic Energy}=\tfrac{1}{2}(\textbf{mass})\times(\textbf{Velocity})^2} \quad ...(2\text{-}50)$$

Equation (2-50) tells us that the work done by a force on an object causes the kinetic energy of that object to change. *Kinetic energy and work are not identical quantities, they are related concepts,* linked by the fact that the imposition of one—work—on any object causes a change in the magnitude of the other—kinetic energy. This link demands that energy is measured in exactly the same units as work. In addition, the two concepts are both *non-directional, scalar quantities*. The fact that energy is a scalar quantity makes it much easier to deal with than, say, forces.

The equation (2-50) also shows that kinetic energy of an object increases if positive work is performed on the object and decreases if the work is negative.

The bullet fired from a rifle, the rotating flywheel of an engine, a falling body, a swinging pendulum, a common ball in motion have all got kinetic energy. Electrons are the smallest particles present in ordinary matter. Electrons constitute a beam in a television picture tube, whose impacts on the tube screen produce tiny flashes of light. Such electrons have high speeds and hence possess large kinetic energy.

Potential energy. (*Energy of Position*). Work can also alter the energy of objects without affecting their speed. Imagine that you pick up an object, say a book, and lift it, very, very slowly without giving it any acceleration. In order to make it move vertically upward, you must apply a force equal to the weight of the book. You must, in other words, do work on the book. This work does not change the speed of the book, however ; for after you have lifted it, the book has zero speed, just as it did before. Rather, the work increases the height of the book, or, to put it more generally, it changes the book's separation from the earth.

We can, therefore, define a new type of energy, **the energy of position. Technically, this is known as potential energy.** With this definition in mind, we can expand our statement concerning the relationship between work and energy. **Work can now be said to change either the kinetic energy of an object or its potential energy or both forms of energy.** By defining as energy any quantity that results from the application of work, we have an extremely broad concept of what energy really is.

Returning to the problem of lifting a book, we see in Fig. 2.59 that the minimum force F required just to lift the book, equals its

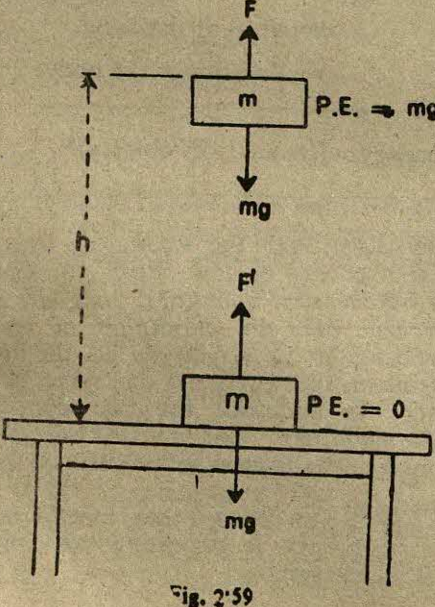


Fig. 2·59

weight mg. Application of any force greater than this will cause the book to accelerate upward and thus increase its speed. So, if we use just the minimum force and lift the book a distance '*h*', we will do an amount of work W on the book given by the equation.

$$\boxed{W = FS = (mg)h = mgh} \qquad \ldots(2\text{-}51)$$

Since this work has increased the gravitational energy of the book, we can write the change in gravitational potential energy (P.E.) as

Change in P.E. = mgh.

The quantity *mgh* represents the gravitational potential energy near the earth's surface. It is the book's energy of position and is the result of the presence of the gravitational force. In equation (2-51), the work done by our effort to lift the book produces a change in the potential energy, since *h* represents a change in the book's vertical position. We can arbitrarily assume the value of *h* to be *zero at its starting point* on the horizontal table and call the potential energy at height *h* above the table *mgh*. Thus

$$\boxed{\textbf{P.E. of a body} = \mathbf{mgh}} \qquad \ldots(2-52)$$

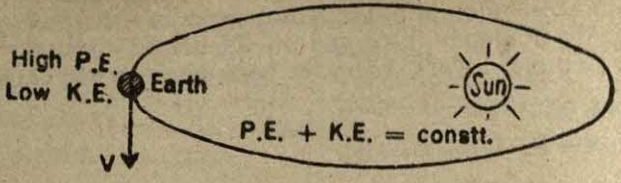
where m = mass of the body

g = Acceleration due to gravity

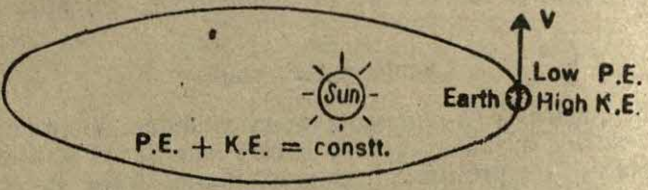
and h = vertical height through which the body is raised from the ground level or zero or starting position.

A lifted weight, like a *Pile-driver*, does work when falling down under the force of gravity. So, it has potential energy. Water stored up in elevated reservoirs in municipal water supply, formations of ice on a mountain top, are also similar instances of potential energy. A coiled spring as in the case of a watch or of a gramophone, a bent or compressed spring, compressed air, etc., have potential energy for, in coming back to the normal configuration (condition), each one of them can do work. The earth itself, for instance, has potential energy with respect to the sun (Fig. 2·60).

We are now in a position to investigate the combined effect of work on the kinetic and potential energies. If, instead of just lifting the book (Fig 2·59) by applying the minimum force, we give it an upward heave that accelerates it as well as lifting it to a new height, both the position and the speed of the book will change.



(*a*) High Potential Energy



(*b*) Low Potential Energy

Fig 2·60

Therefore work$=(\frac{1}{2}mv^2-\frac{1}{2}mu^2)+mgh$ where u is the initial velocity and v is the final velocity.

or $$\boxed{\text{Work}=\text{Change in K.E.}+\text{change in P.E.}} \qquad ...(2\text{-}53)$$

This equation (2-53) is often called **work-energy principle** and it is an extremely useful relation for solving a variety of problems involving mechanical energy. Work done on an object produces a change in the object's kinetic energy or its potential energy or both.

If we consider a situation in which no external forces other than gravity act on an object then no work can be done on the object therefore,

$$0=\text{Change in K.E.}+\text{Change in P.E.}$$

or $$\text{Change in K.E.}=-\text{Change in P.E.}$$

This means that any increase in K.E. is exactly equal to the decrease in P.E. and vice versa. In other words, the total mechanical energy of any system unchanged by external forces remains constant. What this statement means is that *mechanical energy is conserved. Its form may change, but its total amount does not.*

2·25 LAW OF CONSERVATION OF ENERGY

The above example is a specialized case, involving just mechanical energy, of an all embracing law of conservation of energy that applies to all the physical universe.. The law can be stated as ;

"The sum total of energy possessed by a material system remains constant. In the absence of any external force, energy can neither be created nor destroyed ; it can be transformed from one type to another with exact equivalence".

OR

"The total amount of energy in the universe or in any isolated system remains canstant."

The key word in the second statement is *isolated*. The requirement that the system be isolated provides a very important restriction to the law when we come to deal with practical problems. In the case of mechanical systems, isolation requires that no external force can act on the system. But in attacking practical problems, we must also ensure that no other type of energy gets into or out of the sysytem. When heat energy is involved, for example, as it generally is in practical problems, an isolated system is one that neither takes in nor emits heat.

The Law of Conservation or Energy states that the mechanical energy lost is not really destroyed but reappears in some other form or forms in equivalent quantity. Thus the loss of mechanical energy in the case of a body sliding down an inclined plane or that in the case of two bodies making a collision, is really not a loss, for an equivalent energy reappears in each case mostly as heat and partly as sound. When a falling body touches the ground, the mechanical energy is reduced to zero but is transformed in equivalent quantity mostly into heat and partly into sound. Thus, we find that whatever be the system of forces acting on a body, the total energy of the system will be found to remain constant, if we take into account all the different forms of energy to which energy is transformable namely, mechanical, thermal, magnetic, electrical, sound and light energies etc. Sometimes it becomes really difficult to trace out the different forms into which energy transforms itself and makes us doubt the validity of the law but when closely examined, it will be found that the situation arises not due to any defect in the universal character of the law but due to our nadequate knowledge of the transformations. Consider the various transformations of energy in the case of an ordinary steam engine connected to a dynamo for the generation of electricity. When the coal burns, we get *heat energy*. The heat does work in changing water to steam, which then expands. The expanding steam exerts force and causes the piston to move, and thus runs the engine. Thus heat energy is transformed into *mechanical energy*, and when the engine drives a dynamo, which generates electricity, the mechanical energy is converted into *electrical energy*. This energy can be transmitted by wires and made to do useful work such as driving trains where electrical energy is reconverted into *mechanical energy* ; lighting lamps in houses, where electrical energy is reconverted into *light energy* and in this way various other transformations may take place, but whatever are the transformations, the total energy of the whole system will be constant. Again the Hydro-electric power plants such as Bhakra-Nangal, Baira Siul, Srisailam etc., water is stored at a certain height in large reservoirs and thus it possesses potential energy. It is made to fall through a certain height, converting its potential energy into the kinetic energy of falling water. At the end of the fall, the water is made to turn a turbine which drives an electric generator, giving electrical energy which may disappear in one form and reappear in another.

The law of conservation of mechanical energy is best illustrated by a body falling freely under the action of gravity.

Body Falling Freely under Gravity

Let a body of mass m be raised from the ground level 0 to A at a height 'h' (Fig. 2.61). The work done on the body to raise it against the force of gravity is mgh.

This work is stored in the body as Potential Energy (P.E.). Let the body at A be at rest. At A,

Kinetic Energy (K.E.)$=0$

$$\text{P.E.}=mgh$$

$$\therefore \quad \text{K.E.}+\text{P.E.}=mgh \qquad \ldots(i)$$

Let the body now fall through a distance x and come to B. To obtain the velocity (v) at B, we have

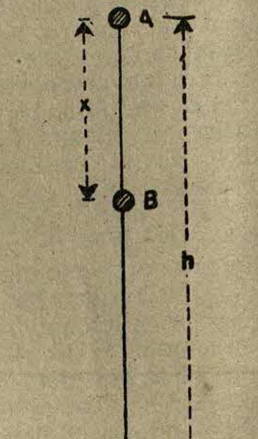


Fig 2·61

$$u=0$$

$$v=?$$

$$a=g$$

$$s=x$$

$$\because \quad v^2-u^2=2as$$

$$\therefore \quad v^2-0=2gx$$

$$\therefore \quad v^2=2gx$$

Hence at B,

$$\text{K.E.}=\tfrac{1}{2}mv^2=\tfrac{1}{2}m(2gx)=mgx$$

$$\text{P.E.}=mg(h-x)=mgh-mgx$$

$$\therefore \quad \text{K.E}+\text{P.E.}=mgx+(mgh-mgx)=mgh \qquad \ldots(ii)$$

On reaching O, let the velocity of the body be V, then

$$u=0 \quad v=V=?$$

$$a=g, \quad S=h$$

$$\therefore \quad V^2-O=2gh \qquad (\because \ v^2-u^2=2as)$$

Hence at O,

$$K.E.=\tfrac{1}{2}mV^2=mgh$$

$$P.E.=0$$

$$\therefore \ K.E.+P.E.=mgh \qquad \ldots(iii)$$

The equations (*i*), (*ii*) and (*iii*) show that the total energy of a body falling freely under gravity remains constant and is always equal to *mgh* which is independent of *x* or the position of the body during its fall. On striking the ground, the body finally comes to rest. The K.E. possessed by the body is thus lost, being converted into other forms.

Atomic bomb has, however, shown that mass can be transformed into energy in accordance with the following relation

$$\boxed{E=mc^2} \qquad \ldots (2\text{-}54)$$

where E = Energy equivalent of the mass

m = mass converted into energy

c = Speed of light $=3\times10^8$ ms^{-1}

This relation was established by Albert Einstein in 1905 in his Special Theory of Relativity. Thus mass, like heat, sound, light, etc., is also a form of energy. This relation is known as *mass-energy relation*. If m is measured in kg, c in ms^{-1}, E is then given in *joules*.

Suppose $\frac{1}{1000}$ kg of mass is converted into Energy. The amount of energy available will be

$$E=\frac{1}{1000}\times(3\times10^8)^2=9\times10^{13} \text{ joules}$$

which is a very large value.

The relation (2·54) shows that mass and energy are interconvertible. In this universe, most of the energy is in the form of mass energy. But we do not know how to convert mass completely into energy, though during nuclear reactions including fusion and fission reactions, a very small portion of mass is converted into energy. The energy which the sun continuously emits is derived as a result of conversion of a small quantity of mass into energy during the fusion of hydrogen nuclei taking place continuously in the sun. Almost the whole of our available energy comes from the sun. A vast store of energy can be made available from the matter in the universe.

Example 2·22. A bullet of mass 10 g travelling horizontally at a speed of $1{\cdot}0\times10^2$ ms^{-1} embeds itself in a block of wood of mass $9{\cdot}9\times10^2$ g suspended by strings so that it can swing freely. Find (*a*) the vertical height through which the block rises, (*b*) how much of the bullet's energy becomes internal energy ($g=9{\cdot}8$ ms^{-2}).

Solution.

(*a*) The bullet is brought to rest very quickly due to the resistance offered by the block and we shall assume that the block (with the bullet embedded), hardly moves until the bullet is at rest. Momentum is conserved in the collision and so

$$mu=(M+m)v$$

where m and M are the masses of the bullet and block respectively, u is the velocity of the bullet before impact and v is the velocity of the block and bullet as they move off.

$$\therefore \quad 10\times10^{-3}\times1{\cdot}0\times10^2=(990+10)\times10^{-3}\times v$$

$$\therefore \quad v=1{\cdot}0 \text{ ms}^{-1}$$

When the block has swung to its maximum height 'h', all its kinetic energy has become potential energy—if frictional forces are neglected. *Conservation of energy therefore holds* and we can say

$$\tfrac{1}{2}(M+m)v^2=(M+m)gh$$

$$\therefore \quad h=\frac{v^2}{2g}$$

or

$$h=\frac{(1)^2}{2\times9{\cdot}8}\simeq5\times10^{-2} \text{ m.}$$

(*b*) Original K.E. of the bullet

$$mu^2=\tfrac{1}{2}(10\times10^{-3})\times(1{\cdot}0\times10^2)^2=50 \text{ J}$$

[illegible] of block + bullet after impact

$$=\tfrac{1}{2}(M+m)v^2=\tfrac{1}{2}(1000\times10^{-3})\times(1)^2=0{\cdot}50 \text{ J}$$

Therefore, internal energy produced

=loss of K.E.

$=50-0{\cdot}50=49{\cdot}5$ J

2.26 POWER.

Like the term work, the term *power* has a scientific meaning that differs from its every day meaning. When we say a person has great power, we usually mean he has great strength or he wields great authority. In physics, the term **power means the rate of doing work.**

A man does the same amount of work whether he climbs a flight of stairs in one minute or in one hour, but he does not use the same amount of power. Power depends upon *three* factors :

(*i*) displacement of the object

(*ii*) the component of the force in the direction of the displacement, and

(*iii*) the time required.

Since power is the time rate of doing work,

$$\boxed{P=\frac{W}{t}} \qquad ...(2\text{-}55)$$

where P = Power

W = work done by an agent

t = time in seconds

Since $W=|\vec{F}|\,|\vec{S}|\cos\theta=\vec{F}\cdot\vec{S}$

where $\vec{F}$ is the force and $\vec{S}$ is the displacement and θ is the angle between $\vec{F}$ and $\vec{S}$.

$$\boxed{P=\frac{|\vec{F}|\,|\vec{S}|\cos\theta}{t}=\frac{\vec{F}\cdot\vec{S}}{t}=\vec{F}\cdot\vec{v}} \qquad ...(2\text{-}56)$$

where $\vec{v}$ is the velocity.

Units of Power.

The power of an agent is unit if it does unit work in unit time.

The unit of power in *SI* system is **joule per Second** (Js^{-1}) or **watt**. When work is measured in joules and time is measured in seconds, power is expressed in watts. *A watt is a joule per second.* This unit is named in honour of James Watt (1736–1819), who designed the first practical steam engine.

Since the watt is an inconveniently small unit, power is more commonly measured in units of 1000 watts, called **kilowatts (kW)**

$$\boxed{1 \text{ kilowatt (kW)} = 10^3 \text{ watts.}}$$

Because we have heard the terms watt and kilowatt used frequently in connection with electricity, we may think that these terms apply only to electric power. This definitely is not so. The watt is the basic unit of power in SI system and may be used to express quantities of mechanical as well as electric power.

An earlier unit, still in common use, is the **horse power,** which is a somewhat arbitrary estimate James Watt established for the power of an average horse. **One horse power (HP) is equal to 746 watts.** An average man can work at the rate of about $\frac{1}{8}$ HP or approximately 100 watts.

In the C.G.S. system, the unit of power is 1 *erg per second.*

$$\boxed{1 \text{ watt} = 1 \text{ J s}^{-1} = 10^7 \text{ ergs/sec.}}$$

The rating of electric lamps, fans etc., is generally given in watts. Thus a 100 watt lamp consumes electrical energy at the rate of 100 Js^{-1}. If this lamp is lighted for an hour, the energy consumed is $100 \times 60 \times 60$ joules. This is expressed in units of (power) × (time). In this case, the energy consumed is stated as 100 watt-hour. It should be noted that *watt-hour* is a unit for energy and not for power.

$$\boxed{\textbf{1 watt-hour} = \textbf{3600 joules}}$$

Another large unit of energy is called **kilowatt hour** (*KWH*) which is equal to *1000 watt-hour*.

Thus *1 KWH* = 1000 watt-hour

$$= 1000 \times 3600 \text{ joules} = 36 \times 10^5 \text{ joules.}$$

Example 2·23. A well 20 m deep and 3·5 m in diameter contains water to a depth of 10 *m*. A pumping engine empties it in one hour. Find the power of the engine.

Solution :

$$P = \frac{W}{t}$$

Now volume of water (V) $= \left(\frac{\pi d^2}{4}\right) \times h$

where *d* is the diameter of the well and *h* is the depth

$$(V) = \frac{22 \times (3·5)^2}{7 \times 4} \times 10$$

$$= \frac{192·5}{2} \text{ m}^3$$

Now $\quad \text{density } (\rho)=\dfrac{\text{Mass (M)}}{\text{Volume}}$

$$\therefore \quad \text{Mass (M)}=\rho\times V$$

$$=1000\times\frac{192{\cdot}5}{2}=\frac{192500}{2}\text{ kg}$$

Now the water is at a depth of 20 m and water is upto a depth of 10 m therefore, the water can be considered to be an average depth (h') of $\left(\dfrac{20+10}{2}\right)=15$ m from the top.

$$\therefore \quad \text{Work down (W)}=m\,gh'$$

and $\quad \text{Power}=\dfrac{W}{t}=\dfrac{mgh'}{t}$

$$=\frac{192500\times9{\cdot}8\times15}{3600\times2}$$

$$=\frac{94325}{2}\text{ watts}=\frac{94{\cdot}325}{2}=47{\cdot}162\text{ kW}$$

Example 2·24. A pump can hoist 10 000 kg of coal per hour from a coal mine 120 m deep. Calculate the power of the engine, assuming that the efficiency is 80%.

Solution.

$$\text{Mass}=m=10\,000\text{ kg}$$

$$\text{Depth}=h=120\text{ m}$$

$$g=9{\cdot}8\text{ ms}^{-2}$$

$$\text{Time}=t=1\text{ hr.}=3600\text{ s}$$

$$\text{Efficiency}=\eta=80\%$$

$$\text{Power}=P=\frac{W}{t}=\frac{mgh}{t}$$

$$=\frac{10\,000\times9{\cdot}8\times120}{3600}=\frac{9800}{3}\text{ watts}$$

$$\text{Now efficiency } (\eta)=\frac{\text{Output Power}}{\text{Input Power}}\times100\%$$

$$\therefore \quad \text{Input Power}=\frac{\text{Output Power}}{\eta}\times100$$

$$=\frac{9800}{3\times80}\times100$$

$$=4083{\cdot}33\text{ watts}=4{\cdot}08333\text{ kW}$$

Example 2·25. An elevator motor is used to lift an elevator and its load (total 1500 kg) to a height of 20 m. The time taken for the job is 20 s. What is the work done? What is the rate at which the work is done? If the efficiency of the motor is 75%, at what rate is the energy supplied to the motor?

Solution.

Force required to lift the elevator $= 1500 \times 9{\cdot}8 = 14700$ N

Height through which it is lifted $= 20$ m

$$\text{Work done} = 14700 \times 20 = 2{\cdot}94 \times 10^5 \text{ joules}$$

$$\text{Time taken} = 20 \text{ s}$$

$$\text{Rate of doing work} = \frac{2{\cdot}94 \times 10^5}{20} = 1{\cdot}47 \times 10^4 \text{ watts}$$

Efficiency (η) of the motor $= 75\%$

$$\text{But efficiency } (\eta) = \frac{\text{Output}}{\text{Input}} \times 100\%$$

$$\therefore \quad 75 = \frac{1{\cdot}47 \times 10^4}{\text{Input}} \times 100$$

$$\therefore \quad \text{Input} = \frac{1{\cdot}47 \times 10^4}{75} \times 100 = 1{\cdot}96 \times 10^4 \text{ watts}$$

which is the rate at which energy is being supplied.

2·27 FRICTION

We have already learnt that a body lying on a horizontal surface is in equilibrium under the action of two forces, the weight of the body acting vertically *downwards*, and the reaction of the surface, equal to the weight of the body, acting vertically *upwards*. As these forces have no component acting along the surface (being at right angles to the line of action of the forces), the body ought to move, if a force, however, small, acts along the surface. But, as a matter of fact, we know from experience that the body does not move unless a definite force is applied in that direction. The force with which we try to move the body is resisted by some other force which comes into play when we try to move the body. Thus, if there are two bodies in contact, any attempt to move one over the other will set up forces in between them which tend to oppose their relative motion. Such opposing forces are known as **friction forces.**

"**Friction may thus be defined as the temporary resistance (*opposing force*) offered to a body sliding on the surface of another body or moving in another medium.**"

2·28 ORIGIN OF FRICTION

The surfaces of bodies are never perfectly smooth. Even a highly polished surface when seen under a powerful microscope reveals dents and irregularities [Fig. 2·62] of the surface. When one body is placed upon a level surface of the same or different material, there is a certain degree of *interlocking* of these irregular projections on the two surfaces.

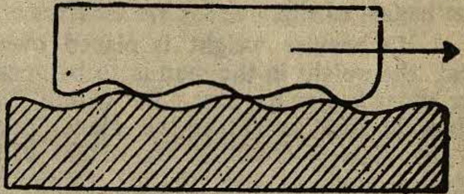


Fig. 2·62

When a force is applied to make a body move over the surface of another body, the motion is resisted due to the interlocking of the large number of irregularities and thus an opposing force develops which is the force of friction.

Thus friction is due t *roughness of the surfaces in contact* and arises on account of *adhesion i.e.*, the mutual force of attraction between the molecules of the two contacting surfaces. If the surfaces are highly polished, this force will be small, though it will never be equal to zero.

2.29 EXPERIMENTS ON FRICTION (*Nature of Friction*)

A wooden block S, known as the slider (Fig. 2·63), is attached to a string which passes over a pulley, carrying a scale pan at the end. The slider is placed on a smooth horizontal wooden plane *AC*. It will be seen that it does not move, though a force equal to the weight of the pan acts horizontally on it. This is due to the fact that the force of friction, which comes into operation between the two surfaces is quite sufficient to neutralise its effect, that is, the force of friction is *equal* and *opposite* to the weight of the pan.

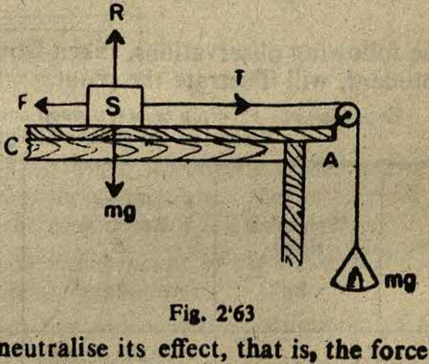


Fig. 2·63

Place a small weight in the pan. The slider is still found to be at rest. This shows that the frictional force has also increased proportionately. But, if the weights in the pan are gradually increased, a point will come when the slider will *just* begin to move. Evidently, up to this point, the frictional force between the surfaces goes on increasing proportionally with the weights placed in the scale pan and is quite sufficient to balance the weight of the pan and its contents. But when it reaches this point, it cannot increase any further. This *maximum or limiting value of friction* is known as the **limiting friction** and is obviously equal to the weight of the scale pan, together with the weights placed in it, when the slider *just* begins to slip.

If a heavier weight is placed over the slider it will be seen that the weight in the pan is to be increased considerably before the slider begins to slip. By careful experiments it is seen that when the total weight on the plane, and hence the *normal reaction* of the plane is doubled, the weight of the pan with its contents, which is equal to the limiting friction is also doubled. Thus, we see that *the limiting friction is proportional to the normal reaction.*

If the slider be now placed on its side instead of on its base, it will be seen that the same force will be required to move it as before so long as the weights placed on it remain the same. *The force of friction is, therefore, independent of the area of the surfaces in contact.* But if a slider or a plate of some other material be placed on the plane with the same weights on it (so that the normal reaction of the plane remains the same as before), the limiting friction will be altogether different. Thus, friction depends upon the *nature* of the surfaces in contact.

The ratio between the limiting value of the friction and the normal reaction for a given pair of surfaces is constant, known as the **coefficient of friction,** and is generally represented by the letter μ. Thus, if F be the *limiting friction*, and R, the *normal reaction*, we have

$$\frac{F}{R}=\mu, \text{ whence } \boxed{F=\mu R.} \qquad \ldots(2\text{-}57)$$

The following observations, taken from the laboratory note-book of a student, will illustrate the point.

Experiment—*Friction between wood and wood.*
Weight of the slider=100 gm=0·1 kg

No.	Weight on slider kg	Normal Reaction R kg	Weight of hanger + Weight on it F kg	$\mu=\frac{F}{R}$
1	0·180	·180+·1=·280	0·077	·274
2	0·850	0·850+·1=0·950	0·258	·272
3	1·150	1·150+0·1=1·250	0·346	·277
	Mean			·275

The value of the coefficient of friction varies with the nature and the actual state of the surfaces. For wood upon wood, when smooth, it is between ·25 and ·5 ; for two polished metal surfaces, it is about ·2, but when the surfaces are lubricated, the value may be reduced to half of this.

Thus, if a wooden block weighing 200 g is resting on a wooden surface, the force required to put the block into motion would be equal to 200 × ·5 = 100 g weight, if the coefficient of friction between the surfaces is ·5. Similarly, if a railway train of weight W is at rest on a railway track, then the force which is necessary to start the train must be greater than ·2 × W, where ·2 is the coefficient of friction between the wheels and the track, and if the wheels are lubricated, then a force of one-half this magnitude would be sufficient. All these experimental results can be summarised into laws, known as the *laws of friction.*

2·30 LAWS OF FRICTION

1. *The direction of the force of friction is always opposite to that in which motion takes place or tends to take place.*

2. *The force of friction is always equal and opposite to the applied force, before the motion takes place, but for a given pair of surfaces, it has a maximum value, known as the* **limiting friction.**

If, however, the value of the force applied is greater than this limiting friction, the motion will ensue.

Thus, if P is the applied force acting on a mass m, and F, the limiting friction, we have

$$\boxed{\textit{acceleration},\ a=\frac{P-F}{m}} \qquad P>F$$

3. *The limiting friction between two given surfaces is proportional to the normal reaction between them, that is,* $F \propto R$, *the quotient* F/R *being known as the coefficient of friction.*

4. *The limiting friction is independent of the areas of the surfaces in contact but depends upon their nature.*

The following table gives the value of the coefficients of friction for some important pairs of substances :

1.	Wood and wood	0·25 to 0·5
2.	,, ,, metal	0·5 to 0·6
3.	Metals ,, metals	0·15 to 0·20
4.	,, ,, stone	0·5

2·31 STATIC AND DYNAMIC FRICTION

The force required to make a body just slide over the surface of another, measures the *static* or *starting* friction, whereas the force necessary to maintain a body in steady motion over the surface of another body, after the motion has started, measures the *dynamic* or *sliding* or ***kinetic*** friction between the two surfaces, the former being always greater than the latter.

Further, *the dynamic friction is independent of the velocity of the body, provided it is neither too great nor too small.* Evidently then, there is a *second* coefficient of friction, corresponding to the dynamic or kinetic friction, called the *coefficient of dynamic* or *kinetic friction.* It is given by the relation

$$\boxed{\mu_k = \frac{F_k}{R},}$$

where μ_k is the *coefficient of kinetic friction* and F_k, the *frictional force called into play when the bodies are moving.*

The following table gives an idea of the differences between the dynamic (or kinetic and) the static frictions for some pairs of surfaces. *These are only typical values.* They *vary greatly* depending upon the condition of surfaces, grain of wood, contamination, moisture etc.

Material	*Coefficient of Friction*	
	Kinetic (μ_k)	*Static* (μ_s)
Wood on wood (dry)	0·4	0·7
Steel on steel (dry)	0·4	0·5
Steel on steel (oiled)	0·03	0·1
Metal on leather (dry)	0·5	0·6
Wood on leather (dry)	0·4	0·5
Rubber on concrete (dry)	0·7	0·9
Rubber on concrete (wet)	0·6	0·7
Waxed wood on snow	0·05	—
Metal on ice	0·02	0·4

2·32 STABILITY ON A ROUGH INCLINED PLANE

A body of mass *m* when placed on an inclined plane is acted upon by a force *mg* sin θ, causing the body to move down the plane; therefore we expect that when that plane is slightly tilted, the body must move down. But, we actually find that the body does not slide down until the plane is considerably tilted.

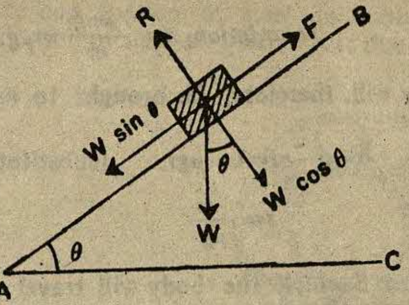


Fig 2·64

Let us suppose that a body of weight W is resting on a plane, which is slowly tilted till the body *just* begins to slip, and let θ be the angle that the plane *now* makes with the horizontal. Then the forces acting on the body are (*i*) *the weight W of the body*, (*ii*) *the reaction R of the plane, and* (*iii*) *the limiting friction F between the two surfaces*. The weight W can be resolved into two components, along and perpendicular to the plane. The component along the plane is W sin θ and this tends to move the body down, while the other component at right angles to the plane, is W cos θ. Since the body is in equilibrium under these forces, clearly,

$$F = W \sin\theta \text{ and } R = W \cos\theta.$$

$$\therefore \text{ Coefficient of friction, } \mu = \frac{F}{R} = \frac{W \sin\theta}{W \cos\theta} = \tan\theta.$$

or

$$\boxed{\mu = \tan\theta} \qquad \dots (2\text{-}58)$$

This angle θ at which the slip is just on the point of commencing is known as the **angle of friction**, and *the coefficient of friction is therefore; equal to tangent of the angle of friction.* The angle of friction is found to be independent of the areas of the surfaces in contact but depends on the nature of the surfaces in contact.

2·33 EFFECT OF FRICTION ON MOTION OF BODIES

As friction always acts in a direction opposite to that of the motion, it acts as a *retarding force* producing negative acceleration. Thus, if a body of mass *m* is moving along a horizontal surface with a velocity *u*, it will be continuously retarded by a force of friction and therefore, ultimately it will come to rest. In this case, the normal reaction of the plane is mg and if μ is the coefficient of dynamic friction, the limiting friction is μmg, and, therefore.

$$\text{retardation, } a=\frac{\mu mg}{m}=\mu g.$$

The body will, therefore, be brought to rest in time t given by the equation

$$0=u-at=u-\mu gt, \qquad \text{[substituting the value of } a\text{]}$$

whence,

$$t=\frac{u}{\mu g}$$

And the distance S which the body will travel during the interval is given by the equation

$$0^2-u^2=-2(\mu g)S \text{ or } u^2=2(\mu g)S$$

or,

$$\boxed{S=\frac{u^2}{2\mu g}}$$

Again, (*i*) if the body is dragged over the horizontal surface through a distance S, then the work done against the force of friction is equal to $F\times S=\mu mg\times S$. *This work is irrecoverable since it is converted into heat and, therefore, it is lost to us for all mechanical purposes.*

(*ii*) If the body is moving down an inclined plane, the force which is tending to move it down in $mg \sin\theta$ and the force of friction which is opposing its motion is $\mu mg\cos\theta$. The resultant force acting on the body is, therefore, $mg\sin\theta-\mu mg\cos\theta$, and the acceleration '$a$' produced will, therefore, be

$$\boxed{a=\frac{mg\sin\theta-\mu mg\cos\theta}{m}=g\sin\theta-\mu g\cos\theta}$$

(*iii*) If however, the body is dragged up an inclined plane, then work has to be done both, against gravity and also against friction. If h be the height of the inclined plane and l its length, the work done against gravity is mgh, and the work done, in overcoming friction is $\mu mgl\cos\theta$. Hence the total work done, in this case is equal to $mgh+\mu mgl\cos\theta$.

2·34 ROLLING FRICTION

When a spherical or a cylindrical body rolls over a plane surface, a certain amount of friction is offered to its motion. This is called *rolling friction and the corresponding coefficient is called coefficient of rolling friction* (μ_r). It is due to the deformation of the body on the surface, or *vice versa* (Fig. 2·65). It is very much less than the sliding friction, for the same pressure between the bodies. Thus, the rolling friction of steel on rubber is about $\frac{1}{100}$th of the sliding friction of steel on steel. It is due to this that

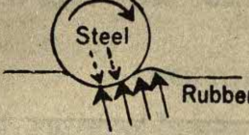


Fig. 2·65

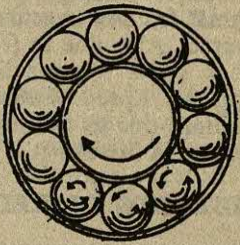


Fig. 2·66

wheeled carriages are used on roads instead of sledges and that ball bearings, (Fig. 2·66), are used in bicycles etc.

2·35 FLUID FRICTION

When a solid body moves slowly through a liquid, the layer of liquid in contact with the solid body also moves along with it. Layers of liquid away from the body move also, but with a smaller velocity. From this it is evident that when a solid body moves in a liquid, a certain amount of kinetic energy is acquired by the liquid, which is to be supplied by whatever keeps the solid body in motion. It thus follows that resistance is offered by fluids, but this resistance is of an entirely different nature and is called **viscocity** of the liquid. Viscous resistance depends upon the velocity of the solid body and varies as the square of the velocity of the moving body, *e.g.*, when the velocity is doubled, the resistance becomes four times.

2·36 IMPORTANCE OF FRICTION

It is evident from the above that friction plays a very important part in our daily lives. All bodies that move on the ground do so without slipping on account of it. When this force becomes less, the bodies are likely to slip. This explains why a cyclist is more likely to slip on a muddy road than on a dry road and a person is more likely to slip on ice or a polished floor than on a rough surface. It is on account of friction that stones or objects can be piled one upon the other, without falling. Again, transmission of power by belts would be impossible without friction. Sand paper, grinding stones, etc., also depend upon friction for their action. Further, since rolling friction is less than sliding friction, we can understand the use of sledges (wheelless carriages) in arctic regions and the use of wheeled carriages in other places. This also explains why sand is spread over tarred roads in wet weather, why a cricketer uses shoes with spikes and why it is more difficult to walk on sand than on hard ground. But it must be remembered that the check of motion by friction is often a disadvantage, for it results in a lot of wastage of work in machines. While moving any part of a machinery on another part, force must be exerted not only to move the load but also to over-come friction. This work done against friction is generally converted into heat and it cannot be recovered. Also due to friction there is wear and tear between the various parts of a machine and hence the rubbed surfaces have to be repaired. Speed of automobiles cannot be increased beyond a certain limit due to friction. Thus we find that though friction is a *necessity* in life but it is an *evil* as well.

2·37 METHODS OF REDUCING FRICTION

As explained above, friction can be reduced by having highly polished surfaces and wherever possible by substituting rolling friction for sliding friction. For the same reason ball bearings are used at the axle of the wheel. In actual practice, the wheel is attached to the axle whereas the bearing is made a part of the frame of the car, and consequently the axle rolls on the balls and the balls roll on the bearings.

To still further reduce friction, *grease* or *an oil lubricant* is used. This holds the surfaces apart so that inequalities or projections do not interlock tightly and the rubbing takes place between the layers of the lubricants instead of between the solid surfaces. In order that the lubricants may not be squeezed out of the bearings, it is necessary that in heavy machinery thick oil must be used, whereas in light machinery, like watches etc., thin oil is used.

When a body moves in air with small speeds, it is the mechanical friction that matters most ; for the friction due to the resistance of air is small. At high speeds, on the other hand, it is the air resistance that is predominant. This explains why the shape of fast moving bodies such as railway trains are streamlined.

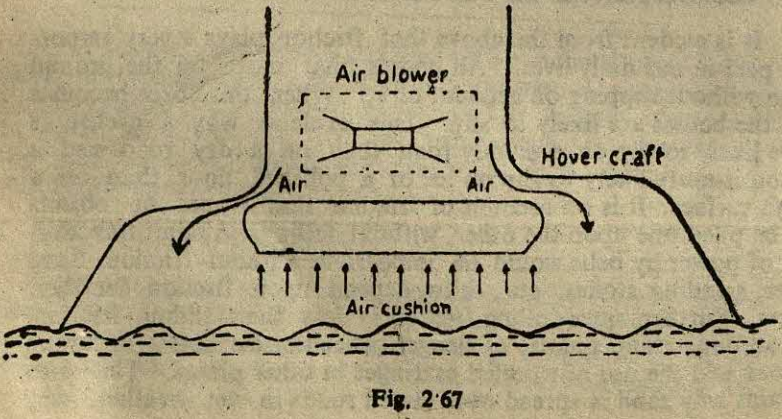


Fig. 2·67

The use of air as a **lubricant** has great potentialities. Purified air is compressed and employed as a lubricant. It forms an elastic cushion between the moving parts and eliminates the problem of friction and heating. The outflow of air from the machine also prevents the entry of dust into the machine. The principle of floating a vehicle on a cushion of air from a compressor to prevent ground friction is used for efficient locomotion in *Hoover Craft* (Fig. 2·67).

Metals of low coefficient of friction (**Anti-friction metals**) are used for lining the moving parts of the machinery to reduce friction. When steel slides over an alloy, such as bronze or brass, the coefficient of friction is much less than when steel slides over steel.

2·38 TERMINAL VELOCITY

When a body falls under the action of gravity in air, its velocity depends on its weight and the resistance of the air, which in turn, depends on the area of the surface of the body. This at once explains why a small bird can fall through a great height without being injured, whereas a man may be killed in the same fall ; for, proportionately, the bird offers much greater resistance to the air. The air resistance increases with the velocity with the result that as the velocity of the falling body increases, the air resistance also increases till a stage is reached when the air resistance becomes equal to the weight of the body. The body then begins to move down with a uniform velocity. This velocity is called the **terminal velocity**. This terminal velocity of a man is about 193 km/hour but with a parachute it is only 32·2 km/hour which corresponds to a jump from a roof, less than 4·8 m high. The terminal velocity of snowfall is only a few metre per second.

Example 2·26. A bullet of mass 0·01 kg is fired horizontally into a 4 kg wooden block at rest on a horizontal surface. The coefficient of kinetic friction between the block and the surface is 0·25. The bullet mains embedded in the block and the combination moves 20 m before coming to rest. With what speed did the bullet strike the block ?

Solution.

Mass of the bullet $=0·01$ kg

Let u be the initial velocity of the bullet

$\therefore$ Kinetic energy of the bullet $=\frac{1}{2}(·01)\ u^2$ *joules*

Coefficient of kinetic friction $=\mu_k=0·25$

Normal Reaction $=R=$ Weight $=4\times 9·8$ N

Now $\mu_k=\dfrac{F}{R}$ where F = Frictional force.

$$\therefore \qquad F=\mu_k R$$
$$=0{\cdot}25\times4\times9{\cdot}8$$
$$=9{\cdot}8\text{N}$$

Since the final velocity of the block=0 after covering 20 m, so change in Kinetic Energy $=\frac{1}{2}(0{\cdot}01)u^2$ joules.

But change in K.E. is equal to work done by the frictional force against the block.

$$\therefore \quad \tfrac{1}{2}(0.01)u^2=F\times S=9{\cdot}8\times20$$

or $$u=197{\cdot}96 \text{ ms}^{-1}$$

Example 2·27. A truck tows a trailer of mass 1200 kg at a speed of 10 ms^{-1} on a level road. The tension in the coupling is 1000 N. What is the power expended on the trailer? Find the tension in the coupling when the truck ascends a road having an inclination of 1 in 6. Assume that the frictional resistance on the incline is the same as that on the level road.

Solution.

Force applied by the truck=1000 N

Distance moved per second=10 m

∴ Work done per second or power expended

$$=1000\times10=10^4 \text{ Js}^{-1}$$
$$=10^4 \text{ watts}$$

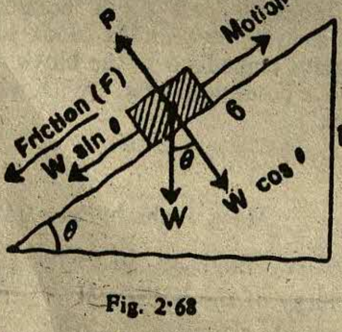


Fig. 2·68

When the truck is ascending a road having an inclination of 1 in 6, it has not only to apply a force of 1000 N to overcome friction (F) but has also to balance the component of the weight (W) of the trailor down the inclined plane.

Now $$\sin\theta=1/6$$

$\therefore$ Total force applied $= F + W \sin \theta$

$$= 1000 + 1200 \times 9{\cdot}8 \times 1/6$$

$$= 1000 + 1960 = 2960\ N$$

Example 2·28. Find the distance travelled by a body before coming to rest if it is moving with a speed of 10 ms^{-1} and the coefficient of friction between the ground and the body is 0·4.

Solution.

Here $\mu = \dfrac{F}{R}$

$\therefore$ $F = \mu R$

But $F = ma$ where a = Retardation

and $R = mg$

$\therefore$ $ma = mg\,\mu$

or $a = \mu g = 0{\cdot}4 \times 9{\cdot}8 = 3{\cdot}92\ ms^{-2}$

Hence $v^2 - u^2 = 2as$

$0 - (10)^2 = 2(-3{\cdot}92) \times 5$ ($\because$ a is retardation)

$\therefore$ $S = 12{\cdot}75$ m

SUMMARY

1. *Scalars* are quantities that are independent of direction. *Vectors* depend on direction. The vector equivalent of distance is called *displacement*.
2. Vectors not in a straight line are added and substracted using simple trigonometry. In *vector addition*, the tail of each successive vector is placed at the head of the previous one, with their lengths and original directions kept unchanged. The sum is vector drawn from the tail of the first vector to the head of the last.
3. A vector can be resolved into two or more other vectors whose sum is equal to the original vector. The new vectors are called the *components* of the original vector and are normally chosen to be perpendicular to one another.
4. Speed is a scalar quantity, the ratio of the distance an object travels to the time of travel. Velocity is a vector, the ratio of an objects' displacement to the time interval during which the displacement occurs.
5. The average velocity is the slope of the line connecting the starting and finishing points on a graph of displacement against time. The instantaneous velocity at any point is the tangent to the curve of displacement against time at that point.
6. Acceleration is the rate at which velocity changes An important example of constant acceleration is free fall, the motion of any object under the influence of gravity alone.
7. Force is that agency which produces or tends to produce ; destroys or tends to destroy motion in a body.
8. Objects continue to move at constant speed, according to Newton's First Law of Motion, unless a net force is applied to them. The tendency to resist a change in motion is known as inertia.

9. Application of any net force accelerates an object ; Newton's Second Law states that the amount of acceleration is proportional to the strength of the force.

10. The proportionality factor between force and the acceleration it causes in any object is the object's *inertial mass*. This quantity indicates the object's inertia. It also gives a measure of the amount of material inside the object.

11. Weight is the force with which an object is attracted towards the earth.

12. For every action, there is an equal and opposite reaction. This is Newton's Third Law.

13. Projectiles are real-life examples of free fall motion.

14. The momentum of any object is the product of object's mass and velocity.

15. The impulse of a force is the product of the force and the time during which it acts.

16. The change of momentum produced by any force on an object equals the impulse of that force.

17. In *elastic* collisions, the total Kinetic Energy is conserved. In *inelastic* collisions, at least some of the kinetic energy is lost as other forms of energy, such as heat and sound. Momentum is conserved in all types of collision, however.

18. Work is the product of the displacement of an object and the component of the force that is moving the object in the direction of that displacement.

19. Negative work occurs when a force acts in the direction opposite to that of the object's movement. Friction is an example of a force that always does negative work.

20. Kinetic energy is the energy that any object possesses by virtue of its motion. Its numerical value is half the product of the object's mass and the square of its velocity.

21. Potential energy is the energy that objects possess as a result of their positions. The gravitational potential energy of an object is the product of its mass, its height above an *appropriate reference level*, and the local acceleration due to gravity.

22. The effect of work on any object is to alter the object's Kinetic energy, or its potential energy or both.

23. The total amount of energy in any isolated system remains constant whatever happens to the system. But energy can and does change from one form to another, notably from K.E. to P.E. and *vice versa*.

24. Frictional forces always oppose motion. The amount of frictional force between any two surfaces depends on the condition and type of those surfaces and on whether or not they are moving relative to one another.

25. The coefficient of friction is the constant of proportionality for a given pair of contacting surfaces that relates the frictional force between them to the normal force holding them together.

QUESTIONS

1. Distinguish between Scalar and Vector quantities. How is a vector quantity represented ?

2. Explain the meaning of equal vectors and the negative of a vector.

3. A vector has zero magnitude. Is it necessary to specify its direction ?

4. Can a vector have zero magnitude if one of its components is not zero ?

5. Several vectors, which are not all in the same plane, add together to give a zero resultant. What is the minimum number of vectors which can satisfy this requirement ?

6. Ten vectors add together to give a zero resultant. Is it possible that nine of these vectors are in the same plane but that the tenth is not in this plane ?

7. Two vectors $\vec{P}$ and $\vec{Q}$ are added. Show that the magnitude of the resultant cannot be greater than $|P+Q|$ or smaller than $|P-Q|$, where the vertical bars signify absolute value

8. What are the properties of two vectors $\vec{A}$ and $\vec{B}$ such that

(*a*) $\vec{A}+\vec{B}=\vec{C}$ and $A+B=C$

(*b*) $\vec{A}+\vec{B}=\vec{A}-\vec{B}$

9. Two vectors $\vec{P}$ and $\vec{Q}$ make an angle θ with each other when placed tail to tail. Prove that the length of the resultant vector is

$$|\vec{R}|=\sqrt{|\vec{P}|^2+|\vec{Q}|^2+2|\vec{P}|\,|\vec{Q}|\cos\theta}$$

10. **Define scalar product of two vectors and vector product of two vectors.** Show for any vector $\vec{a}$ that

$$\vec{a}\cdot\vec{a}=|a|^2$$

and $$\vec{a}\times\vec{a}=0.$$

11. If $\vec{A}\times\vec{B}=\vec{C}\times\vec{B}$ show that $\vec{C}$ need not be equal to $\vec{A}$. When will $\vec{A}$ be equal to $\vec{C}$?

12 Justify with an example that Commutative law does *not* apply to vector products, that is ; $\vec{a}\times\vec{b}$ does not equal $\vec{b}\times\vec{a}$. Show that the commutative law does apply to scalar products, that is ; $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ [*A.I.S.S.C. 1986*]

[**Hints** : Example of a right hand screw]

13. Show that the distributive law applies to both scalar products and vector products, that is, show that

$$\vec{a}\cdot(\vec{b}+\vec{c})=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c} \text{ and that}$$

$$\vec{a}\times(\vec{b}+\vec{c})=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}.$$

14. What do you mean by the resolution of a vector into its components ? Resolve a given vector into its rectangular components.

15. What do you mean by velocity, uniform velocity, average velocity, instantaneous velocity ; acceleration, uniform acceleration and instantaneous acceleration.

16. Derive the equations

(*i*) $v=u+at$

(*ii*) $S=ut+\frac{1}{2}at^2$ and

(*iii*) $2as=v^2-u^2$ Using integral calculus.

17. Average speed can mean the magnitude of the average velocity vector. Another meaning given to it is that average speed is the total length of path travelled divided by the elapsed time. Are these meanings different ? If so, give an example.

18. Does the speedometer on an automobile register speed ? Can a body have zero velocity and still be accelerating ? Can a body have a constant speed and still have a varying velocity ? Can a body have a constant velocity and still have a varying speed ? Give a simple example of each. [*A.I.S.S.C. 1986*]

19. Consider a ball thrown vertically up. Taking air resistance into account, would you expect the time during which the ball rises to be longer or shorter than the time during which it falls ?

20. Can there be motion in two dimensions with an acceleration in only one dimension ?

21. State Newton's First Law of Motion. Why is it also called Law of Inertia ? Illustrate your answer by a few examples. How does Newton's First Law of Motion gives the definition of force ?

22. State Newton's Second Law of Motion. How does it help to measure force ? Define the units of force in the SI and C.G.S. systems and find the relationship between them.

23. State the law of conservation of linear momentum. Prove it mathematically and give a few of its illustrations.

24. State Newton's Third Law of Motion. How do you derive it from the Law of Conservation of Linear Momentum ? Give some examples to illustrate this law.

25. Distinguish between Inertial mass and Gravitational mass of a body. How do you compare the inertial mass of two bodies ? What does a spring balance measure ?

26. Define Impulse of a force. Show that Impulse of a force is equal to the total change in momentum of a body.

27. What do you mean by a collision ? What are its various types ? Discuss the elastic collision between two bodies along one dimension and derive expressions for the velocities after the collision. Discuss the special cases when (*i*) masses of the bodies are equal, (*ii*) the body struck against is at rest and the masses of the bodies are equal.

28. What do you mean by a projectile ? Show that a body is projected horizontally, its path is a parabola. Also derive an expression for the magnitude and direction of its velocity at any instant of its motion.

29. A projectile is fired at a certain angle with the horizontal

    (*a*) Show that its trajectory is a parabola

    (*b*) Also derive expressions for (*i*) the maximum height attained, (*ii*) the time of its flight, and (*iii*) the horizontal range.

    (*c*) At what angle is the horizontal range maximum ?

    (*d*) Also prove that for a given velocity of projection, there are two directions equally inclined to that horizontal and vertical for which the range is maximum.

30. What do you mean by work ? How is it measured ? Discuss the special cases when the angle between force and displacement is (*a*) 0°, (*b*) 90°. What are the units of work in C.G.S. and S.I. units ?

31. Prove that the work done by a constant force in displacing a body through a certain distance measures the change in the Kinetic Energy of the body.

32. What do you mean by the Law of Conservation of Energy ? Prove the conservation of mechanical energy by taking the case of a (*i*) Freely falling body (*ii*) Simple Pendulum (*iii*) Projectile.

33. What do you mean by power ? Give its units in C.G.S. and SI systems. Also explain the meaning of kilowatt hour.

34. What do you mean by friction ? What is its origin ? Define Static friction; Limiting friction: Kinetic friction and Rolling friction.

35. What are the laws of Limiting friction ? How will you verify them experimentally.

36. "Friction is a necessary evil." Explain.

37. How will you reduce friction ?

38. Derive an expression for the work done against friction when body is made to move. (*i*) over a level surface. (*ii*) up an inclined plane.

39. What are the factors on which the coefficient of friction between two surfaces depends ? How does lubrication help to minimise friction ?

40. When a tennis ball strikes your head, why are you aware of the total mass of the ball, rather than just the mass of a few atoms in contact with your skin ?

41. How does the momentum of a Satellite in a circular orbit vary with time ?

42. A firework is projected vertically upward. At its highest position it explodes into three pieces with equal masses and equal speeds. What can be said about the direction of their velocities.

43. How does the law of conservation of momentum apply to a car accelerating along a straight horizontal road ?

44. Can you imagine circumstances under which (*i*) the total momentum of a system of bodies is conserved, but its total energy is not ? (*ii*) the total energy of a system of bodies is conserved, but its total momentum is not ?

45. If the total energy of a system of bodies remains constant, is the resultant external force necessarily zero ?

46. A boy slides down a slope and ends up in a snow bank. Where did his kinetic energy come from and where does it go to ?

47. If we identify one force acting on an object and yet the object is at rest, what can we conclude ?

48. You are in the middle of a pond standing with a bag of groceries on the ice which is so slippery that no frictional force exists. How can you get off to the shore ?

49. Acceleration is defined as the rate of change of velocity. Suppose we call the rate of change of acceleration SLAP :
(*i*) What is the unit of SLAP ?
(*ii*) How can we calculate instantaneous SLAP ?

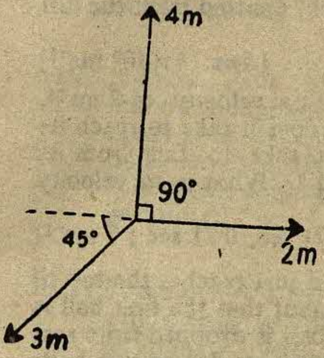
(*Delhi Senior Sec. 1980*)

50. Give an example of a physical quantity which can be represented as a vector product of two vectors.

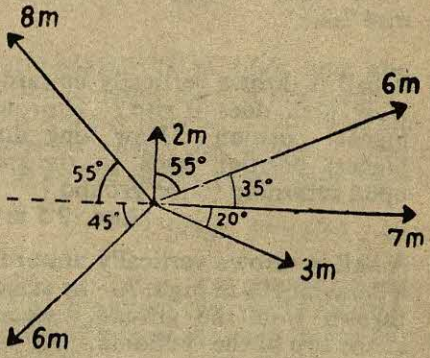
(*Delhi Senior Sec. 1980*)

PROBLEMS

1. Draw diagrams to find the sum of the vectors in each part of the figure.



(*a*)



(*b*)

2. A train is travelling at 50 km h^{-1} in a direction 30° east of north. Resolve its velocity into two rectangular component due north and due east.

[**Ans.** 25 kmh^{-1} towards east ; 43·3 kmh^{-1} towards the north].

3. A racing pigeon can fly with a speed of 30 ms^{-1}. The wind is blowing from the west with a speed of 15 ms^{-1}. In what direction must the bird fly in order to reach a destination north east of his starting point ? If the distance to be covered is 25 km, how long will it take ?

[**Ans** : 65·7° north of east, 10·7 min.]

4. A river has a velocity of 2 ms^{-1}. The bow of a row boat is pointed directly toward the opposite bank and it is rowed with a speed of 5 ms^{-1} relative to the water. If the river is 50 m wide, how long will it take to row across and how far downstream will the boat have drifted in this time. What is the angle between the bank and the direction of motion of the boat ?

[**Ans.** 10 sec ; 20 m : 68·2°]

5. A rifle is fired from a jeep travelling with a speed of 10 ms^{-1}. The target is stationary 100 m away in a direction at right angle to the velocity of the jeep. The muzzle velocity of the bullet is 200 ms^{-1}. If the rifle is aimed directly at the centre of the target, how far to one side will the bullet strike ? How far to one side must it be aimed in order to hit the centre ?

[**Ans.** 5 m ; 5 m]

6. An electron is emitted from a hot metal wire with an unknown velocity. It then enters a device 4 cm long which accelerates it at a rate of 3×10^{14} ms^{-2} in the same direction in which it is moving. At the far end of this 4 cm it enters another device which measures its velocity and finds it to be 7×10^6 ms^{-1}. With what velocity was the electron emitted from the hot wire ?

[**Ans.** 5×10^6 ms^{-1}]

7. A ball is thrown vertically upward with a velocity of 7 ms^{-1}. How high does it rise ? How long does it take to reach its highest position ? How long does it take to fall from its highest position back to the ground ? What is its velocity upon returning to the ground ?

[**Ans.** 2·5 m ; 0·71 sec, 0·71 sec ; 7 ms^{-1}]

8. A ball is thrown vertically upward and just reaches the top of a building 100 m high. At the same instant that the first ball is thrown from the ground, a second ball is dropped from rest at the top of the building. At what height do the two balls pass one another ?

[**Ans.** 75 m above the ground]

9. A body of mass 5 kg is subject to horizontal forces of 3 N pointed due east and also 4 N pointed due North. What is the magnitude and direction of its acceleration ?

[**Ans.** 1 ms^{-2}, 37° east of north]

10. A mass of 3 kg is acted on by a force of 25 N. If it starts from rest, how far will it have travelled after 5 sec ?

[**Ans.** 104·16 m]

11. A lump of clay of mass 30 g travelling with a velocity of 0·25 ms^{-1} collides head on with another lump of clay of mass 0·05 kg travelling with a velocity of 0·40 ms^{-1} in exactly the opposite direction. If the two lumps coalesce, what is the velocity of the combined lump after the collision, assuming that no external forces act on the system ?

[**Ans.** 0·156 ms^{-1}].

12. A star of mass 2×10^{30} kg moving with a velocity of 2×10^4 ms^{-1} collides with a second star of mass 5×10^{30} kg moving with a velocity of 3×10^4 ms^{-1} in a direction at right angle to the first star. If the two join together, what is their common velocity ?

Ans. $2·22 \times 10^4$ ms^{-1} in a direction making an angle of 14·93° with the direction in which the second star was initially moving.]

13. A tractor is dragging a 400 kg log. The chain from the tractor to the log is at 30° to the horizontal and exerts a force of 120 N. How much work is done by the tractor in dragging the log through a distance of 25 m ?

[**Ans.** $2·6 \times 10^3$ joules].

14. A skier takes off from a ski jump with a velocity of 50 ms^{-1} at an unknown angle to the horizontal and lands at a point whose vertical distance below the point of take off is 100 m. What is his speed just before landing, if the friction of the air can be ignored ?

[**Ans.** 66·8 ms^{-1}]

15 Niagara Falls is approximately 50 m high and about one million kilograms of water flows over it per second. How much potential energy is lost per second ? An electric light bulb requires 100 joules of energy per second. How many such bulbs could be lit if all the energy of Niagara Falls could be utilized ?

[**Ans.** 49×10^7J ; 49×10^5]

16. A Batsman hits a ball of mass $\frac{1}{4}$ kg that was thrown toward him with a speed of 10 ms^{-1} back toward the pitcher with a speed of 20 ms^{-1}. If the process of hitting the ball takes 0·05 sec., compute the impulse of the force and the average force exerted on the ball by the bat.

[**Ans.** 7·5 kg ms^{-1} ; 150 N]

17. A 100 kg man is standing in an elevator. What force does the floor exert on his feet

(*a*) when the elevator has a constant velocity,

(*b*) when it has an acceleration of 1·0 ms^{-2} downwards, and

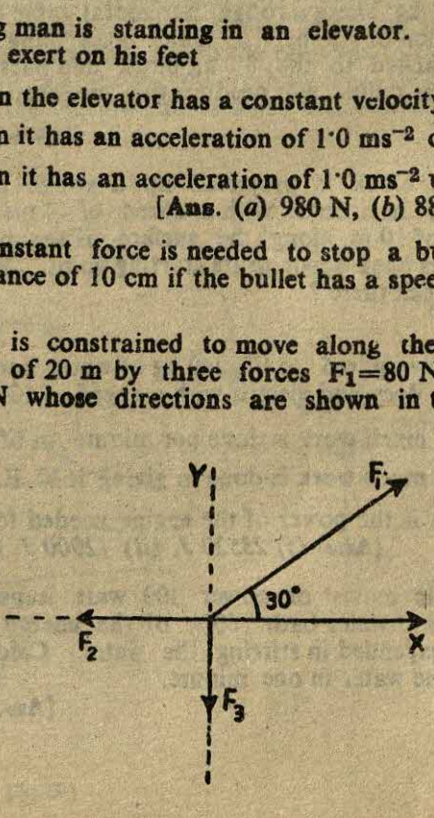
(*c*) when it has an acceleration of 1·0 ms^{-2} upwards ?

[**Ans.** (*a*) 980 N, (*b*) 880 N, (*c*) 1080 N]

18. What constant force is needed to stop a bullet of mass 25g in a distance of 10 cm if the bullet has a speed of 300 ms^{-1} ?

[**Ans.** 11 250 N].

19. A body is constrained to move along the *x*-axis through a distance of 20 m by three forces $F_1=80$ N ; $F_2=30$ N and $F_3=40$ N whose directions are shown in the following dia-



gram. What is the work done by each of these forces, and what is the change in kinetic energy of the body ?

[**Ans.** 1386 J ; −600 J ; 0 ; 786 J]

20. A coconut falls to earth from a height of 10 m. What is its kinetic energy when it is 4 m from the ground if its mass is 1·0 kg ? What is its speed at that height ?

[**Ans.** 58·8 J ; 10·8 ms^{-1}]

21. A block of mass 5·0 kg is projected up a frictionless plane inclined at 37° to the horizontal. It slides a distance of 10 m along the surface, stops and then slides down to its original starting point. With what speed did it start up the plane, and with what speed does it regain its starting point ?

[**Ans.** 10·8 ms^{-1} ; 10·8 ms^{-1}]

22. The coefficient of friction between the tyres and the road for a particular automobile is 0·8 and its rear wheels support 40% of its weight. What is the maximum possible acceleration that can be achieved on starting ?

[**Ans.** 3·136 ms^{-2}]

23. What constant horizontal force is needed to drag a block of mass 8·0 kg along a rough, horizontal table ($\mu=0·5$) with an acceleration of 1·8 ms^{-2} ? What mass hanging from a light cord attached to the 8·0 kg block and passing over a small frictionless pulley, will produce this acceleration ?

[**Ans.** 53·6 N ; 6·7 kg]

24. A 5·0 kg block is projected up a rough plane inclined at 37° to the horizontal with a speed of 14 ms^{-1}. It slides a distance of 10 m along the surface of the plane, stops, and then slides back down to its starting point. With what speed does it reach there ?

[**Ans.** 6·3 ms^{-1}]

25. A pump is required to lift 240 kg of water per minute from a well 10 m deep and eject it with a speed of 10 ms^{-1}.

(*i*) How much work is done per minute in lifting the water ?

(*ii*) How much work is done in giving it K. E. ?

(*iii*) What is the power of the engine needed for this ?

[**Ans.** (*i*) *23520 J*, (*ii*) *12000 J*, (*iii*) **592 watt**]

26. An electric motor of power 300 watt is used to drive the stirrer in a water bath. 60% of the energy supplied to the motor is expended in stirring the water. Calculate the work done on the water in one minute.

[**Ans.** 10800 joules]

UNIT-3

CIRCULAR MOTION

3·1 MOTION OF A BODY IN CIRCULAR ORBIT

In everyday life, in atomic Physics and in astronomy and space travel there are many examples of bodies moving in paths which if not exactly circular are nearly so. We find athletes running in circular tracks. We keep to a circular path now and then when we go on a scooter or a bicycle. Fig. 3·1 shows the twisting and turning path of a crystal of barium sulphate as it moves through the small intestine of an X-ray patient. The actual velocities of the crystal at various points along the intestine are represented by arrows pointing along the tangents to the curve at those points. In this chapter we will see how ideas developed for dealing with straight line motion enable us to tackle circular motion.

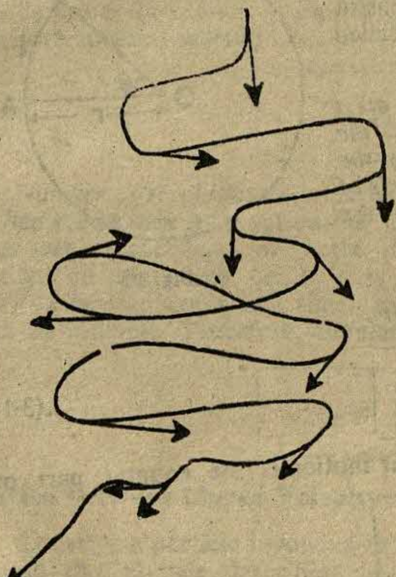


Fig. 3·1

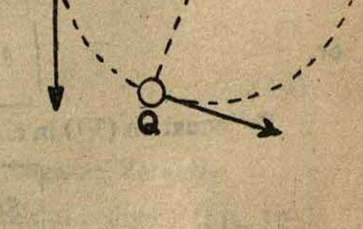


Fig. 3·2

A body which travels equal distances in equal times along a circular path has constant speed but *not* constant velocity. This is due to the way we have defined speed and velocity ; speed is a scalar quantity, velocity is a vector quantity. Fig. 3·2 shows a ball attached to a string being whirled round in a horizontal circle. The velocity of the ball at P is directed along the tangent at P ; when it reaches Q its velocity is directed along the tangent at Q. If the speed is constant the *magnitudes* of the velocities at P and Q are the same but their *directions* are different and so the velocity of the ball has changed. A change of velocity is an acceleration and a body moving uniformly in a circular path or arc is therefore *accelerating*.

In everyday language acceleration usually means going faster and faster *i.e.*, involves a change of speed. However, in Physics it means a change of velocity and since the velocity changes not only when the speed changes, but also when the direction of motion changes. For example, a car rounding a bend (even at constant speed) is accelerating.

3·2 ANGULAR VELOCITY

Consider a particle rotating in a circle of radius 'r' about an axis through O and perpendicular to the plane of the paper Fig. 3·3. Let the particle move from A to B in time t. As the particle moves from A to B, the radius OA describes the angle AOB$=\theta$. Thus the angle described by the radius in unit time will be θ/t. This is called **angular velocity**.

Thus "*the angular velocity of a body rotating about an axis is the angle described in unit time by the line joining the body to the axis of rotation*". It is denoted by the Greek letter ω (omega). Thus

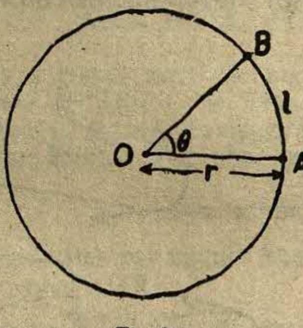


Fig. 3·3

$$\omega=\theta/t$$

or

$$\boxed{\theta=\omega t} \qquad ...(3\text{-}1)$$

The equation (3-1) in circular motion is the counter part of

$$\boxed{S=ut}$$

in linear motion, the symbols having the usual meanings.

The units of angular velocity depend on the units in which the angle θ and time t are measured. The angle θ is measured either in *degrees* or in *radians* (rad). If the time t is in seconds, angular velocity (ω) will then be in **degrees/sec** or **radians/sec** (**rad s^{-1}**). Its dimensions are T^{-1}

One radian is the angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.

In general, if θ (Fig. 3·3) is the angle subtended by an arc of length 'l' at the centre O of the circle of radius 'r', then

$$\theta(\text{in radians})=l/r$$

The angle θ in radians being the ratio of one length to a length is a *pure number*. As the circumference of the circle $=2\pi r$, there are $\frac{2\pi r}{r}=2\pi$ radians *in one complete rotation* or 360°.

Hence,

$$\boxed{1 \text{ radian}=\frac{360^\circ}{2\pi}=57{\cdot}3 \text{ degrees.}}$$

It must be noted that θ/t gives the average value, if the angular velocity (ω) is variable. If it is not variable, θ/t gives the constant value of the angular velocity.

Now time taken by a body to complete one revolution or one rotation is called **time period**. Thus if, T is the time period, the body describes 2π radians in time T and $2\pi/T$ radians in unit time. Therefore, angular velocity

$$\boxed{\omega=(2\pi/T)} \qquad (3\text{-}2)$$

Number of revolutions or rotations made by a body in unit time is called **frequency** of rotation of the body. Its units are s^{-1} or *cycles per second* (**c/s**) or hertz (**Hz**). Thus if a body makes n rotations in unit time *i.e.*, if n is the frequency of rotation, it describes $2\pi n$ radians in unit time, as in one rotation body describes 2π radians. Therefore, angular velocity

$$\boxed{\omega=2\pi n} \qquad ...(3\text{-}3)$$

Relation between Linear Velocity and Angular Velocity

Consider a particle revolving in a circle of radius r (Fig. 3·3). Suppose the particle describes the arc AB of length l in time t. Then in the same time t the particle also rotates through angle θ about O.

Let v be the linear velocity of the particle ω its angular velocity, then

$$\text{Linear velocity } v = \frac{\text{Distance covered}}{\text{Time taken}} = \frac{l}{t}$$

i.e. $$v = \frac{l}{t} \qquad \ldots(i)$$

and angular velocity $$(\omega) = \frac{\text{Angle described}}{\text{Time taken}} = \frac{\theta}{t}$$

But $$\theta\,(\text{radians}) = \frac{l}{r}$$

$$\therefore \qquad l = r.\theta. \qquad \ldots(ii)$$

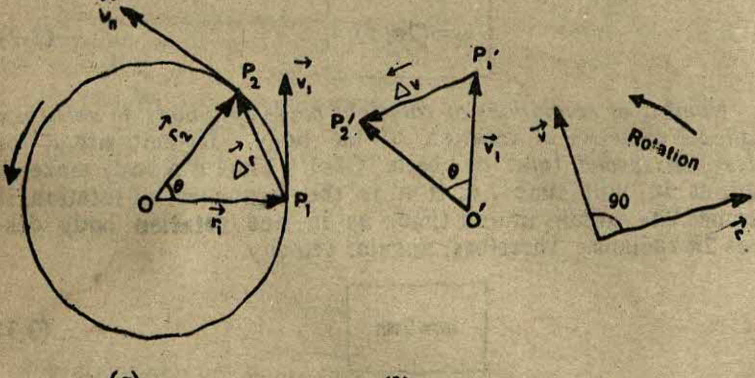
Putting equation (*ii*) in (*i*),

$$v = \frac{r\,.\,\theta}{t} = r\,.\,\frac{\theta}{t} = r.\omega. \qquad \left[\because \ \frac{\theta}{t} = \omega\right]$$

or $$\boxed{v = r\omega} \qquad \ldots(3\text{-}4)$$

i.e., **Linear Velocity = Radius × Angular Velocity**

3·3 UNIFORM CIRCULAR MOTION (*Centripetal Force*)



(*a*) (*b*) (*c*)

Fig. 3·4

Consider a particle of mass '*m*' travelling round a circular path with constant speed V and uniform angular velocity ω. Let '*r*' be the radius of the circular path. Then the speed of the particle is given by $V = r\omega$.

Consider the position vectors $\vec{r_1}$ and $\vec{r_2}$ of the particle (which are simply directed lines from the centre to the particle) for two successive instants of time say t_1 and t_2 as shown in [Fig. 3·4(*a*)]. These vectors are, of course, both of length r, equal to the radius of the circle, and so there is no change in the magnitude of $\vec{r}$ with time. But there is a change of direction, and this means that the vector $\vec{r}$ is changing with time. The displacement of the particle between the two instants is the difference

$$\overrightarrow{\Delta r} = \vec{r_2} - \vec{r_1} = \overrightarrow{P_1P_2}$$

It will be apparent that as the interval of time $(t_2-t_1)=\Delta t$ between the two instants is shrunk to zero, then this vector displacement will tend to become perpendicular to radius. Since the velocity vector is defined by the relation

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

the conclusion is that, for circular motion, the velocity vector is perpendicular to the position vector. The velocity vector is *always* along the tangent line to the path, but it is only perpendicular to the position vector for the circular path, since this is a unique geometrical property of a circle. This is true whether the speed is constant or not. This is illustrated in [Fig. 3·4(*c*)]. The velocity vector $\vec{V}$ and the position vector $\vec{r}$ have been drawn from a common point, which may be taken as the centre of the circle

As the particle moves with constant speed round the circle, the position vector rotates with constant angular velocity ω, and the velocity vector also rotates with constant angular velocity ω but is 90° ahead of the position vector as explained above. In [Fig. 3·4(*a*)] the position vectors $\vec{r_1}$ and $\vec{r_2}$ at two instants of time have been drawn and the velocity vectors $\vec{V_1}$ and $\vec{V_2}$ for the same two instants have also been shown. The difference between the vectors $\vec{V_1}$ and $\vec{V_2}$, which represents the change in velocity, can be found as explained below :

Take an external point O′ [Fig. 3·4 (*b*)] and draw $\overrightarrow{O'P'_1}$ and $\overrightarrow{O'P'_2}$ equal and parallel to the vectors $\vec{V_1}$ and $\vec{V_2}$. Draw also the vector $\overrightarrow{P'_1P'_2}$ so as to close the vector triangle O′P′₁P′₂. Now according to *triangle Law of Vectors.*

$$\overrightarrow{O'P_2'} = \overrightarrow{O'P'_1} + \overrightarrow{P'_1P'_2}$$

or

$$\vec{V_2} = \vec{V_1} + \overrightarrow{P'_1P'_2}$$

$\therefore$ Change in velocity $\overrightarrow{\Delta V} = \overrightarrow{P'_1P'_2} - \vec{V_2} - \vec{V_1}$ and so the average acceleration over the interval

$$= \frac{\overrightarrow{\Delta V}}{\Delta t} = \frac{(\vec{V_2} - \vec{V_1})}{(t_2 - t_1)}$$

As the interval Δt is shrunk to zero, $\overrightarrow{\Delta V}$ becomes perpendicular to $\vec{V}$ in exactly the same way as $\overrightarrow{\Delta r}$ became perpendicular to $\vec{r}$. Note that the magnitude of the velocity does not change with time, only its direction. It follows that the instantaneous acceleration defined by

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta V}}{\Delta t} = \frac{\overrightarrow{dV}}{dt}$$

is *perpendicular* to the velocity, just as the instantaneous velocity is perpendicular to the position vector.

When angle θ (angle traced out by the radius vector in Δt seconds) is small, the chord, $P_1 P_2$ becomes nearly equal to the arc P_1P_2 so that P_1OP_2 can be considered to be a triangle. Now as $O'P'_1$ and $O'P'_2$ are perpendicular to OP_1 and OP_2 respectively and $\angle P_1OP_2 = \angle P'_1O'P'_2 = \angle\theta$ so isosceles Δs P_1OP_2 and $P'_1O'P'_2$ are similar. Therefore, by the properties of similar triangles, the ratio of the corresponding sides are equal. That is,

$$\frac{P_1P_2}{OP_1} = \frac{P'_1P'_2}{O'P'_1}$$

or $\frac{\Delta r}{r} = \frac{\Delta V}{V}$ since $r_1 = r_2 = r$, and $V_1 = V_2 = V$.

Dividing both sides of this equation by Δt, we have

$$\frac{1}{r}\frac{\Delta r}{\Delta t} = \frac{1}{V}\frac{\Delta V}{\Delta t}$$

Proceeding to the limit by shrinking the interval Δt to very, very near to zero, we have

$$\lim_{\Delta t \to 0}\left(\frac{1}{r}\frac{\Delta r}{\Delta t}\right) = \lim_{\Delta t \to 0}\left(\frac{1}{V}\frac{\Delta V}{\Delta t}\right)$$

or $$\frac{1}{r}\,\frac{dr}{dt}=\frac{1}{V}\,\frac{dV}{dt}$$

or $$\frac{V}{r}=\frac{|\,a\,|}{V} \qquad \left[\because\ \frac{dr}{dt}=V \text{ and } \frac{dV}{dt}=|\,a\,|\right]$$

or Instantaneous acceleration

$$=\boxed{|\,a\,|=\frac{V^2}{r}} \qquad \text{...(3-5)}$$

Since $$V=r\omega$$

$$\therefore \qquad \boxed{|\,a\,|=r\omega^2=\omega V} \qquad \text{...(3-6)}$$

Equations (3-5) and (3-6) give the *magnitude* of the acceleration. The *direction* of the acceleration is **perpendicular** to the velocity just as the velocity is perpendicular to the position vector. Fig. [3·4(*a*)] and [3·4(*b*)] show that acceleration *leads* the velocity by 90° just as the velocity leads the position vector by 90°

Fig. (3·5) shows the three vectors $\vec{r}$, $\vec{V}$ and $\vec{a}$ for the *same* instant,

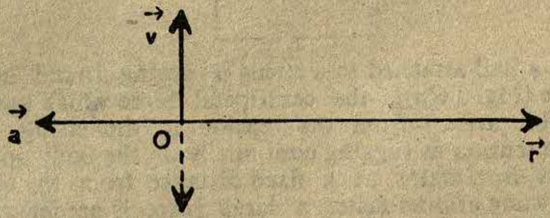


Fig. 3·5

all drawn from the centre of the circle O. **The acceleration is 180° ahead of the position vector**, which is another way of saying **that it acts towards the centre of the circle**. This *centre-seeking* acceleration is called the **centripetal acceleration**. The magnitudes of the three vectors are constant in time, but the vectors themselves are not, since their directions are constantly changing. Does a body moving uniformly in a circle have *constant* acceleration ? (Remember that acceleration is vector.)

Centripetal Force

Since a body moving in a circle (or a circular arc) is accelerating, it follows from Newton's first law of motion that there must be a force acting on it to cause the acceleration. This force, like the centripetal acceleration, will also be directed towards the centre and is called the **Centripetal Force**. It causes the body to deviate from the straight line motion which it would naturally follow if the force were absent. Thus the centripetal force may be defined as that "*force which deflects a body from its linear path to make it move along a circle and is always directed radially inwards*".

The *magnitude* (F) of the centripetal force is given by Newton's Second Law of motion, that is

$$\boxed{F = ma = \frac{mV^2}{r}} \qquad \text{...(3-7)}$$

where m = mass of the body

V = speed of the body

r = radius of circular path.

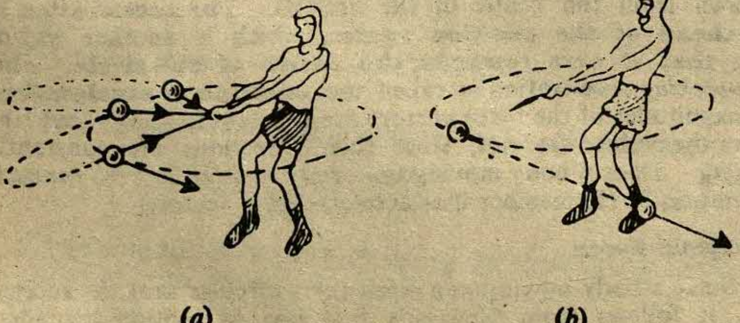
If the angular velocity of the body moving along the circle is ω radians per second, then

$$F = \frac{mV^2}{r} = \frac{m(r\omega)^2}{r}$$

or

$$\boxed{F = mr\omega^2} \qquad \text{...(3-8)}$$

When a ball attached to a string is swung round in a horizontal circle [Fig. 3·6(*a*)], the centripetal force which keeps it in a circular orbit arises from the tension in the string. We can think of the tension as tugging continually on the body and turning it in so that it remains at a fixed distance from the centre. If the ball is swung around faster, a large force is needed and if it is



(*a*) (*b*)

Fig. 3·6

greater than the tension the string can bear, it breaks and the ball continues to travel along a tangent to the circle at the point of breaking [Fig. 3·6(*b*)] due to *directional inertia* (the inability of a body to change by itself its direction of motion in the absence of the external force).

Other Examples of Circular Motion

In all cases it is important to appreciate that the forces acting on a body must provide a resultant force of magnitude $\frac{mV^2}{r}$ towards the centre.

(*a*) Banking of the Curve

When a car (or a bus or a truck) is travelling round a circular bend on a *perfectly level road* with uniform speed, the resultant force acting on it must be directed to the centre of its circular path *i.e.*, it must be the centripetal force. This force arises from the interaction of the car with the air and the ground. The direction of the force exerted by the air on the car will be more or less opposite to the instantaneous direction of motion. The other and more important horizontal force is the *frictional force* exerted *inwards* by the ground on the tyres of the car (Fig. 3·7). The resultant of these two forces is the centripetal force (Fig. 3·8). The successful negotiation of a bend on a flat road therefore depends on the tyres and the road surface being in a condition that enables them to provide a sufficiently high frictional force—otherwise skidding occurs.

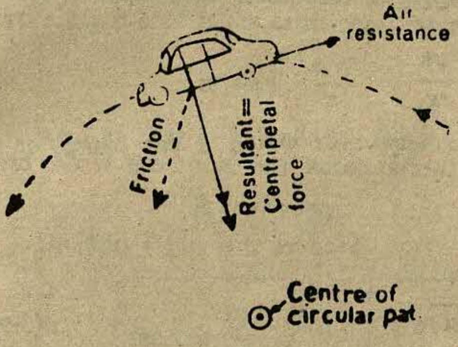


Fig. 3·7

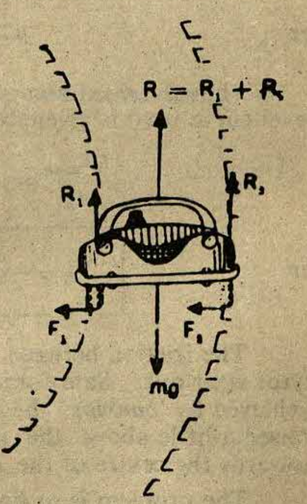


Fig. 3·8

Let mg be the weight of the car, F_1 and F_2 the force of friction at each tyre and R_1 and R_2 the normal reaction due to the ground exerted at each tyre (Fig. 3·7). Since the weight of the car is supported by the normal reaction (R) due to the ground, then for equilibrium,

$$F_1+F_2=\frac{mV^2}{r}$$

where V = Speed of the car

and r = Radius of the curved path

$$R=R_1+R_2=mg$$

If μ is the coefficient of friction, the maximum force of friction between the tyres and the road is given by

$$\mu=\frac{F}{R}=\frac{F}{mg}$$

where F = Maximum force of friction.

$$\therefore \quad \boxed{F=\mu\, mg}$$

The car will, therefore, stay on the road if the maximum force of friction is greater than the centripetal force $\frac{mV^2}{r}$ *i.e.*,

$$\mu\, mg > \frac{mV^2}{r}$$

or

$$\mu > \frac{V^2}{rg}$$

Thus the maximum velocity with which a car can go on a level curve road is given by

$$\frac{V^2_{max}}{rg}=\mu$$

or

$$\boxed{V_{max}=\sqrt{\mu\, rg}} \qquad \text{...(3-9)}$$

The friction between the tyres and the road soon makes the tyres wear out. Safe cornering that does not rely on friction is achieved by '*banking*' the road *i.e.*, the outer part of the road is raised a little above the inner side so that the road is sloping towards the centre of the curve.

The problem is to find the angle θ at which a bend should be banked so that the centripetal force acting on the car arises entirely from the component of the normal force R of the road

[Fig 3·9 (*a*)]. The normal reaction R due to the road bed is equal to the sum of the reactions at each tyre and is thus inclined to the vertical at an angle θ. Treating the car as a particle and resolving R into its rectangular components vertically and horizontally we have, since R sin θ is the centripetal force,

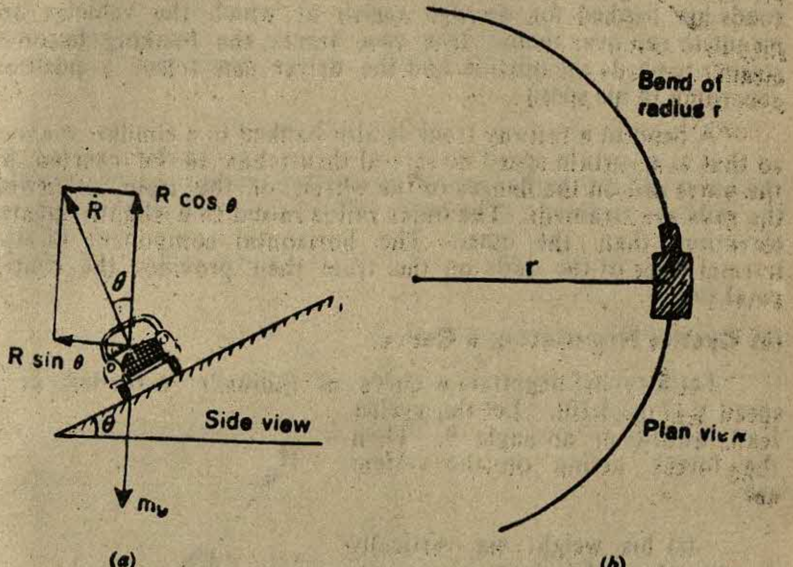


Fig. 3·9

$$R \sin \theta = \frac{mV^2}{r} \qquad ...(i)$$

Also, the car is assumed to remain in the same horizontal plane and so has no vertical acceleration, thus

$$R \cos \theta = mg \qquad ...(ii)$$

Dividing (*i*) by (*ii*), we have

$$\boxed{\tan \theta = \frac{V^2}{rg}} \qquad ...(3\text{-}10)$$

or

$$\boxed{\theta = \tan^{-1}\left(\frac{V^2}{rg}\right)} \qquad ...(3\text{-}11)$$

It is clear from equation (3-10) that the angle of banking (θ) is directly proportional to the square of the velocity, and inversely proportional to the radius of the curve. For a given radius of bend, the angle of banking is *only correct* for one speed. Since one particular angle of banking is not correct for all velocities, the roads are banked for *average speeds* at which the vehicles are meant to run over them. In a race track, the banking becomes steeper towards the outside and the driver can select a position according to his speed.

A bend in a railway track is also banked in a similar manner so that at a certain speed no lateral thrust has to be exerted by the outer rail on the flanges of the wheels of the train, otherwise the rails are strained. The outer rail is raised to a slightly greater elevation than the inner. The horizontal component of the normal force of the rails on the train then provides the centripetal force.

(*b*) Cyclist Negotiating a Curve.

Let a cyclist negotiate a curve of radius 'r' to the left at a speed V [Fig. 3·10]. Let the cyclist lean inwards at an angle θ. Then the forces acting on the system are

(*i*) his weight mg vertically downwards at the centre of gravity G.

(*ii*) the normal reaction R of the ground which passes through G and inclined to the vertical at an angle θ.

(*iii*) the force of friction F between the tyre and the ground along the ground at A.

The horizontal component of the reaction, R $\sin\theta$, provides the centripetal force $\frac{mV^2}{r}$, where m is the mass of the cyclist ; V is the speed of the cyclist and r is the radius of the curved path ; whereas the vertical component R $\cos\theta$, supports the weight. The friction (F) between the tyres and the ground soon makes the tyres wear out. Thus for a cyclist going round a curve, we have

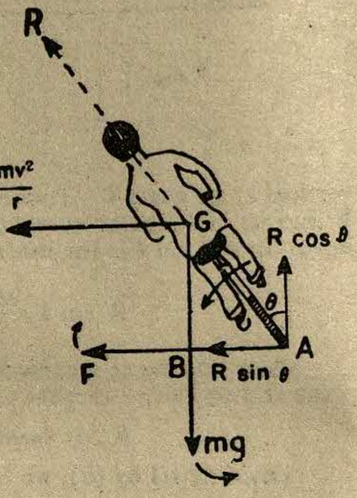


Fig. 3·10

$$R\sin\theta = \frac{mV^2}{r}$$

and
$$R\cos\theta = mg$$

$$\therefore \quad \frac{R\sin\theta}{R\cos\theta} = \frac{mV^2}{r.\,mg}$$

or $$\tan\theta=\frac{V^2}{rg}$$

or $$\theta=\tan^{-1}\left(\frac{V^2}{rg}\right)$$

Thus to keep himself in equilibrium, therefore, the cyclist must lean inwards from the vertical at an angle

$$\theta=\tan^{-1}\left(\frac{V^2}{rg}\right)$$

If, he were to remain vertical or upright, mg would act through A and he will topple over under the action of the forces F and $\frac{mV^2}{r}$.

Now, as we know $\frac{F}{R}=\mu$, the coefficient of friction between the tyres and the ground, so that $F=\mu R=\mu mg$. The cyclist will, therefore, *skid* when F is less than $\frac{mV^2}{r}$ or when $\frac{mV^2}{r}>F$ or when $\frac{mV^2}{r}>\mu\, mg$ or $V^2>\mu rg$.

In other words, skidding will occur in the event of

(*i*) V, the speed of the cyclist, being large,

(*ii*) μ being small *i.e.*, the road surface beng slippery, and

(*iii*) r being small *i.e.*, the curve being sharp.

(*e*) Looping the loop (*Motion in a Vertical Circle*)

A pilot who is not strapped into his aircraft can loop the loop without falling out at the top of the loop. A bucket of water can be swung around in a vertical circle without spilling A ball bearing can loop the loop on a length of curtain rail in a vertical plane. All these effects have similar explanations.

Let a body of mass 'm' tied to a string be whirled round in a verticle circle with O as its centre (Fig. 3.11). As the body moves, its speed is maximum at the lowest point B and minimum at the highest point A. At any other point, the value lies in between the maximum and the minimum values.

Let P be the position of the body at any time and PN the perpendicular from P on the vertical diameter AOB. Further let $\angle POB=\theta$ and $PO=r$, the radius of the circle. Then

$$\begin{aligned}BN&=OB-ON\\&=r-r\cos\theta\\&=r(1-\cos\theta)\end{aligned}$$

The forces acting at P are the tension T along the string and the weight mg acting vertically downwards. The force mg may be

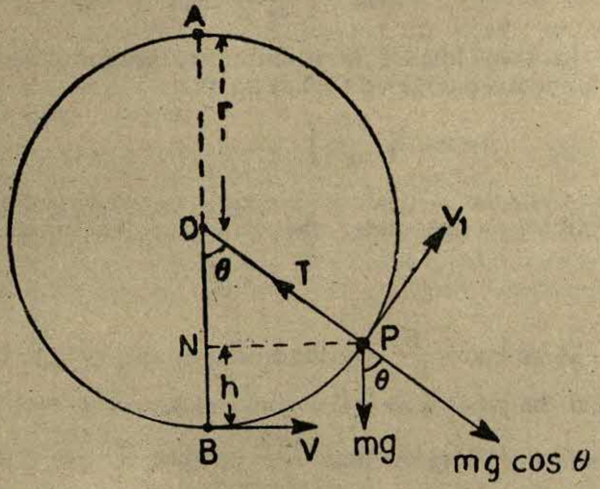


Fig. 3·11

resolved into two components $mg\cos\theta$ along the string and $mg\sin\theta$ perpendicular to it. Therefore, the resultant force along the string is $(T-mg\cos\theta)$. This provides the necessary centripetal force which keeps the body in motion. If V_1 is the velocity of the body at P, then

$$T-mg\cos\theta=\frac{mV_1^2}{r}$$

or

$$T=\frac{mV_1^2}{r}+mg\cos\theta$$

If V is the velocity of the body at B, then

$$V_1^2=V^2-2gh$$

and

$$\cos\theta=\frac{r-h}{r}$$

$\therefore$

$$T=\frac{m(V^2-2gh)}{r}+mg\,\frac{(r-h)}{r}$$

$$=\frac{m}{r}\left[V^2-2gh+gr-gh\right]$$

$$=\frac{m}{r}\left[V^2-3gh+gr\right] \qquad (i)$$

At the lowest point B, $h=0$. If T_1 is the tension of the string at this point, then

$$T_1=\frac{m}{r}\left[V^2+gr\right] \qquad (ii)$$

At the highest point, $h=2r$. If T_2 is the tension of the string at this point, then

$$T_2 = \frac{m}{r}\left[V^2 - 5gr\right] \qquad (iii)$$

From equations (*i*), (*ii*) and (*iii*) we observe that the tension is maximum at the lowest point and it gradually decreases to a minimum value at the highest point. In order that the body may continue moving in the circle, the *string should never slacken i.e.*, the tension in the string must remain positive. Therefore,

$$T_2 = \frac{m}{r}\left[V^2 - 5gr\right] > 0$$

or

$$V^2 > 5gr$$

or

$$\boxed{V > \sqrt{5gr}}$$

Thus the minimum velocity which the body must possess at the lowest point to be able to complete the circle is given by

$$V = \sqrt{5gr}$$

So an aircraft must have at least a velocity of $\sqrt{5gr}$ at the bottom of the loop to clear the loop without any mishap. (Fig. 3.12).

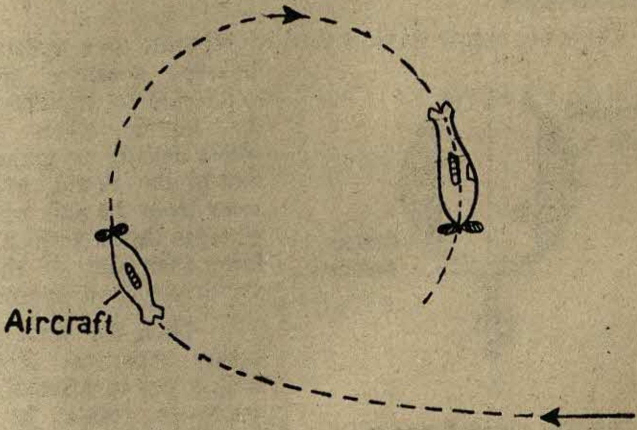


Fig. 3·12

Similarly, for the same reason, if the weight mg of the water in a bucket of water when it is at the top of the loop Fig. (3.13), is *less* than mV^2 the normal force R of the bottom of the bucket on

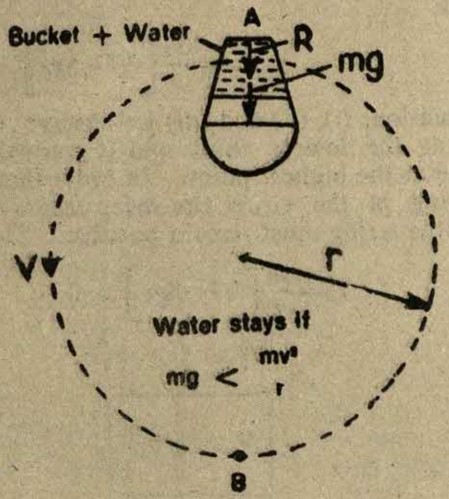


Fig. 3.13

the water provides the rest of the force required to maintain the water in its circular path. However, if the bucket is swung more slowly then mg will be greater than $\frac{mV^2}{r}$ and the *unused* part of the weight causes the water to leave the bucket.

(*d*) Centrifuges

These are simple devices used to separate out substances of different densities suspended in a liquid by rapidly rotating the liquid, when particles whose density is greater than that of the liquid, are driven away from the axis of rotation whereas those, with a density lower than that of the liquid are driven inwards towards it.

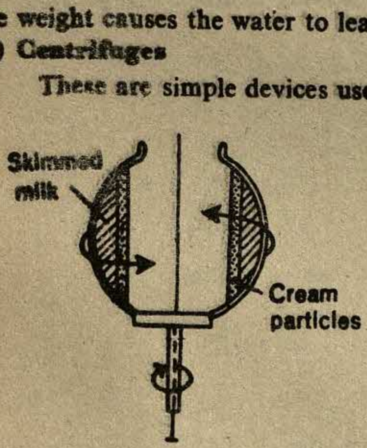


Fig. 3·14 Cream Separator

Thus, for example, in a cream separator (Fig. 3.14), milk is put in a vessel which is made to rotate at a high speed. Every particle of the whirling liquid requires the needed centripetal force, otherwise it cannot make the orbit. This is provided by pressure differences. The outer end of the liquid particles experiences a greater pressure than the inner end. Thus there is a gradation of increasing pressure from the inner to the outer end. The milk revolves and the heavier milk component (skimmed milk) falls to the bottom while the lighter cream component (cream) rises to the surface.

Laundry drier, centrifugal pump etc., also work on the above principle. Doctors use centrifuges for separating colloidal particles in liquids from bacteria. *Ultra centrifuges* are capable of operating at as high a speed as 100 000 rotations per minute. At such high speeds of rotation, the centripetal force acting on any mass exceeds 750 000 times its weight. They are extensively used in medical researches for concentrating solutions containing viruses and harmones, cell fragments and bacteria. An ultra-centrifuge can easily separate lighter molecules from heavier ones.

During the launching and re-entry of space vehicles accelerations of about *8g* occur and the resulting large forces which act on the surface of the astronaut's body cause blood to drain from some parts and congest others. If the brain is deprived, loss of vision and unconsciousness may follow. Tests with large *man-carrying centrifuges* in which passengers are subjected to high centripetal accelerations show that a person will tolerate *15g* for a few minutes when his body is perpendicular to the direction of the acceleration and only *6g* when in the direction of acceleration. What will be the best position for an astronaut to adopt at lift-off and re-entry ?

Example 3·1. What is the period of an earth satellite travelling in a circular orbit 10^5 m above the earth where the acceleration of gravity is 9·5 ms^{-2} ? The radius of the earth is $6{\cdot}4\times10^6$ m.

Solution.

A satellite coasting round the earth is in free fall, since it is under the influence of gravity alone. Hence, the centripetal acceleration of the satellite is just the local gravitational acceleration, or

$$\omega^2 r = g$$

The period (T) of the satellite is given in terms of ω by equation

$$T=\frac{2\pi}{\omega}, \text{ whence we get}$$

$$T=2\pi\sqrt{\frac{r}{g}}$$

From the data given, $g=9{\cdot}5\ \text{ms}^{-2}$ and

$$r=(6{\cdot}4\times10^6+10^5)\ \text{m}=6{\cdot}5\times10^6\ \text{m}$$

Substituting these values gives

$$T=2\pi\sqrt{\frac{6{\cdot}5\times10^6}{9{\cdot}5}}=5196\,\text{s}=1{\cdot}44\ \text{h}$$

Example 3·2. A string can withstand a tension of 25N. What is the greatest speed at which a body of mass 0·5 kg can be whirled in a circle using 0·5 m length of the string ? Neglect the force of gravity on the body.

Solution.

Here $$F = 25 \text{ N}$$

$$r = 0{\cdot}5 \text{ m}$$

$$m = 0{\cdot}5 \text{ kg}$$

Let V be the greatest speed at which the string can be whirled.

Now $$F = \frac{mV^2}{r}$$

$\therefore$ $$25 = \frac{0{\cdot}5 \times V^2}{0{\cdot}5}$$

$\therefore$ $$V = 5 \text{ ms}^{-1}$$

Example 3·3. An aeroplane travelling at a speed of 500 km h^{-1} banks at an angle of 30° as it makes a turn. What is the radius of the curve ? [$g = 9{\cdot}8 \text{ ms}^{-2}$].

Solution.

Here $$V = 500 \text{ km h}^{-1}$$

$$= \frac{500 \times 1000}{60 \times 60} = \frac{1250}{9} \text{ ms}^{-1}$$

$$\theta = 30^\circ$$

$$g = 9{\cdot}8 \text{ ms}^{-2}$$

$$r = ?$$

Since $$\tan\theta = \frac{V^2}{rg}$$

$\therefore$ $$\tan 30^\circ = \left(\frac{1250}{9}\right)^2 \times \frac{1}{9{\cdot}8} \times \frac{1}{r}$$

$\therefore$ $$r = \left(\frac{1250}{9}\right)^2 \times \frac{1}{9{\cdot}8} \times \sqrt{3}$$

$$\left(\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right)$$

$$= 3{\cdot}41 \times 10^3 \text{ m}$$

Example 3·4. A body weighing 0·4 kg is whirled in a vertical circle making 2 revolutions per second. If the radius of the circle is 1·2 m, find the tension in the string when the body is (*a*) at the top of the circle, (*b*) at the bottom of the circle. ($g = 9{\cdot}8 \text{ ms}^{-2}$)

Solution.

Here $m=0{\cdot}4$ kg

$w=2\times 2\pi=4\pi$ rad s^{-1}

$r=1{\cdot}2$ m

(*i*) At the highest point A on the vertical circle (Fig. 3·15); it is evident that both the weight and the tension act downward towards the centre and the sum of the two provides centripetal force *r*

$$\therefore \quad T+mg=\frac{mV^2}{r}=mr\omega^2$$

$$\therefore \quad T=mr\omega^2-mg$$

$$=m[r\omega^2-g]$$

$$=0{\cdot}4[1{\cdot}2\times(4\pi)^2-9{\cdot}8]$$

$$=0{\cdot}4\left[1{\cdot}2\times\left(4\times\frac{22}{7}\right)^2-9{\cdot}8\right]$$

$$=71{\cdot}936 \text{ N}$$

(*ii*) At the bottom B, T and *mg* are opposite to each other

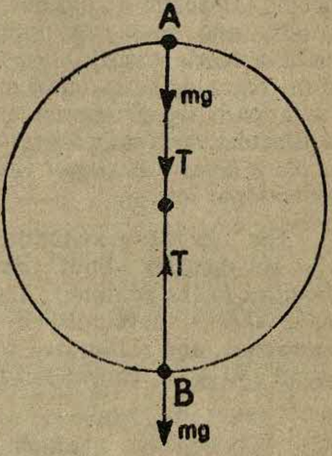


Fig. 3·15

$$\therefore \quad T-mg=mr\omega^2$$

$$\text{or} \quad T=mr\omega^2+mg$$

$$=m[r\omega^2+g]$$

$$=0{\cdot}4\left[1{\cdot}2\times\left(4\times\frac{22}{7}\right)^2+9{\cdot}8\right]$$

$$=79{\cdot}776 \text{ N}$$

3·4 PLANETARY MOTION

If we look up at the heavens on a clear night, we will have much the same view of the sky as did the ancient astronomers of China, Babylon, and Egypt. And if we were to spend a number of nights observing the relative movements of the shimmering stars and steadily shining planets, we would perhaps rediscover some of the sense of mystery that those observers of old experienced when they viewed the cosmos.

The ancient astronomers and astrologers contributed a great deal to what we now recognize as the Sciences of astronomy and Physics, by identifying the planets in the Solar system and carefully plotting the variations in their positions in the sky over periods of many years. These data eventually led to models and theories of the solar system. A mixture of improvements in data, modifications in theory and large leaps in conceptual understanding finally led to the underlying principle that embodies the basic reason for all the movements of celestial objects. Known as the universal Law of Gravitation, it is another product of the immensely fertile mind of the English Physicist Sir Isaac Newton.

Newton's Law of gravitation is not the final word in our understanding of the universe. Albert Einstein modified Newton's concepts (although he did not really negate them) in his theory of relativity, and to-day a number of physicists are trying to modify even Einstein's theory. Furthermore, much of the most exciting and far reaching research in modern physics and astronomy concerns apparently incomprehensible objects undreamed of by Newton and Einstein, such as Quasars (or quasi-stellar radio sources); Pulsars and *black holes*. However the core of our knowledge remains the gravitational law.

The law of gravitation itself was actually the climax of a series of theories about the solar system that ranged from the ridiculous to the sublime. The first major theory, called the *Geocentric theory*, developed by the Greek philosophers Aristotle and Hipparchus, and elaborated by another Greek astronomer, *Ptolemy*, around 140 A.D.; considered the earth as the centre of the universe.

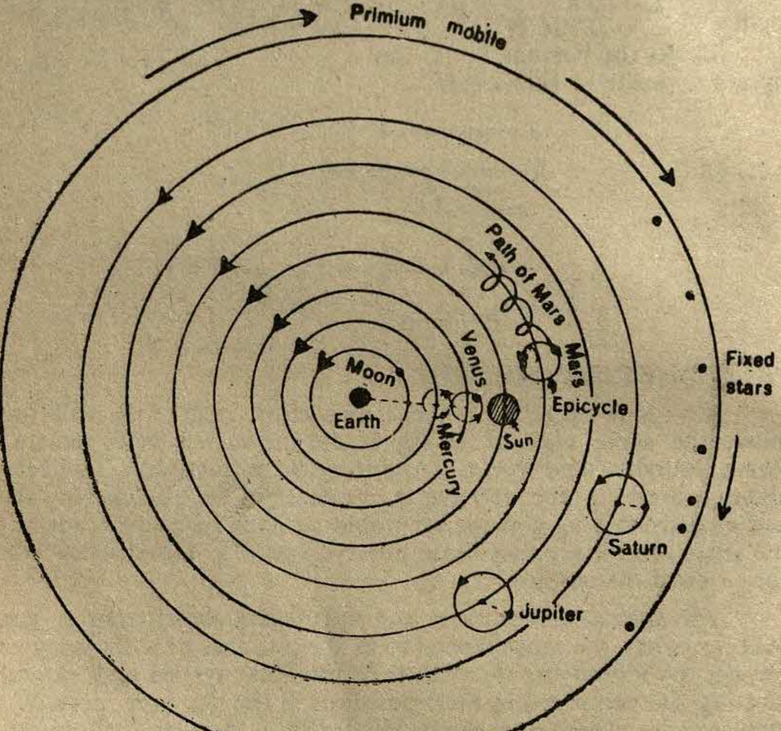


Fig. 3·16 Ptolemy's Geocentric concept of the universe

According to the model, the moon, the sun and the other stars supposedly revolved around the earth in simple circles while the planets were believed to be in more complex orbits, revolving in small circles called epicycles that were imposed on their basic circular revolution around earth. This arrangement is shown in (Fig. 3·16). The moon moved in an orbit nearest the Earth. Mercury in an orbit next farthest out, then Venus, then the Sun, and then, Mars, Jupiter and Saturn. Ptolemy accounted for the long eastward motion of these heavenly bodies with respect to the stars by postulating counter clockwise motion along concentric circles called deferents. He ingeneously explained the loop (retrograde motion) formed by a planet when it moves westward for a short time by assuming that the planets move counterclockwise on small circles called epicycles as the centres of the epicycles move along the deferents. Despite its inherent complexity, this theory agreed so well with the naked eye observations of the time that it was almost universally accepted for more than fourteen centuries.

Eventually, the Ptolemaic theory, as it was known, was superseded by the *heliocentric theory* published in 1543 by *Nicholas Copernicus*, a Polish astronomer who had developed the ideas held many centuries earlier by Greek philosophers. Its precept was that each and every planet, including earth, revolves round the Sun in its orbit. In addition, Copernicus assumed that the earth rotated on its axis once each day. Great Indian Mathematician and astronomer **ARYABHAT** of the 5th century A.D. also from his astronomical observations stated that the earth moves about its axis. (Aryabhat's universe was geocentric). Perhaps due to lack of communication between the scientists of the east and those of the west, his findings did not reach the natural philosophers of the West. The Christian church, which dominated over every sphere of activity in Europe, was very dogmatic with regard to its ideas of the universe and opposed any theory which did not give a central place of importance to the earth. Even, Copernicus, who propounded the heliocentric theory, could not boldly assert it for fear of the church but gave it as a mathematical convenience for understanding the motion of the planets. Two men—one an astronomer observer par excellence, the other a theorist with a splendid mind—brought rigorous scientific realism to the objective of understanding of what makes the universe tick.

The observer was Danish astronomer **TYCHO BRAHE** of the 16th century (1546—1601), whose naked eye observations of the planets during the last Quarter of the sixteenth century were far more accurate than any performed previously. The theorist was the German **JOHANNES KEPLER** (1571—1630) who, because of Brahe's data found it necessary to modify the Copernican theory. Kepler was able to boil down the motions of the planets into three relatively simple statements that enabled observers to produce more accurate mathematical predictions of planetary positions. Kepler's statements took the form of three *empirical laws of motion* that formed the basis of Newton's later statement of his law of universal gravitation.

3·5 KEPLER'S LAWS OF PLANETARY MOTION

Kepler's Laws, as already stated, were the result of a careful analysis of the astronomical data that were available to him from Tycho Brahe's observations over a period of twenty years. Kepler was Tycho Brahe's assistant for sometime before his death. He announced his first two laws in 1609 and the third in 1619.

The three laws are :

(1) **Every planet moves in an elliptical orbit. with the Sun at one focus** (*the law of orbits*).

In planetary motion, as shown in (Fig. 3·17), the *perihelion* is the point P in the Planet's orbit that is closest to the Sun, while the *aphelion*, A, is the point in the orbit furthest from the Sun. In the earth's orbit, the *major axis*—the line that joins the perihelion and aphelion—is about 300 million km in length.

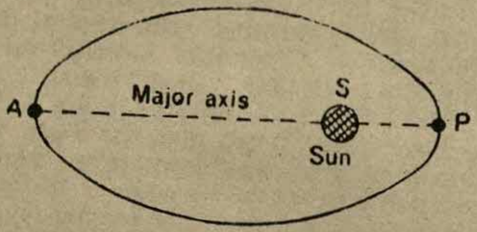


Fig. 3·17

(2) **The radius vector from the Sun to the planet (i.e., the imaginary line that joins a planet to the Sun) sweeps out equal areas in equal intervals of time** (*the law of areas*)

In (Fig. 3·18), if planet P takes the same time to travel from A to B as from C to D, then the shaded areas are *equal*.

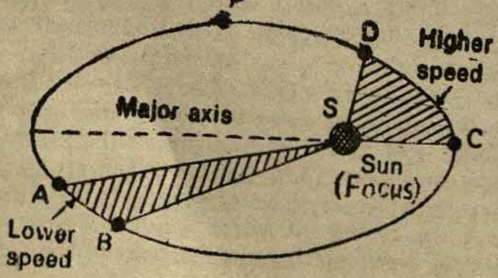


Fig. 3·18

(3) **The squares of the times of revolution of the planets (i.e., their periodic times) about the Sun are proportional to the cubes of their mean distances from it** (*the law of periods*).

Mathematically, this is expressed as

$$T^2 \propto r^3$$

$$\text{or} \quad \boxed{\frac{T^2}{r^3} = \text{constant}}$$

where T = time of one revolution round the Sun *i.e.*, period of the planet,

and r = average distance of that planet from the Sun.

The value of the constant is determined by the units in which T and r are measured. In the case of the earth, for example, we can arbitrarily define our average distance from the Sun as *one astronomical unit*. Since T also has the value of one (in years), the constant is also 1.

Predictions of the positions of the planets according to Kepler's Laws agreed so well with observations that the Copernican theory of a stationary Sun and an orbiting earth was finally accepted despite a violent rearguard action by organized religion. As it happens, the laws apply equally well to a variety of phenomena other than planetary motion. In the solar system, for example, they predict accurately the positions of asteroids, and of comets with elliptical orbits, in planetary systems, they allow scientists to calculate the positions of natural orbiting moons and artificial satellites.

3·6 NEWTON'S LAW OF UNIVERSAL GRAVITATION

Kepler's laws summed up neatly *how* the planets of the solar system behaved without indicating *why* they did so. One of the problems was to find the centripetal force which kept a planet in its orbit round the sun, or the moon round the earth, in a way which agreed with Kepler's Laws.

Newton reflected (perhaps in his garden when the apple fell) that the earth exerts an inward pull on nearby objects causing them to fall. He then speculated whether the same force of gravity might not extend out farther to pull on the moon and keep it in orbit. If it did, might not the sun also pull on the planets in the same way with the same kind of force ? He decided to test the idea first on the moon's motion—as we will do now.

If 'r' is the radius of the moon's orbit round the earth and T is the time it takes to complete one orbit *i.e.*, its period (Fig. 3·19), then using accepted values we have

$r = 3{\cdot}84 \times 10^8 m$

$T = 27{\cdot}3$ days $= 27{\cdot}3 \times 24 \times 3600$ s

The speed V of the moon along its orbit (assumed circular) is

$$V = \frac{\text{Circumference of orbit}}{\text{Period}} = \frac{2\pi r}{T}$$

$$= \frac{2 \times \pi \times 3{\cdot}84 \times 10^8}{27{\cdot}3 \times 24 \times 3600} \text{ ms}^{-1}$$

$$= 1{\cdot}02 \times 10^3 \text{ms}^{-1}$$

The moon's centripetal acceleration 'a' will be

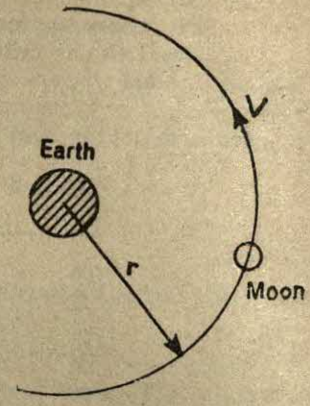


Fig. 3·19

$$a = \frac{V^2}{r} = \frac{(1{\cdot}02 \times 10^3)^2}{3{\cdot}84 \times 10^8} = 2{\cdot}72 \times 10^{-3} \text{ ms}^{-2}$$

The acceleration due to gravity at the earth's surface is 9·81 ms^{-2} and so if gravity is the centripetal force for the moon it must weaken between the earth and the moon. The simplest assumption would be that gravity halves when the distance doubles and at the moon it would be $\frac{1}{60}$ of 9·81 ms^{-2} since the moon is 60 earth-radii from the centre of the earth and an object at the earth's surface is 1 earth-radius from the centre. But $\frac{9{\cdot}81}{60} = 1{\cdot}64 \times 10^{-1}$ ms^{-2}, which is still too large.

The next relation to try would be an *inverse square Law* in which gravity is one Quarter when the distance doubles, one-ninth when it trebles and so on. At the moon it would be $\frac{1}{(60)^2}$ of 9·81 ms^{-2} *i.e.*, $2{\cdot}72 \times 10^{-3}$ ms^{-2}—the value of moon's centripetal acceleration.

Since forces on bodies depended on their mass ($\vec{F} = m\vec{a}$), Newton concluded that the gravitational force between moon and the earth should depend upon both the mass of the earth and the moon, besides being proportional inversely to the square of the distance between them.

Sir Isaac Newton, after he had formulated his relationship between force and motion, that the moon was held in its orbit around earth by precisely the same kind of force that causes objects near the earth to fall to the ground, expressed this concept now established as a theory and known as the **law of universal gravitation**, which may be stated quantitatively as follows :—

"*Every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of their distance apart.*

The gravitational attraction F between two particles of masses m_1 and m_2, distance 'r' apart is thus given by

$$F \propto \frac{m_1 m_2}{r^2}$$

or

$$\boxed{F = \frac{G m_1 m_2}{r^2}} \qquad \ldots(3\text{-}12)$$

where G is a constant, called the **universal gravitational constant**, and assumed to have the same value everywhere for all matter.

If m_1 and m_2 be each equal to 1 kg and r be equal to 1 m, then according to equation (3-12)

$$F = G\,\frac{1 \times 1}{(1)^2} = G$$

Thus, numerically, *universal gravitational constant* (*G*) *is the force of attraction between two masses of 1 kg each separated by a distance of 1 m.* Henry Cavendish succeeded in determining the value of G in 1798. Improved techniques were adopted by those who made subsequent determinations. The value of G accepted now is $\mathbf{6{\cdot}67 \times 10^{-11}\ N\ m^2/kg^2}$.

We do not ordinarily find the bodies moving towards each other for the simple reason that these forces are *small* and other forces like friction etc., predominate over these forces.

To test $F = \frac{G m_1 m_2}{r^2}$ for the sun and planets the numerical values of all quantities on both sides of the equation need to be known. Newton neither had reliable information about the masses of the sun and planets nor did he know the value of G and so he could not adopt this procedure. In a sophisticated form of checking, he was able to derive Kepler's Law of periods by combining the universal law of gravitation with his own laws of motion. The behaviour of the solar system is summarized by Kepler's Laws and any theory which predicts these would, for a start, be in agreement with the facts.

Suppose a planet of mass m moves with a speed V in a circle of radius r round the sun of mass M. Hence

Gravitational attraction of the sun for planet

$$=\frac{G\,M\,m}{r^2}$$

If this is the centripetal force keeping the planet in orbit ; then

$$\frac{G\,M\,m}{r^2}=\frac{m\,V^2}{r}$$

$$\therefore \qquad \frac{GM}{r}=V^2$$

If T is the time for the planet to make one orbit

$$V=\frac{2\pi r}{T}$$

$$\therefore \qquad \frac{GM}{r}=\left(\frac{2\pi r}{T}\right)^2=\frac{4\pi^2 r^2}{T^2}$$

$$\therefore \qquad GM=\frac{4\pi^2 r^3}{T^2}$$

Hence

$$\frac{r^3}{T^2}=\frac{GM}{4\pi^2}$$

Since GM is constant for any planet, $\frac{r^3}{T^2}$ is constant, which is Kepler's third law.

The grand feature of Newton's triumph was that he could explain the motion of the planets as well as the motion of the bodies falling near the surface of the earth by the *same laws*. This gave a death blow to earlier belief that laws determining the celestial and terrestrial motion were different and the heliocentric theory of copernicus was confirmed. It provided an inertial reference frame, fixed with respect to the sun.

Example 3·5. Compute the gravitational force of attraction between two 5 kg balls ⅓ metre apart.

$$[G=6{\cdot}67\times10^{-11}\ \mathrm{N}\,m^2/\mathrm{kg}^2]$$

Solution.

$$F=\frac{G\,m_1\,m_2}{r^2}$$

$$=\frac{6{\cdot}67\times10^{-11}\times5\times5}{(\frac{1}{3})^2}=1{\cdot}5\times10^{-8}\ \mathrm{N}$$

3·7 RELATION BETWEEN 'g' AND 'G'

A body of mass m at a place on the earth's surface where the acceleration due to gravity is g, experiences a force $F=mg$ (*i.e.*, its weight) due to its attraction by the earth (Fig 3·20). Assuming the earth behaves as if its whole mass M were concentrated at its centre O, then, by the law of gravitation, we can also say that F is the gravitational pull of the earth on the body. Hence

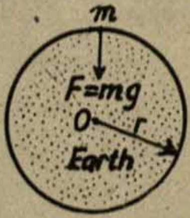


Fig. 3·20

$$F=\frac{G\,M\,m}{r^2}$$

where r is the radius of the earth.

$$\therefore \qquad mg=\frac{GMm}{r^2}$$

$$\therefore \qquad \boxed{g=\frac{GM}{r^2}} \qquad \ldots(3\text{-}13)$$

3·8 ARTIFICIAL EARTH'S SATELLITES AND ORBITAL VELOCITY

The moon is the natural satellite of the earth. For scientists in the space programme the problem of launching an artificial earth satellite (*i.e.*, man-made) is a dual one of over-coming the earth's gravitational force and at the same time providing the satellite with sufficient velocity both in magnitude and in proper direction to begin the desired orbit. Once placed into orbit, the satellite will, of course, be held there by the *centripetal force* provided by the gravitational attraction of the earth. The orbits of real satellites are always elliptical, but we can consider circular orbits.

Figure (3·21) shows the results of four separate efforts to launch satellites. Those taken up along paths 1 and 2 did not have enough speed in the right direction to make it into orbits, while those that went up along routes 3 and 4 did achieve stable orbits by getting above the earth's atmosphere.

Let a satellite of mass m go round the earth in a circular orbit of R with a velocity V. If 'h' is the height of the satellite above the earth's surface and r is the radius of the earth, then $R=(r+h)$. The centripetal force required to keep the satellite in its circular orbit is given by the gravitational pull between the earth and the satellite.

$$\therefore \quad \frac{mV^2}{R} = \frac{GM\,m}{R^2}$$

where M is the mass of the earth and G is the universal gravitational constant.

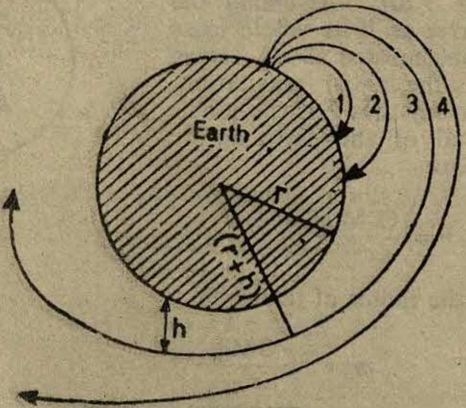


Fig 3·21

or

$$\boxed{V = \sqrt{\frac{GM}{R}} = \sqrt{\frac{GM}{(r+h)}}} \qquad \ldots(3\text{-}14)$$

If g' is the acceleration due to gravity at a height 'h' above the earth's surface then

$$mg' = \frac{GmM}{R^2}$$

or

$$GM = g'\,R^2 = g'\,(r+h)^2 \qquad \ldots(i)$$

Putting equation (i) in (3-14) we have

$$V = \sqrt{\frac{g'\,(r+h)^2}{(r+h)}}$$

or

$$\boxed{V = \sqrt{g'\,(r+h)}} \qquad \ldots(3\text{-}14)$$

Again, if 'g' be the value of acceleration due to gravity at the earth's surface, we have

$$g = \frac{GM}{r^2}$$

Also from eq. (i)

$$g' = \frac{GM}{(r+h)^2}$$

Hence $$\frac{g'}{g}=\frac{GM}{(r+h)^2}\times\frac{r^2}{GM}=\frac{r^2}{(r+h)^2}$$

$\therefore$ $$g'=\frac{gr^2}{(r+h)^2} \qquad \ldots(ii)$$

Putting equation (*ii*) in (3-14) we have

$$V=\sqrt{\frac{gr^2}{(r+h)^2}\cdot(r+h)}$$

or $$V=\sqrt{\frac{gr^2}{(r+h)}}$$

or $$\boxed{V=r\sqrt{\frac{g}{r+h}}} \qquad \ldots(3\text{-}15)$$

If the satellite is close to the earth, say at a height of 100-200 km, then $R\approx r$ and $g\approx g'$

Hence $V=\sqrt{gr}$

Substituting $r=6{\cdot}4\times10^6$ m (6400 km)

and $g=9{\cdot}8\ \text{ms}^{-2}$, we have

$$V=\sqrt{6{\cdot}4\times10^6\times9{\cdot}8}$$
$$=7{\cdot}9\times10^3\ \text{ms}^{-1}\approx 8\ \textit{kms}^{-1}$$

The velocity of about 8 kms^{-1} which is given to an artificial earth's satellite a few hundred kilometres above the earth's surface so that it may start circling the earth is called **Orbital velocity**.

Time Period of the satellite : The time for the satellite to make one complete orbit round the earth *i.e.*, its period T, is

$$T=\frac{\text{circumference of earth}}{\text{speed}}=\frac{2\pi(r+h)}{V}$$

But $$V=\sqrt{\frac{GM}{(r+h)}}$$

$\therefore$ $$T=2\pi(r+h)\sqrt{\frac{(r+h)}{GM}}=2\pi\sqrt{\frac{(r+h)^3}{GM}}$$

or $$\boxed{T=2\pi\sqrt{\frac{(r+h)^3}{gr^2}}} \qquad [\because\ GM=gr^2] \quad \ldots(3\text{-}16)$$

For a satellite close to the earth $(r+h)\approx r$

$$\therefore \quad T=\frac{2\pi r}{V}=\frac{2\pi\times 6{\cdot}4\times 10^6}{7{\cdot}9\times 10^3}$$

$$\approx 83 \text{ minutes.}$$

Launching a satellite

In recent years, several artificial satellites have been launched in orbits round the earth with the help of multistage rockets (Fig 3·22). India has also put in orbit an earth satellite "*ARYABHATA*" on 19th April, 1975 named after the celebrated astronomer Aryabhata of the fifth century A.D.

To be placed in orbit, a satellite must be raised to the desired height and given the correct speed and direction by the launching rocket. A typical launching sequence using a two-stage rocket might be as follows :

At lift-off, the rocket, with a manned or unmanned space capsule on top, is held down by clamps on the launching pad for a few seconds until the exhaust gases have built-up an upward thrust which exceeds the rocket's weight. The clamps are then removed by remote control and the rocket accelerates upwards.

To penetrate the dense lower part of the atmosphere by the shortest possible route, the rocket rises vertically initially and after this is gradually tilted by the guidance system. The first-stage rocket, which may burn for about 2 minutes producing a speed of 3 km s^{-1} or so, lifts the vehicle to a height of around 60 km, then separates and falls back to earth, landing many kilometres from the launching site. [Fig. 3·22 (*b*)]

The vehicle now coasts in free flight (unpowered) to its orbital height, say 160 km, where it is momentarily moving horizontally (*i.e.*, parallel to the earth's surface immediately below). The second stage rocket then fires and increases the speed to that required for a circular orbit at this height (about 8 km s^{-1}). By firing small rockets, the capsule is separated from the second stage which follows behind, also in orbit.

The equation $V^2=g\,\frac{r^2}{(r+h)}$ for circular orbits shows that each orbit requires a certain speed and the greater the orbit radius $(r+h)$, the smaller the speed V.

Some notable space flights are given below. Synchronous satellites have a period of 24 hours, exactly the same as that of the earth, and so remain in the same position above the earth, apparently stationary. By acting as relay stations, they make continuous, world-wide communications *e.g.*, of television programmes possible.

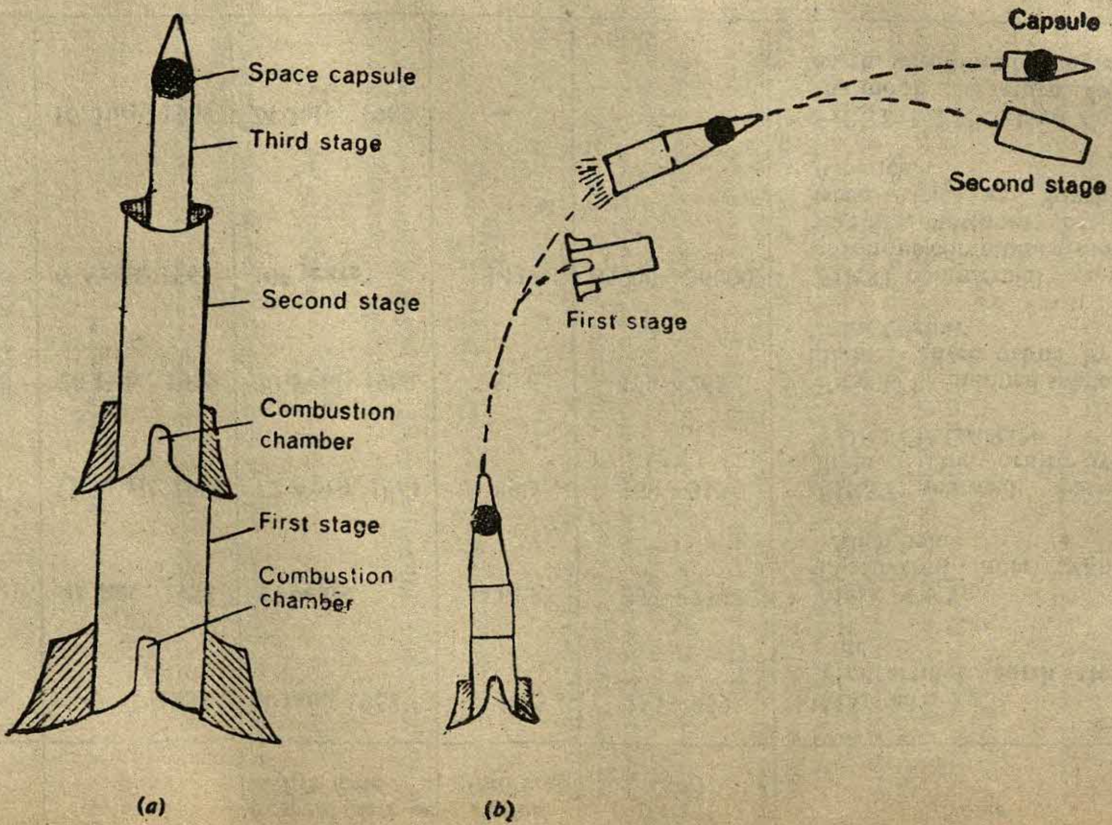


Fig. 3·22 Satellite launching

Name	*Launch date*	*Descent date or life time*	*Period (minute)*	*Height (km)*	*Notes*
Sputnik 1	4 Oct., 1957	4 Jan., 1968	96·2	215—939	Mass 83·6 kg. First artificial earth satellite.
Explorer 1	31 Jan., 1958	11 years	114·8	356—2548	Mass 14·0 kg Discovered inner Van Allen belt.
Vostok 1	12 April, 1961	12 April, 1961	89·3	169—315	FIRST manned space flight. One orbit by YURI GAGARIN
Mercury 6	20 Feb., 1962	20 Feb., 1962	88·6	159—265	First U.S. manned space flight. Three orbits by John Glenn.
Early Bird	6 April, 1965	10^6 years	1437	35000—36000	FIRST commercial synchronous communications satellite, stationary between Africa and South America.
Apollo 11	16 July, 1969	24 July, 1969	—	—	FIRST MEN to land on the moon and return to earth, with 'moon samples'.

3·9 VELOCITY OF ESCAPE

The faster a ball is thrown upwards the higher does it rise before it is stopped and pulled back by gravity. To escape from the earth into outer space we will show shortly that an object must have a speed of just over *11 km* s^{-1}—called the *escape velocity* Thus escape velocity may be defined as *the velocity with which an object must be projected in order that it may escape the earth's gravitational pull.*

The multistage rockets in use at present burn their fuel in a comparatively short time to obtain the best performance. They behave rather like objects thrown upwards *i.e.*, like projectiles. For a journey to, say, the moon, they, therefore, have to attain the escape velocity. In many cases this is done by first putting the final vehicle into a *parking* orbit round the earth with a speed of 8 kms^{-1} and then firing the final rocket again to reach escape velocity in the appropriate direction. The attainment of the escape velocity is not a necessary condition however. The essential thing is that a certain amount of *energy* is required to escape from the earth.

The escape velocity is obtained from the fact that the potential energy gained by the body equals its loss of kinetic energy, if air resistance is neglected. The work done measures the energy change. Let '*m*' be the mass of the escaping body and M the mass of the earth. The force F exerted on the object by the earth when it is distance *x* from the centre of the earth is

$$F=\frac{GMm}{x^2} \text{ where } G=\text{Gravitational constant}$$

Therefore work done *d*W by gravity when the body moves a further short distance *dx* upwards is

$$dW=-Fdx=-\frac{GMm}{x^2}dx.$$

(the negative sign shows the force acts in the opposite direction to the displacement).

Therefore, total work done while body escapes

$$=\int_r^{\infty}\frac{-GMm}{x^2}\cdot dx$$

where r=radius of the earth.

$$=-GMm\left[-\frac{1}{x}\right]_r^{\infty}$$

$$=GMm\left[\frac{1}{x}\right]_{r}^{\propto}$$

$$=\frac{-GMm}{r}$$

If the body leaves the earth with speed V and *iust* escapes from its gravitational field

$$\tfrac{1}{2}mV^2=\frac{GMm}{r}$$

$$\therefore \qquad \boxed{V=\sqrt{\frac{2GM}{r}}} \qquad \text{... (3-17)}$$

But $g=\frac{GM}{r^2}$ where g=Acceleration due to gravity.

$$\therefore \qquad \boxed{V=\sqrt{2gr}} \qquad \text{... (3-18)}$$

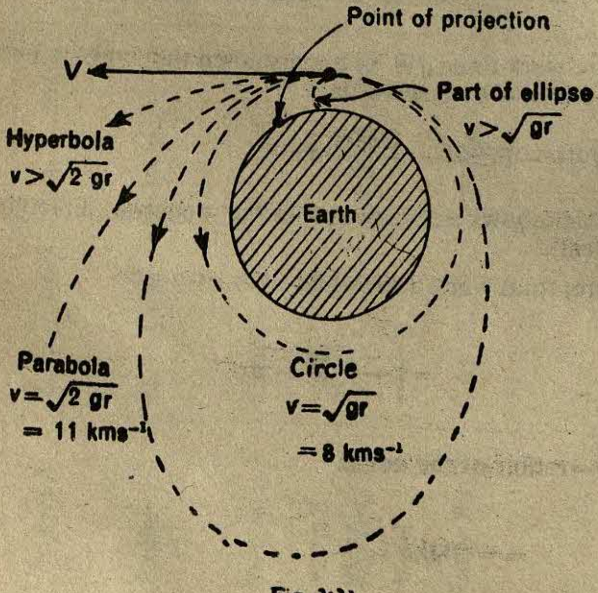


Fig. 3·23

Substituting $r=6{\cdot}4\times10^6\text{m}$

and $g=9{\cdot}8\ \text{ms}^{-2}$, we get

$$V=\sqrt{2\times9{\cdot}8\times6{\cdot}4\times10^6}$$
$$=11{\cdot}2\ kms^{-1}$$

Possible paths for a body projected at different speeds from the earth are shown in (Fig. 3·23).

The earth has atmosphere whereas the moon has no atmosphere. Can you suggest a possible reason ?

Example 3·6. Calculate the escape velocity on the surface of the moon. Given the radius of the moon $=1{\cdot}7\times10^6$ m and mass of the moon $=7{\cdot}35\times10^{22}$ kg and $G=6{\cdot}67\times10^{-11}\ \text{Nm}^2/\text{kg}^2$.

Solution.

According to equation (3-17), the escape velocity is given by

$$V=\sqrt{\frac{2GM}{r}}$$

where M = mass of the moon

and r = Radius of the moon

$$=\sqrt{\frac{2\times6{\cdot}67\times10^{-11}\times7{\cdot}35\times10^{22}}{1{\cdot}7\times10^6}}$$
$$=2{\cdot}4\times10^3 ms^{-1}$$

Example 3·7. A satellite is revolving in circular orbit at a height of 1000 km from the surface of the earth. Calculate the orbital velocity and time of revolution. Given, radius of the earth $=6380$ km; mass of earth $=6\times10^{24}$kg and $G=6{\cdot}67\times10^{-11}\ \text{N}m^2/\text{kg}^2$.

Solution.

Orbital velocity of the satellite is given by

$$V=\sqrt{\frac{GM}{(r+h)}}$$

in which $M=6\times10^{24}$ kg

$r=6380$ km

$h=1000$ km

$$\therefore \quad V=\sqrt{\frac{6{\cdot}67\times10^{-11}\times6\times10^{24}}{7380\times10^3}}$$
$$=7{\cdot}35\times10^3 ms^{-1}$$

$$\text{Time Period } T=\frac{2\pi(r+h)}{V}=\frac{2\times\frac{22}{7}\times7380\times10^3}{7{\cdot}35\times10^3}$$
$$=6305 \text{ sec}$$
$$=1 \text{ hour } 45 \text{ minutes } 5 \text{ sec.}$$

3·10 RIGID BODY ROTATION

Rigid Body. A body is said to be rigid if it does not undergo any change in shape under the action of forces. It offers resistance to a force tending to change its shape.

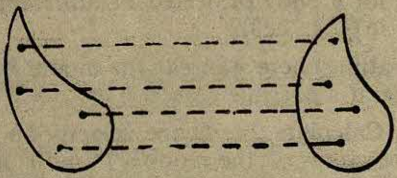


Fig. 3·24

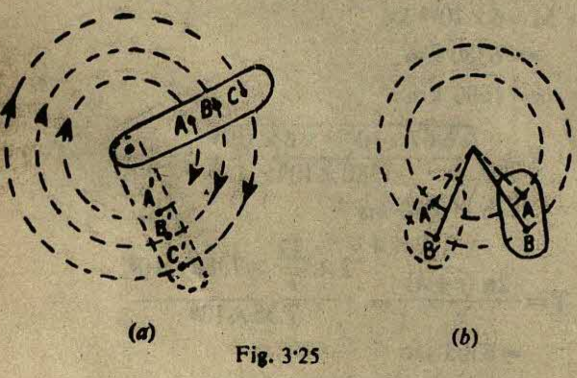
A rigid body may also be defined as follows :

A perfectly rigid body is one in which the distance between any pair of particles or atoms of which it is composed of remains same when forces, however great, act on it.

If the body suffers a simple displacement, all the atoms undergo the same displacement.

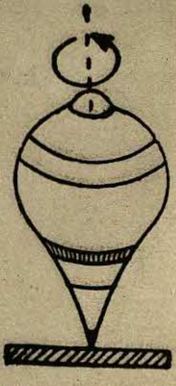
There is not a single body which conforms to the definition given above. But for all practical purposes all solids may be regarded as rigid bodies because the yielding is small. Thus, a railway bridge girder is perfectly rigid though it does bend to a slight extent under its own weight and to a slightly greater extent under the weight of a running train. Similarly a wooden metre rod is considered to be perfectly rigid when forces of the order of say 2N are applied on it.

Translatory Motion. A rigid body is said to exectue **translatory** motion if it moves bodily form one place to another



(a) (b)

Fig. 3·25



(A spinning top is a rotating rigid body)

Fig. 3·26

in such a way that all its particles generate parallel lines with the same speed as shown in (Fig 3·24). The motion of a train along its track is translatory in character.

Rotatory Motion. A rigid body is said to execute **rotatory** motion if it rotates about a fixed axis in such a way that its particles generate concentric circles with the *same angular velocity* but with *different linear velocities* (Fig. 3·25). The motion of a wheel of a train about its axle is rotatory in character. The motion of the axle, however, is translatory. A spinning top (Fig. 3·26) is a rotating rigid body. The reader should visualize the earth rotating about a line through the north and south poles.

3·11 ANGULAR ACCELERATION

When the angular velocity of a rotating body is not uniform, it is said to possess angular acceleration. **The angular acceleration of a rigid body in pure rotation is defined as the rate of change of angular velocity.**

If the angular velocity increases uniformly and ω_1 and ω_2 are the angular velocities at time intervals t_1 and t_2, the uniform angular acceleration is given by

$$\boxed{\alpha=\frac{\omega_2-\omega_1}{t_2-t_1}=\frac{\Delta\omega}{\Delta t}} \quad ...(i)$$

or

$$\boxed{\omega_2=\omega_1+\alpha(t_2-t_1)} \quad ...(ii)$$

Equation (*ii*) in rotatory motion is the counterpart of

$$V=u+at$$

in translatory or linear motion where the symbols have their usual meanings. The instantaneous angular acceleration is the limit when Δt approaches zero.

∴ Instantaneous angular accleration

$$=\alpha=\lim_{\Delta t\to 0}\left(\frac{\Delta\omega}{\Delta t}\right)=\frac{d\omega}{dt}.$$

$$=\frac{d}{dt}\left(\frac{d\theta}{dt}\right)=\frac{d^2\theta}{dt^2}$$

or

$$\boxed{\alpha=\frac{d\omega}{dt}=\frac{d^2\theta}{dt^2}} \quad ...(3\text{-}19)$$

Angular acceleation is measured in *rad sec^{-2}*.

Relation between Angular Acceleration and Linear Acceleration.

Let us consider a rigid body rotating about the axis through O with variable angular velocity (Fig. 3·27). Any point such as B distant r from the axis of rotation moves along a circular arc of radius r. Let the linear velocity of B at any instant t_1 be V_1 corresponding to its angular velocity ω_1. At another instant t_2, when B takes up the position B′, let its linear velocity be V_2 corresponding to angular velocity ω_2. Then linear acceleration

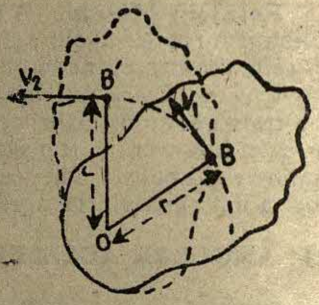


Fig. 3 27

$$a=\frac{V_2-V_1}{(t_2-t_1)}$$

But $V_1=r\omega_1$ and $V_2=r\omega_2$

$$\therefore \quad a=\frac{\omega_2 r-\omega_1 r}{(t_2-t_1)}$$

$$=\frac{r(\omega_2-\omega_1)}{(t_2-t_1)}$$

$$\therefore \quad a=r.\,\frac{\Delta\omega}{\Delta t}$$

or

$$\boxed{a=r\alpha} \qquad \ldots(32\text{-}0)$$

∴ *Linear Acceleration = radius × angular acceleration*

3·12 MOMENT OF INERTIA (Rotational Kinetic Energy)

Let us consider a body rotating with a uniform angular velocity ω about an axis through O perpendicular to the plane of the paper (Fig. 3·28). Each particle of the body possesses some kinetic energy by virtue of its motion, their sum total being equal to the kinetic energy of the rotating body as a whole. To calculate this energy, let a particle of mass m_1 distant r_1 from the axis of rotation, move with a linear velocity V_1. Its kinetic energy then is $\frac{1}{2}m_1V_1^2$. But $V_1=\omega r_1$. Therefore, the kinetic energy of the particle is $\frac{1}{2}m_1(V_1)^2=\frac{1}{2}m_1r_1^2\omega^2$. Similarly, a particle of mass m_2 at a distance r_2 from O and moving with a linear velocity V_2 has kinetic energy equal to $\frac{1}{2}m_2V_2^2=\frac{1}{2}m_2(\omega r_2)^2=\frac{1}{2}m_2r_2^2\omega^2$ and so on. Hence the total kinetic energy of the body is given by

$$\tfrac{1}{2}m_1r_1^2\omega^2+\tfrac{1}{2}m_2r_2^2\omega^2+\ldots\ldots$$

$$\frac{1}{2}m_n\,r_n^2\,\omega^2$$

$$=\tfrac{1}{2}(m_1r_1^2+m_2r_2^2+\ldots\quad)\omega^2$$

The term (Σmr^2) is the sum of the products of the masses of the various particles and the squares of their respective distances from the axis of rotation. It is called **the moment of inertia or the rotational inertia of the given body and it depends upon the particular axis about which the body rotates and the distribution of the mass of the body about that axis. Hence moment of inertia of a rigid body about a fixed axis may be defined as the sum of the products of the masses of all the particles constituting the body and the squares of their respective distances from the axis of rotation.**

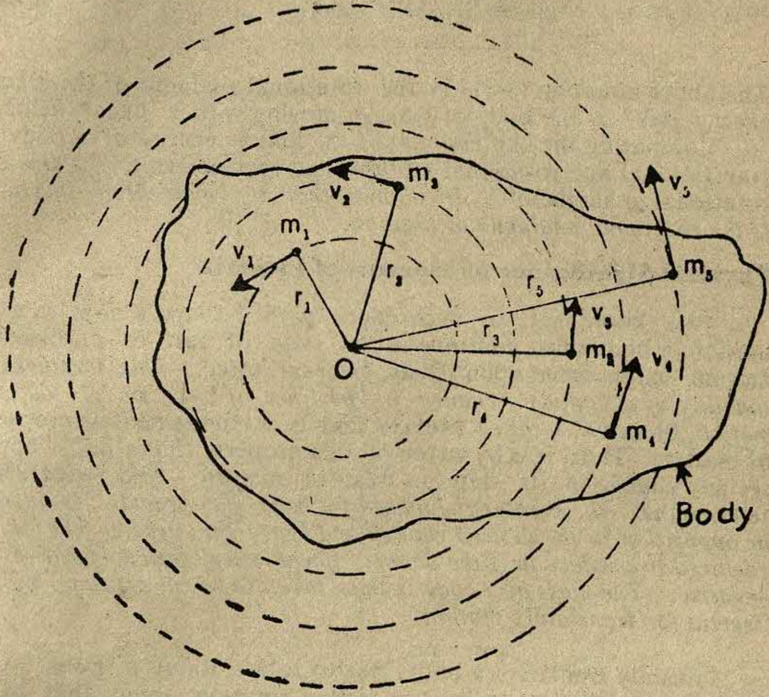


Fig. 3·28

In calculus notation, the expression for the moment of inertia may be written as

$$I = \int r^2 \, dm$$

where dm is the mass element at a perpendicular distance r from the axis of rotation and the integral is extended over all the mass elements of the body.

Denoting the moment of inertia by I, the expression for kinetic energy of rotation becomes

$$\boxed{\text{K.E.} = \tfrac{1}{2}\, I\omega^2} \qquad \text{..(3-21)}$$

The above equation (3-21) is the rotational analogue of the kinetic energy $\frac{1}{2}mV^2$ or the body of mass m moving with a linear velocity V. Comparing the expressions for the kinetic energy of a body in translational and rotational motions and remembering that ω is the rotational analogue of V, *it follows that the moment of Inertia I is the rotational analogue of mass m.*

Physical Significance of Moment of Inenrtia

We know that, in accordance with Newton's first law of motion, a body must continue in its state of rest or of uniform motion, unless acted upon by an external force. *This inertness or inability of a body to change its position of rest or of uniform motion, by itself, is called* **inertia** and is a fundamental property of matter. Thus, it is by virtue of this property that a body opposes any change in its state of rest or motion. And, experience tells us that *the greater the mass of a body, the greater its inertia or opposition to the desired change and hence the greater the force required to produce in it the desired linear acceleration, positive or negative. The mass of a body is thus taken to be a measure of its 'inertia for translatory motion.'*

Exactly similarly, a body, free to rotate about a given axis, opposes both the state of rest as well as rotation about that axis. It thus possesses *inertia for rotational motion.* And, clearly, the greater the couple required to produce in it a desired angular acceleration (positive or negative), the greater its *inertia for rotational motion.* This *rotational inertia of the body is called its* **moment of inertia,** on the analogy of the moment of the couple, which it opposes.

It will thus be clear that *the moment of inertia of a body plays the same part as, or is analogous to, its mass in the case of translatory motion.* So that we may define *mass* as the '*coefficient of inertia for translatory motion*' and *moment of inertia* as the '*coefficient of rotational inertia.*'

With all this seeming similarity, however, there is yet an important difference between the two cases, *viz.*, that whereas in the case of translatory motion, *the inertia of the body depends wholly upon its mass* and is, therefore, measured in terms of mass (M) alone, in the case of rotational motion, *the moment of inertia depends upon mass (M) of the body as well as the effective distance (K) of its particles from the axis of rotation, i.e., on the distribution of* *mass about the axis*, and is measured by MK^2. Thus, for example, the moment of inertia of a ring will be greater than that of a disc about a perpendicular axis through the centre, even though the mass and radius of either be the same. For, obviously, the effective distance (K) of the particles from the axis of rotation is greater in the case of the ring than in the case of the disc.

Radius of Gyration

Radius of gyration may be defined as *the distance from the axis of rotation at which, if the whole mass of the body is concentrated, its moment of inertia about the axis is the same as that with the actual distribution of mass.*

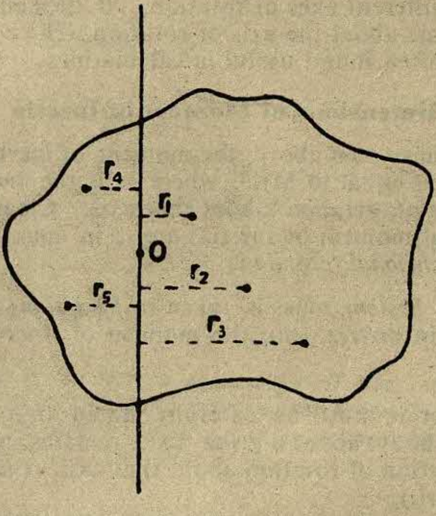


Fig. 3·29

To determine the radius of gyration, let us consider a body of mass M, consisting of n particles of mass m each, situated at distances $r_1, r_2, r_3 \ldots\ldots, r_n$ from the axis of rotation (Fig. 3.29). Then the moment of inertia of the body about the given axis is given by

$$
\begin{aligned}
I &= mr_1^2 + mr_2^2 + mr_3^2 + \ldots\ldots + mr_n^2 \\
&= m(r_1^2 + r_2^2 + r_3^2 + \ldots\ldots + r_n^2) \\
&= m \times n \left(\frac{r_1^2 + r_2^2 + r_3^2 + \ldots\ldots + r_n^2}{n} \right) \\
& M \left(\frac{r_1^2 + r_2^2 + r_3^2 + \ldots + r_n^2}{n} \right)
\end{aligned}
$$

where $M = m \times n =$ total mass

If K is the radius of gyration about the given axis, we also have $I = MK^2$

$$\therefore \qquad MK^2=M\left(\frac{r_1^2+r_2^2+r_3^2+\ldots\ldots r_n^2}{n}\right)$$

or

$$K=\sqrt{\frac{r_1^2+r_2^2+r_3^2+\ldots\ldots+r_n^2}{n}}$$

$$=\sqrt{\text{Mean square distance}}$$

$$=\text{Root mean square distance}$$

Thus the radius of gyration of a body about a given axis is equal to the root mean square distance of its particles about the axis of rotation. The radius of gyration for a given body will be different about different axes of rotation. It depends upon the distribution of mass about the axis of rotation. The concept of radius of gyration is often found useful in calculations.

Units and dimensions of Moment of Inertia

As we have seen above, the moment of inertia of a body about a given axis is equal to MK^2; where M is the *mass* of the body and K, its *radius* of *gyration* about that axis. Clearly, therefore, the *dimensions* of moment of Inertia are 1 in mass and 2 in length, so that its *dimensional formula* is M^1L^2.

In *SI system*, mass is taken in *kilograms* (kg) and the radius of gyration in *metres* (m), the moment of Inertia is expressed in **kg m²**.

Further it must be carefully noted that since moment of Inertia of a body, about a given axis, remains unaffected by reversing its direction of rotation about that axis, *it is a pure and simple Scalar Quantity.*

3·13 TWO GENERAL THEOREMS ON MOMENT OF INERTIA

1. **The law (or theorem) of perpendicular axis.** This theorem states that *the moment of inertia of a plane lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about two axes at right angles to each other, in its own plane, and intersecting each other at the point where the perpendicular axis passes through it.*

Proof. Let I_x and I_y be the respective moments of inertia of a plane lamina about the axes *OX* and *OY*, at right angles to each other *lying in the plane of the lamina* and intersecting each other at *O*. (Fig. 3·30). Then, if I be moment of inertia of the lamina about an axis *perpendicular to the plane of the lamina* and passing through *O*, we have

$$I=I_x+I_y.$$

For, considering a particle of mass m at P, distant x and y from OY and OX and r from O respectively, we clearly have

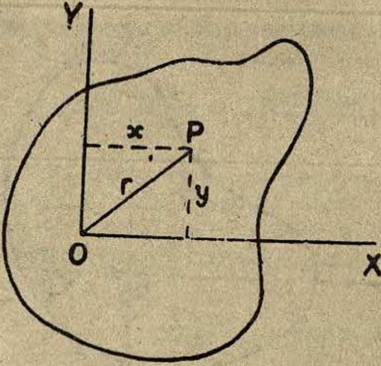


Fig. 3·30

$$I = \Sigma mr^2,$$

$$I_y = \Sigma my^2 \quad \text{and} \quad I_x = \Sigma mx^2.$$

So that,

$$I_x + I_y = \Sigma my^2 + \Sigma mx^2 = \Sigma m(y^2 + x^2)$$

$$= \Sigma mr^2. \qquad [\because y^2 + x^2 = r^2]$$

Or,

$$\boxed{\mathbf{I_x + I_y = I}} \qquad \text{...(3-22)}$$

i.e., moment of inertia of the lamina about axis OY plus its moment of inertia about axis OX, both in the plane of the lamina, is equal to moment of inertia of the lamina about an axis perpendicular to its plane and passing through O, the point of intersection of the two axes OY and OX.

2. **The principle (or theorem) of parallel axes.** This theorem, due to *Steiner*, states that *the moment of inertia of a body about any axis is equal to its moment of inertia about a parallel axis through its centre of mass (or c.g.) plus the product of its mass and the square of the distance between the two axes.*

Proof. Let AB be the axis, in the plane of the paper, about which the moment of inertia of the body is to be determined and CD, a parallel axis through O, the centre of mass of the body, at a distance r from AB.

Consider a particle or a small element of the body, of *mass m* at P, at a distance x from CD. Then, clearly, moment of inertia of m about $AB = m(x+r)^2$.

And $\therefore$ *moment of inertia of the whole body* about AB, *i.e.,*

$$I = \Sigma m(x+r)^2.$$

Or,

$$I = \Sigma mx^2 + \Sigma mr^2 + 2\Sigma mxr.$$

Obviously, $\Sigma mx^2 = I_g$, the moment of inertia of the body about the axis CD through the centre of mass.

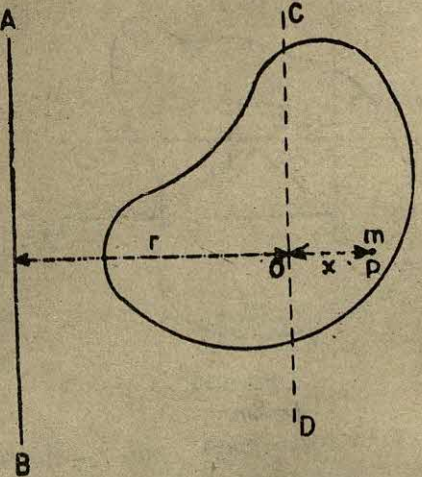


Fig. 3·31

So that, $$I = I_g + \Sigma mr^2 + 2\Sigma mxr.$$

Now $\Sigma mr^2 = Mr^2$, where M is the mass of the whole body, and Σmx, the moment of all its particles about the axis CD and, therefore, equal to *zero* because, as we know, a body always balances about its centre of mass, indicating that the algebraic sum of the moments about it is zero. We, therefore, have

$$\boxed{\mathbf{I = I_g + Mr^2}.} \qquad ...(3\text{-}23)$$

Or, *moment of inertia of the body about axis AB*
= its moment of inertia about the parallel axis CD through its centre of mass + mass of the body × (distance between the two axes)².

3·14 MOMENT OF INERTIA OF A UNIFORM CIRCULAR DISC

(*a*) *Moment of inertia of a disc about an axis through its centre and perpendicular to its plane.* Let M be the mass of the disc and R, its radius, [Fig. 3·32(*a*)] so that its *mass per unit area* $= M/\pi R^2$.

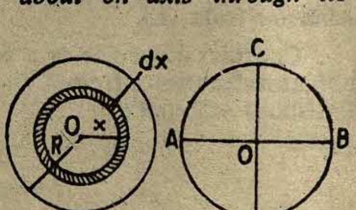


Fig. 3·32

Consider a ring of the disc, distant x from its centre O, *i.e.*, *of radius* x and of *width* dx. Then, clearly, the *area of this ring = its* *circumference* × *its width*, or equal to $2\pi x.dx$.

This is because when open up, it can be considered to be a rectangle of length $= 2\pi x$ and width $= dx$ $\quad \therefore$ Area $= 2\pi x\, dx$

$$\text{Hence, } \textit{its mass} = \frac{M}{\pi R^2} \times 2\pi x.dx = \frac{2Mx.dx}{R^2}.$$

$\therefore$ *moment of inertia of this ring about an axis through O and perpendicular to its plane* (*i.e., the plane of the disc*)

$$= \frac{2Mx.dx}{R^2} \cdot x^2 = \frac{2Mx^3.dx}{R^2}.$$

Since the whole disc may be supposed to be made up of such like concentric rings, of radii ranging from O to R, we can obtain the moment of inertia (I) of the disc by integrating the above expression for the moment of inertia of the ring between the limits $x=0$ and $x=R$.

$$\text{Thus,} \qquad I = \int_0^R \frac{2Mx^3}{R^2}.dx = \frac{2M}{R^2}\left[\frac{x^4}{4}\right]_0^R = \frac{2M}{R^2} \cdot \frac{R^4}{4}.$$

$$\text{Or,} \qquad \boxed{\mathbf{I = MR^2/2.}} \qquad \text{...(3-24)}$$

(*b*) *Moment of inertia of the disc about a diameter.* Let AB and CD be two perpendicular diameters of the circular disc, [Fig. 3·32 (*b*)]. Then, since all diameters are alike, the moment of inertia of the disc about one diameter is the same as that about another. So that, its *moment of inertia about AB = its moment of inertia about CD = I*, say,

Now, *by the principle of perpendicular axes*, we have

moment of inertia of the disc about AB + its moment of inertia about CD = its moment of inertia about the axis through O and perpendicular to its plane.

$$\text{So that,} \qquad I + I = MR^2/2 \quad \text{or} \quad 2I = MR^2/2,$$

$$\text{whence,} \qquad \boxed{\mathbf{I = MR^2/4}} \qquad \text{...(3-25)}$$

Note. The formula for the moment of Inertia of a right cylinder about its axis is the same as that obtained above for a uniform circular disc, namely $\frac{1}{2}MR^2$, because a cylinder is a disc of large thickness.

3·15 MOMENT OF INERTIA OF AN ANNULAR RING AND A UNIFORM CIRCULAR RING (OR HOOP)

(*a*) **Moment of inertia of an annular ring about a perpendicular axis through its centre.** An annular ring is just a circular disc from which a smaller, concentric disc has been removed, leaving a hole in its place, as shown in Fig (3·33).

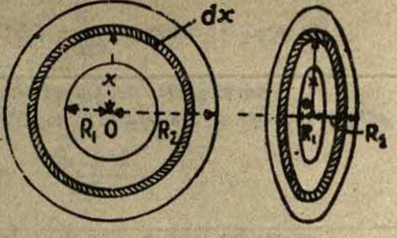


Fig 3·33

Let R_1 and R_2 be the inner and outer radii of the annular disc and O, its centre. Then, clearly, *face area of the disc* $= \pi(R_2^2 - R_1^2)$ and, therefore, if M be its mass, we have

$$\textit{mass per unit area of the disc} = M/\pi(R_2^2 - R_1^2).$$

Now, consider a co-axial ring of radius x and width dx, shown dotted in the figure. We clearly have

face area of this ring $= 2\pi x.dx$ and, therefore, its *mass*

$$= \frac{M}{\pi(R_2^2 - R_1^2)} \times 2\pi x.dx = \frac{2Mx}{(R_2^2 - R_1^2)}.dx.$$

Hence, *moment of inertia of this ring about an axis through O and perpendicular to its plane*

$$= \frac{2Mx}{(R_2^2 - R_1^2)} \cdot dx \times x^2 = \frac{2Mx^3}{(R_2^2 - R_1^2)} \cdot dx.$$

The *moment of inertia* (*I*) *of the whole annular disc about this axis through O and perpendicular to its plane* is thus easily obtained by integrating the above expression for the moment of inertia of the ring, between the limits $x = R_1$ and $x = R_2$. So that, we have

$$I = \int_{R_1}^{R_2} \frac{2Mx^3}{(R_2^2 - R_1^2)}.dx = \frac{2M}{(R_2^2 - R_1^2)} \int_{R_1}^{R_2} x^3.dx$$

$$= \frac{2M}{(R_2^2 - R_1^2)} \left[\frac{R_2^4 - R_1^4}{4}\right] = \frac{2M}{R_2^2 - R_1^2} \left[\frac{(R_2^2 + R_1^2)(R_2^2 - R_1^2)}{4}\right]$$

or,

$$\boxed{I = \frac{M(R_2^2 + R_1^2)}{2}} \qquad \ldots(3\text{-}26)$$

or, *moment of inertia of the annular disc about a perpendicular axis through its centre*

$$= M(R_2^2 + R_1^2)/2.$$

(*b*) **Moment of inertia of an annular disc about its diameter.** From the symmetrical shape of an annular disc it is clear that its moment of inertia about one diameter will be the same as about another. Let it be I. Then, by the principle of perpendicular axes, we have *sum of the moments of inertia about two perpendicular diameters =moment of inertia about a perpendicular diameter through O.*

or
$$I+I=\frac{M(R_2^2+R_1^2)}{2}$$

or
$$2I=\frac{M(R_2^2+R_1^2)}{2}$$

whence,
$$\boxed{I=\frac{M(R_2^2+R_1^2)}{4}} \qquad \text{...(3-27)}$$

or Moment of Inertia of an annular disc about a diameter
$$=\frac{M(R_2^2+R_1^2)}{4}$$

(*c*) **Moment of Inertia of a circular disc.**

(*i*) *About a perpendicular axis through its centre :*

If $R_1=0$, we have a plane circular disc. So that, putting $R_1=0$ and $R_2=R$, the *radius of the disc*, we have from equation (3-26)

$$\boxed{I=\frac{MR^2}{2}} \text{ i.e.} \qquad \text{...(3-28)}$$

Moment of Inertia of a plane circular disc about a perpendicular axis through its centre$=\frac{1}{2}MR^2$

(*ii*) *About a diameter*

Putting $R_2=R$ and $R_1=0$ in equation (3-27), we have

$$\boxed{I=\frac{MR^2}{4}} \qquad \text{...(3-29)}$$

(*d*) **Moment of Inertia of a circular ring :**

(*i*) *About an axis through its centre and perpendicular to its plane :*

If $R_1=R_2$, we have a **circular ring** or **hoop**. So that, putting $R_1=R_2=R$, the radius of the ring, we have from equation (3-26)

$$I=\frac{M(R^2+R^2)}{2}$$

or $$\boxed{I = MR^2} \qquad \ldots(3\text{-}30)$$

i.e., Moment of Inertia of a ring about a perpendicular axis through its centre $= MR^2$.

(*ii*) ***About a diameter :***

Putting $R_1 = R_2 = R$ in equation (3-27), we have

$$I = \frac{M(R^2 + R^2)}{4}$$

or $$\boxed{I = \frac{MR^2}{2}} \qquad \ldots(3\text{-}31)$$

Here are the formulae for moments of Inertia in a few simple cases :—

(1) The moment of inertia of a uniform rod of mass 'M' and length '*l*' about an axis through its centre and perpendicular to the length of the rod

$$= \frac{Ml^2}{12} \qquad \ldots(3\text{-}32)$$

(2) The moment of Inertia of a uniform rod of mass M and length '*l*' about an axis through one end and perpendicular to the length of the rod

$$= \frac{Ml^2}{3} \qquad \ldots(3\text{-}33)$$

(3) The moment of Inertia of a solid sphere of mass M and radius '*r*' about any diameter

$$= \frac{2}{5} Mr^2 \qquad \ldots(3\text{-}34)$$

Example 3·8. A thin metal hoop of radius 0·25 m and mass 2 kg starts from rest and rolls down an inclined plane. If its linear velocity on reaching the foot of the plane is 2 ms^{-1}, what is its rotational kinetic energy at that instant.

Solution.

Here $M = 2$ kg

$R = 0{\cdot}25$ m

$V = 2$ ms^{-1}

The moment of inertia of a thin circular hoop (or ring) about an axis passing through its centre and perpendicular to the plane of the hoop is

$$
\begin{aligned}
I &= MR^2 \\
&= 2\times(0{\cdot}25)^2 \\
&= \tfrac{1}{8}\ \text{kgm}^2
\end{aligned}
$$

and $\omega = \frac{V}{R} = \frac{2}{0{\cdot}25} = 8\ \text{rad s}^{-1}$

So, rotational Kinetic Energy $= \frac{1}{2} I\omega^2$

$$
\begin{aligned}
&= \tfrac{1}{2}\times(\tfrac{1}{8})\times(8)^2 \\
&= 4\ \text{joules}
\end{aligned}
$$

Example 3·9. A solid cylinder rolls down an inclined plane. Its mass is 2 kg and radius 0·1 m. If the height of the inclined plane is 4 m, what is its rotational kinetic energy when it reaches the foot of the plane.

Solution.

Since $Mgh = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$

Angular velocity $(\omega) = \sqrt{\frac{4gh}{3R^2}}$ as $I = \frac{MR^2}{2}$

and $V = \omega R$

$\therefore$ Rotational K.E. $= \frac{1}{2}I\omega^2$

$$
\begin{aligned}
&= \frac{Mg\,h}{3} \\
&= \frac{2\times 9.8\times 4}{3} \\
&= 26.13\ \text{joules}
\end{aligned}
$$

3·16 TORQUE AND ROTATORY MOTION

Consider a rigid body capable of rotating about a fixed axis passing through O [Fig. 3·34] and perpendicular to the plane of the diagram. A force F applied at P will cause the body to rotate with some angular acceleration. If the same force is applied at the point Q farther from the axis of rotation, the body is found to rotate with greater angular acceleration. *It thus follows that the rotational acceleration produced in a body by a given force depends not only upon the magnitude of the force but also on the point of its application.*

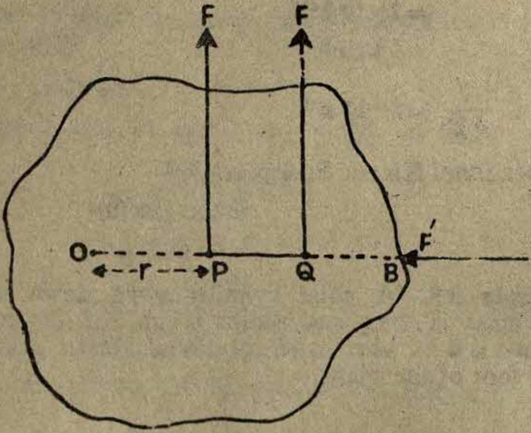


Fig. 3·34 Torque and Rotatory Motion

Now apply another force say F′ at a point B such that the line of action of the force passes through the point of rotation O. The force in this case fails to produce any rotation at all. Thus the rotational acceleration produced in a body by a given force depends also upon its line of action. *No rotational acceleration is produced if the line of action of the force passes through the point of rotation.*

This tendency of a force to rotate a body about a given point is called moment of the force or **torque** *and is given by the product of the force and the perpendicular distance between the line of action of the force and the axis of rotation.*

The perpendicular distance is called lever arm or moment arm. In (Fig. 3·34) the *torque* exerted by the force $\vec{F}$ acting at P is given by

$$\boxed{\vec{\tau} = \vec{OP} \times \vec{F}}$$

or

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}} \quad ...(3\text{-}35)$$

where $\vec{\tau}$ = torque

$\vec{r}$ = lever arm

Since the force $\vec{F}$ and the lever arm $\vec{r}$ are vectors, the torque $\vec{\tau}$ can be written as the *cross-product* of these two vectors. The magnitude of the torque is $|rF|$ and its direction is perpendicular to the plane of $\vec{r}$ and $\vec{F}$ and is given by *right hand screw rule.*

Units of Torque :

Torque is measured in *N-m* (*newton-metre*) in SI system. Its dimensions are ML^2T^{-2}.

It must be noted that though torque has the same dimensions as that of work. Yet they are different physical quantities. In measuring work, force and displacement are along the *same* direction whereas in the case of torque, they are perpendicular to each other.

3·17 WORK DONE BY TORQUE

A pair of equal, unlike, parallel forces having different lines of action is called a **couple**. The vector sum of the forces in a couple is *zero* so that a couple cannot produce any motion of translation. A couple only tends to rotate a body about a given axis and the *torque* exerted by couple is measured by the product of either force and perpendicular distance between their lines of action *i.e.*, $\vec{\tau} = \vec{F} \times \vec{r}$. The perpendicular distance $\vec{r}$ is known as the *arm of the couple.*

Consider a rigid body capable of rotating about a fixed axis through O and perpendicular to the plane of the diagram. Let the body be subjected to the action of a couple, consisting of two equal and opposite forces F, F, one acting at O and the other at P distance r from O (Fig. 3·35). Further, the forces are assumed to be perpendicular to OP. As a result of the applied couple, let the body get displaced through a very small angle $d\theta$ so that P covers along the circular arc a corresponding small distance PP′ = ds.

The work done by the force F acting at P during small angular displacement $d\theta$ is given by

$$dW = F.ds = F.rd\theta \qquad (\because\ ds = r.d\theta)$$

Since the point O remains fixed, no work is done by the force acting at O. Thus the total work done by the two forces or the couple = $F.r.d\theta$. But $F.r.$ is the moment of the force F or its torque about O.

If τ is the torque, then

$$dW = \tau . d\theta.$$

If the body is displaced through an angle θ, the total work done is given by

$$W = \int_0^\theta \tau d\theta.$$

or $$\boxed{W = \tau\theta} \qquad ...(3\text{-}36)$$

or *Work done by a couple (or torque)*

= Torque of the couple × Angle of rotation

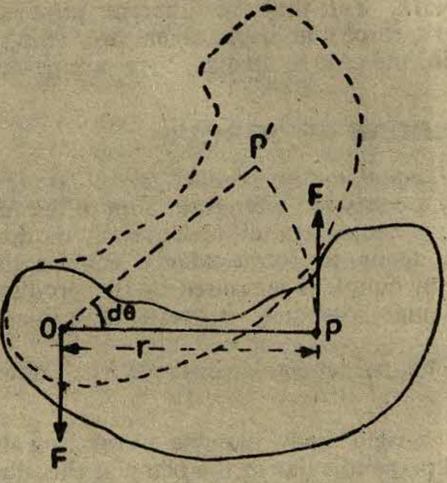


Fig. 3·35

The equation (3-36) is the counter part of $W = F \times S$ or Work done = Force × displacement in translatory motion where torque replaces the force and the angular displacement replaces the linear displacement.

3·18 TORQUE AND ANGULAR ACCELERATION

If we wish to accelerate the rotation of a body about the axis of rotation, we have to apply to it a couple and we know that the moment of the couple, so applied is called *torque*.

Clearly, the angular acceleration of all the particles of the body is the *same*, but the linear accelerations of different particles are different, depending upon their respective distances from the axis of rotation. If $d\omega/dt=\alpha$ be the angular acceleration of the body (and hence of its particles), we have [Fig. 3·28].

Acceleration of the particle distant r_1 from the axis

$$=r_1\frac{d\omega}{dt}$$

Acceleration of the particle distance r_2 from the axis

$$=r_2\frac{d\omega}{dt} \text{ and so on.}$$

Now, force = mass × acceleration. So that the *forces* acting on the different particles are $m_1r_1\frac{d\omega}{dt}$, $m_2r_2\frac{d\omega}{dt}$, $m_3r_3\frac{d\omega}{dt}$ .... $m_nr_n\frac{d\omega}{dt}$. The *moments* of these forces about the axis of rotation are thus equal to $\left(m_1r_1\frac{d\omega}{dt}\right).r_1$, $\left(m_2r_2\frac{d\omega}{dt}\right).r_2$, $\left(m_3r_3\frac{d\omega}{dt}\right).r_3$, ..., $\left(m_nr_n\frac{d\omega}{dt}\right).r_n$.

So that *moment of the whole body about the axis of rotation*

$$=m_1r_1^2\frac{d\omega}{dt}+m_2r_2^2\frac{d\omega}{dt}+m_3r_3^2\frac{d\omega}{dt}+\dots\dots\dots+m_nr_n^2\frac{d\omega}{dt}$$

$$=\sum mr^2.\frac{d\omega}{dt}=\frac{d\omega}{dt}\sum mr^2=\text{I}.\frac{d\omega}{dt}$$

$=\text{I}.\alpha$ because $\Sigma mr^2=\text{I}$, the moment of Inertia of the body about the given axis.

This must clearly be equal to the *moment of the couple* or the *torque* (τ) applied to the body. Therefore,

$$\boxed{\text{torque }(\tau)=\text{I}\frac{d\omega}{dt}=\text{I}.\alpha} \qquad \dots(3\text{-}37)$$

or **Torque = moment of Inertia × angular acceleration**

Clearly, therefore, if $\alpha=1$, we have

$$\boxed{\tau=\text{I}}$$

Thus the moment of Inertia of a body about an axis may also be defined as the torque required to produce unit angular acceleration in it.

The relation (3-37) corresponds to $\vec{F} = m\vec{a}$ for translatory motion.

3·19 ANGULAR MOMENTUM AND ANGULAR IMPULSE

In the case of *linear motion*, we have, as we know,

momentum of a body=mass of the body × its velocity

Since, as we have seen, *moment of inertia*, in the case of rotational motion, is analogous to *mass* in linear motion and angular velocity to linear velocity, we have

angular momentum $(\vec{L})$=*moment of inertia × angular velocity*=$I.\vec{\omega}$.

This may be easily seen from the following :

If $\vec{\omega}$ be the *angular velocity* of a body rotating about an axis, the angular velocity of all its particles is also the *same*, *viz*., $\vec{\omega}$, but their linear velocities are different, depending upon their respective distances from the axis of rotation. Thus, for a particle distant $\vec{r_1}$ from the axis, it is $\vec{r_1}\,\omega$, for a particle distant $\vec{r_2}$ from the axis, it is $\vec{r_2}\omega$ and so on, and their linear momenta are respectively $m_1\vec{r_1\omega}$, $m_2\vec{r_2\omega}$ *etc*., where m_1 and m_2 are the masses of the particles.

So that, moment of momentum of particle m_1 about the axis

$$=m_1\vec{r_1.\omega} \times r_1 = m_1 r_1^2.\vec{\omega}$$

and moment of momentum of particle $m_2 = m_2 r_2^2.\vec{\omega}$ and so on.

∴ *moment of momentum of the whole body about the axis or angular momentum*

$$\vec{L} = m_1 r_1^2.\vec{\omega} + m_2 r_2^2.\vec{\omega} + \ldots\ldots$$

or $$\boxed{\vec{L} = \Sigma m r^2.\vec{\omega} = I\omega,} \qquad \ldots(3\text{-}38)$$

$[\because\ \Sigma m r^2 = I]$

where I is the *moment of inertia* of the body about the axis of rotation.

Angular Momentum = Moment of Inertia × Angular Velocity.

The **angular momentum of a rotating body about its axis of rotation is thus equal to the sum of the moments of the linear momenta of its particles about that axis.** This is the reason why it is also called the *moment of momentum.*

If angular momentum $\vec{L}$ is analogous to linear momentum $\vec{p}$, it must be a *vector quantity* whose direction can be obtained by employing the vector cross product of the momentum $\vec{p}$ and $\vec{r}$ the moment arm. In vector notation

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{V}$$

The dimensions of $\vec{L}$ are $M^1L^2T^{-1}$

Now, as we know

$$torque\ (\tau) = I.d\omega/dt,$$

where $d\omega/dt$ is clearly the rate of change of angular velocity of the body and, therefore, $I.d\omega/dt$ is *the rate of change of angular momentum of the body about the axis of rotation.* So that

$$\boxed{torque = rate\ of\ change\ of\ angular\ momentum}$$

i.e.,
$$\boxed{\tau = \frac{dL}{dt}} \qquad ...(3\text{-}39)$$

Again, it follows from the above that $I.d\omega = \tau.dt$, integrating which we have *angular momentum* $I\omega = \int_0^t \tau.\,dt$, —an expression which is true, howsoever τ may vary with time.

If τ be constant, we have

$I\omega = \int_0^t \tau.dt = \tau.t$, which clearly gives the angular momentum acquired by the body in time t.

And, if τ be *quite large* and t, *very small*, the expression $\int_0^t \tau.\,dt$ measures the *angular impulse* given to the body, which becomes τt, if τ be constant. Thus, we have

angular impulse = torque × time

on the same analogy as we have linear impulse = force × time, in the case of linear motion.

3·20 CONSERVATION OF ANGULAR MOMENTUM

We know that,

External torque = Rate of change of angular momentum

i.e., $$\tau=\frac{dL}{dt}$$

If the external torque acting on a body is zero, then $\tau=0$ and hence $(dL/dt)=0$. This means $L=$ constant

Just as in the case of linear momentum, the importance of this expression is that angular momentum (L) is a *conserved quantity.*

"The total angular momentum of an isolated system remains constant or the angular momentum of a system remains constant as long as no net external torque is applied to it."

It is known as the **principle of conservation of angular momentum**. This principle is quite general and holds true whatever the nature of interaction between particles of a body. It applies not only to rigid bodies but to non-rigid bodies also. During its rotation, a body may change the distribution of mass and hence its moment of inertia about the axis of rotation. If the moment of inertia changes from I_1 to I_2, the angular velocity must also change from ω_1 to ω_2 so that the angular momentum L remains unchanged

i.e., $$\boxed{L=I_1\omega_1=I_2\omega_2} \qquad \text{...(3-40)}$$

Ice skaters, ballet dancers, acrobats and divers use the principle. The diver in (Fig. 3·36) leaves the high diving board with outstretched arms and legs and some initial angular velocity about his centre of gravity. His angular momentum ($I\omega$) remains constant since no external torques act on him (gravity exerts no torque about his centre of gravity). To make a somersault he must increase his angular velocity. He does this by pulling in his legs and arms so that I decreases and ω therefore increases. By extending his arms and legs again, his angular velocity falls to its original value.

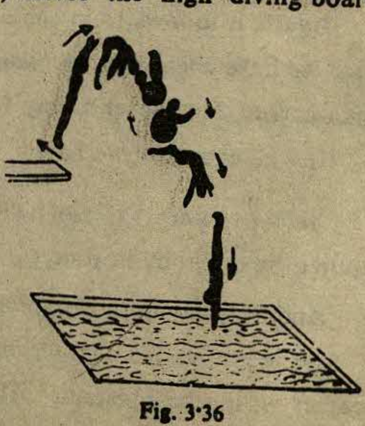


Fig. 3·36

Similarly a skater can whirl faster on ice by folding his arms (Fig. 3·37) Satellites moving in orbit about the earth and planets drifting around the sun speed up when they get closer to the body around which they rotate. The principle of conservation of angular momentum is useful for dealing with large rotating bodies such as the earth and the planets, as well as tiny spinning particles such as electrons.

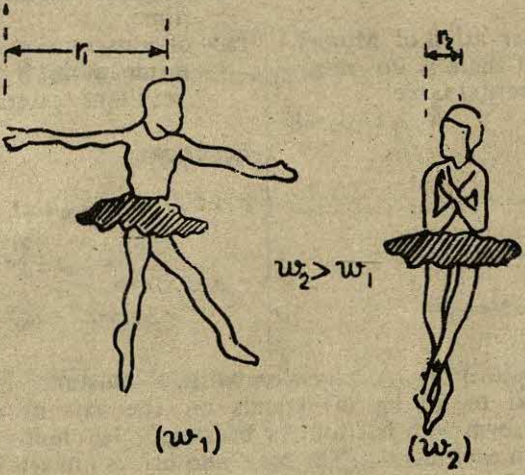


Fig. 3·37 Skater increasing rate of rotation.

3·21 THE ANALOGY BETWEEN ROTATION AND LINEAR MOTION

The following table summarizes the analogy between various quantities that are relevant to linear motion and corresponding quantities that play a similar role in rotational motion.

Linear Motion	**Rotational Motion**
Distance (S)	Angle (θ)
Velocity (V)	Angular velocity (ω)
Acceleration (a)	Angular acceleration (α)
Mass (M)	Moment of Inertia (I)
Force (F)	Moment or torque of a couple (τ)
$F = ma = m\frac{d^2s}{dt^2}$	$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$

Kinetic Energy $=\frac{1}{2}MV^2$	Kinetic Energy $=\frac{1}{2}I\omega^2$
Work done $=FS$	Work done $=\tau\theta$
Linear Momentum $p=MV$	**Angular Momentum** $L=I\omega$
Force = Rate of change of momentum with time	Torque = Rate of change of angular momentum with time.
Law of conservation of Momentum, if there is no resultant external force.	Law of conservation of Angular Momentum, if there is no resultant external torque.
Equations	*Equations*
$V=u+at$	$\omega=\omega_0+\alpha t$
$S=ut+\frac{1}{2}at^2$	$\theta=\omega_0 t+\frac{1}{2}\alpha t^2$
$2aS=V^2-u^2$	$2\alpha\theta=\omega^2-\omega_0^2$

Example 3·10. A physicist with a constant moment of Inertia equal to 20 kg m² stands on the axis of a massless, rotating platform with frictionless bearings. He holds two 4 kg bricks, one in each hand. The man and bricks are set in rotation with an angular speed of one revolution per second with the bricks held out at a distance of 1½ m from the axis of rotation. Calculate the angular speed when the man brings both the bricks toward his body, a distance of 0·5 m from the axis of rotation.

Solution.

Assuming no external torques act upon the system, angular momentum will be conserved, that is,

$$I_1\omega_1=I_2\omega_2$$

Now $\quad I=I_{man}+I_{masses}=20 \text{ kg } m^2+2mr^2$

Initially,

$$I_1=20+2\times4\times\left(\tfrac{3}{2}\right)^2=38 \text{ kg m}^2$$

$$\omega_1=2\pi n=2\pi\times1=2\pi \text{ rad s}^{-1}$$

Finally,

$$I_2=20+2\times4\times\left(\tfrac{1}{2}\right)^2=22 \text{ kg m}^2$$

Now $\quad \omega_2=\dfrac{I_1\omega_1}{I_2}$

or $\quad \omega_2=\dfrac{38\times2\pi}{22}=10{\cdot}85 \text{ rad sec}^{-1}$

Example 3·11. On an application of a constant torque, a wheel is turned from rest through an angle of 200 radians in 8 seconds (*a*) what is its angular acceleration ? (*b*) If the same torque continues to act, what will be the angular velocity of the wheel after 16 s from the start ?

Solution.

(*a*) Initial angular velocity $(\omega_0)=0$

angular displacement $(\theta)=200$ rad

time $(t)=8$ s

Angular acceleration $(\alpha)=?$

Now $\theta=\omega_0 t+\frac{1}{2}\alpha t^2$

$\therefore$ $200=0+\frac{1}{2}\alpha\times 64$

$\therefore$ $\alpha=6·25 \text{ rad s}^{-2}$

(*b*) Here $\omega_0=0$

$\alpha=6·25 \text{ rad s}^{-2}$

$t=16$ s

Final angular velocity $=\omega=?$

Since $\omega=\omega_0+\alpha t$

$\therefore$ $\omega=0+6·25\times 16$

$=100 \text{ rad s}^{-1}$

Example 3·12. A cord is wound round the circumference of a wheel of diameter 0·3 m. The axis of the wheel is horizontal. A mass equal to 0·5 kg is attached to the end of the cord and it is allowed to fall from rest. If the weight falls 1·5 m in 4 s, what is the angular acceleration of the wheel ? Also find the moment of Inertia of the wheel.

Salution

The data given is as follows :

$u=0$

$S=1·5$ m

$t=4$ s

$a=?$

Since, $S=ut+\frac{1}{2}at^2$

$\therefore$ $1·5=0+\frac{1}{2}a(4)^2$

$\therefore$ $a=\frac{3}{16} \text{ ms}^{-2}$

Now $$a=\alpha r$$

where α=angular acceleration and r=radius

$$\therefore \quad \alpha=\frac{a}{r}$$

$$=\frac{3}{16}\times\frac{1}{(0{\cdot}3/2)} \qquad (\because \text{ diameter}=0{\cdot}3 \text{ m})$$

$$=1{\cdot}25 \text{ rad s}^{-2}$$

Let I be the moment of Inertia of the wheel about the axis of rotation. Since

$$\text{Torque } \tau=I\alpha$$

$$\therefore \quad I=\frac{\tau}{\alpha}$$

Now torque produced by the weight of 0·5 kg is

$$\tau=(0{\cdot}5\times9{\cdot}8)\times\frac{0{\cdot}3}{2}=\frac{9{\cdot}8\times0{\cdot}3}{4} \text{ Nm}$$

$$[\because \quad \tau=\text{moment of the force}]$$

Hence $$I=\frac{\tau}{\alpha}=\frac{9{\cdot}8\times0{\cdot}3}{4\times1{\cdot}25}=0{\cdot}588 \text{ kg m}^2$$

Example 3·13. A flywheel of mass 25 kg has a radius of 0·2 m. It is making 240 r.p.m. What is the torque necessary to bring it to rest in 20 s ? If the torque is due to a force applied tangentially on the rim of the flywheel, what is the magnitude of the force ?

Solution

The data given is as follows :

$$\text{Initial angular velocity } (\omega_0)=\frac{240}{60}\times2\pi$$

$$=8\pi \text{ rad s}^{-1}$$

$$\text{Final angular velocity } (\omega)=0$$

$$\text{Time } (t)=20 \text{ s}$$

$$\text{Angular Acceleration } (\alpha)=?$$

Since $$\omega=\omega_0+\alpha t$$

$$\therefore \quad 0=8\pi+20\,\alpha$$

$$\therefore \quad \alpha=\frac{-2\pi}{5} \text{ rad s}^{-2}$$

Moment of Inertia of the wheel about the axis passing through its centre and perpendicular to its plane is

$$I = \frac{MR^2}{2}$$

But $M = 25$ kg

and $R = 0{\cdot}2$ m

$$\therefore \quad I = \frac{25 \times 0{\cdot}2 \times 0{\cdot}2}{2} = 0{\cdot}5 \text{ kg m}^2$$

Now Torque $\tau = I\alpha$

or $$\tau = 0{\cdot}5\left(\frac{-2\pi}{5}\right) = \left(\frac{-\pi}{5}\right) \text{ Nm}$$

Again, Torque $= \tau =$ External force × perpendicular distance

$$\therefore \quad \frac{-\pi}{5} = F \times 0{\cdot}2$$

or $F = \pi$ N (neglecting $-$ve sign).

SUMMARY

1. Objects moving around curved paths are accelerating even when their speeds are constant.
2. The acceleration of any object moving in a circle at constant speed is directed towards the centre of the circle. It is known as centripetal acceleration.
3. According to Kepler's laws of motion, each planet moves in an elliptical orbit around its sun and sweeps out equal areas in equal times during its movement. The square of any planet's time of revolution around its Sun is proportional to the cube of its average distance from the Sun.
4. Newton's Law of Universal Gravitation indicates that every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
5. The constant G that relates the force of attraction to the masses of gravitationally attracting objects and the distance between them is a fundamental invariant that has the same value for all pairs of objects in the universe.
6. The acceleration due to gravity for all objects at the earth's surface is proportional to the mass of the earth and inversely proportional to the square of the earth's radius at that point. The constant of proportionality is G.
7. The centripetal force maintains planets in orbit around their Suns and satellites around their planets. The speed required for stable orbit of any satellite can be obtained from the expression that equals the centripetal force with the gravitational force of attraction on the satellite. The orbital velocity of the Earth's satillite is about 8 km s^{-1} and its velocity of escape is 11·2 km s^{-1}.
8. The moment of Inertia of any object about a particular axis is the sum of the product of the masses and the square of the distances from the axis of all particles that make up the object. It is a scalar quantity.
9. A torque is a twist that tends to rotate an object. Torque is the rotational equivalent of force. It is a vector quantity.

10. **The magnitude of a torque is the product of the force that tends to rotate an object and the perpendicular distance between that force's line of action and the axis of rotation.**

11. **Rotational motion produced by torques is studied in terms of angular quantities such as angular displacements (θ) in radians, angular velocities (ω), and angular acceleration (α).**

12. **Angular acceleration (α) is related to the linear acceleration as follows :**

$$\boxed{a = r\alpha}$$

13. **When the basic laws of motion are rewritten in angular terms, the moment of Inertia substitutes for the mass of translational motion and angular velocity for linear velocity.**

14. **Angular Momentum is the product of moment of Intertia and angular velocity. It is a vector quantity.**

$$\boxed{L = I\omega}$$

QUESTIONS

1. Define angular velocity. Derive a relation between angular velocity and linear velocity.
   Two cars are going around two concentric circular paths at the same angular speed. Does the inner or the outer car have the larger linear speed ?
   (*Delhi Senior Sec.* ; *1980*)
2. What do you mean by centripetal force ? Derive an expression for it when a body moves in a circle with a uniform speed.
3. Explain why curved roads are generally banked ?
   Explain with the help of a neat diagram how banking provides the centripetal force necessary for a car to go in a circular track.
   (*Delhi Senior Sec.* ; *1980*)
4. State Kepler's Laws of Planetary Motion. [*A.I.S.S.C. 1986*]
5. State Newton's Law of Gravitation and hence define Universal Gravitational constant. What are the dimensions and units of G ? Why do we not see the bodies moving towards each other due to gravitational attraction ?
6. How do we derive Kepler's Third Law from Newton's Law of Universal Gravitation ?
7. What do you mean by orbital velocity ? Derive an expression for the magnitude of orbital velocity and the time period of a satellite circling the earth.
8. Briefly explain how an artificial Satellite is launched.
9. What do you mean by "Velocity of Escape" of an object ?
10. Explain the meanings of

    (*a*) Rigid body (*b*) Translatory Motion (*c*) Rotatory Motion (*d*) Angular Acceleration.

Derive the relationship between linear and angular acceleration.

11. Derive an expression for the Kinetic Energy of rotation of a body and hence define moment of Inertia. What is the Physical significance of moment of Inertia? What are its units and dimensions? Is it a vector quantity?

12. Determine moment of Inertia of *(Delhi Senior Sec. 1980)*

    (*i*) a uniform *disc* about an axis passing through its centre and perpendicular to its plane. [*A.I.S.S.C. 1986*]

    (*ii*) a *uniform ring* about a perpendicular axis passing through its centre. ***(Delhi Senior Sec. 1980)***

13. What do you mean by a torque? State its units and dimensions. Is it a scalar quantity?

14. Derive an expression for the work done by a torque (or a couple) in rotating a body through a certain angle.

15. How is torque related to angular acceleration? Derive the relationship and hence define Moment of Inertia of a body in terms of the torque acting on it.

16. What do you mean by angular momentum? Derive an expression for it and hence show that the rate of change of angular momentum is proportional to the external torque.

17. State the Principle of conservation of Angular Momentum and give two examples to illustrate it.

18. When you switch on an electric fan, where does its angular momentum come from?

19. Describe motion in a vertical circle. Show that when a body is projected with a velocity which just takes it to the top of a vertical circle, the tension in the string at the lowest point is *six* times the weight of the body.

20. Why does a cyclist lean to one side while going along a curve? In which direction does he lean?

21. A car is taking a sudden turn to the left. A passenger in the front seat finds himself sliding towards the door. Explain indicating the forces acting on the passenger and on the car at this instant.

22. Imagine a space craft going from the earth to the moon. How does the weight vary as it goes from the earth to the moon? Will there be any change in its mass?

    [**Hint.** Since $g \propto \frac{1}{r^2}$; at a certain stage the weight will be zero (*weightlessness*) and then it increases in the gravitational field of the moon.]

23. If the earth Satellite is put in orbit at a height where the resistance due to the atmosphere cannot be neglected, how will the motion of the Satellite be affected ?

24. The melting of the polar ice caps is given as a possible cause of the variation in the period of rotation of the earth about its axis. Explain.

[**Hint.** Since moment of Inertia I decreases ; the earth begins to rotate faster as ω increases so that Iω=constant.]

PROBLEMS

1. A body moves uniformally in a circle of diameter 10 cm with an angular velocity of 3·5 rad s^{-1}. What is its speed ? What is its radial acceleration ?

[**Ans.** 0·175 ms^{-1} ; 0·6125 ms^{-2}]

2. A simple theory of the hydrogen atom concludes that the electron moves round the proton with a velocity of $2 \cdot 2 \times 10^8$ cms^{-1} in a circular orbit of radius $5 \cdot 3 \times 10^{-11}$ m. Calculate the acceleration of the electron and compare it with the acceleration due to gravity '*g*'.

[**Ans.** $9 \cdot 132 \times 10^{22}$ ms^{-2} ; $\simeq 10^{22}$ g]

3. A body moves with a constant angular velocity of 2·5 rad s^{-1} in a circle of radius 15 cm contained in a verticle plane in the earth's gravitational field. What is its acceleration (*a*) in its highest position (*b*) in its lowest position ? Explain carefully why there is, or is not, a difference between the two answers.

[**Ans.** 0·94 ms^{-2} in both cases. The acceleration of a moving body is completely determined by the way in which it moves along its path.]

4. What is the acceleration due to gravity at a distance from the centre of the earth equal to the diameter of the earth ?

[**Ans.** 2·45 ms^{-2}]

5. A planet moves around a star in a circular orbit of radius 5×10^{11} m with a speed of 3×10^5 ms^{-1}. What is the mass of star ?

[**Ans.** $6 \cdot 7 \times 10^{32}$ kg]

6. A satellite of mass 1000 kg moves around a planet with a speed of 5×10^3 ms^{-1} in a circular orbit of radius $1 \cdot 6 \times 10^7$ m. If the radius of the planet is 7×10^6 m, what is the acceleration due to gravity on its surface ?

[**Ans.** 8·163 ms^{-2}]

7. A flywheel has a moment of inertia of 2×10^4 gm cm^2 about its axis of rotation. When its angular velocity is 2·5 rad s^{-1}, what is its Kinetic Energy ?

[**Ans.** $6 \cdot 25 \times 10^{-3}$ J]

8. Calculate the moment of the couple which must be applied to a wheel with a moment of Inertia 2·5 kg m^2 to accelerate it from rest to 1000 r.p.m. in 5 sec.

[**Ans.** 52·38 Nm]

9. A racing car of mass 800 kg is travelling around a circular track of radius 200 m with a steady speed of 96 km h^{-1}. Find its angular momentum.

[**Ans.** $4·3 \times 10^6$ kg m^2 sec^{-1}]

10. A stone of mass 4 kg is attached to a string 10 m long and is whirled in a horizontal circle. The string can stand a maximum tension of 160 N. Calculate the maximum velocity of revolution that can be given to the stone without breaking the string.

[**Ans.** 20 ms^{-1}]

11. A bend in a level road has a radius of 80 m. Find the maximum speed which a car, turning this bend may have without skidding, if the coefficient of friction between the road and the car is 0·25.

[**Ans.** 14 ms^{-1}]

12. At what angle must a railway track with a bend of radius 196 m be banked for the safe running of trains at a speed of 9·8 ms^{-1} ?

[**Ans.** 2·85°]

13. Calculate the mass of the earth if radius of the earth $=6·38 \times 10^6$ m ; $g=9·8$ ms^{-2} and $G=6·67 \times 10^{-11}$ Nm^2/kg^2.

[**Ans.** $5·98 \times 10^{24}$ kg]

14. The mass of the sun is 2×10^{30} kg and of the planet Jupiter is 2×10^{27} kg. The mean distance between the sun and the Jupiter is $7·8 \times 10^{11}$ m. Calculate the value of the universal gravitational constant if the force of attraction between the Sun and the Jupiter is $4·276 \times 10^{23}$ N.

[**Ans.** $6·50 \times 10^{-11}$ Nm^2/kg^2]

15. An artificial satellite is revolving around the earth at a height of 30 km above the surface of the earth. Find the orbital velocity and the time period of revolution if radius of earth $=6370$ km and $g=9·80$ ms^{-2}. Neglect the variation of 'g' with height.

[**Ans.** 7·882 kms^{-1} ; 5103 sec.]

16. A uniform circular disc of mass 1 kg and radius 5 cm rotates about an axis passing through its centre and perpendicular to its plane. Calculate its moment of Inertia.

[**Ans.** $1·25 \times 10^{-3}$ kg m^2]

17. A thin metal ring of diameter 0·6 m and mass 1 kg starts from rest and rolls down an inclined plane. Its linear velocity on reaching the foot of the plane is 5 ms^{-1}. Calculate (*a*) the moment of Inertia of the ring (*b*) K.E. of rotation at that instant.

[**Ans.** 0·09 kg m^2 ; 12·5 J]

18. A flywheel of an engine starting from rest attains an angular velocity of 100 rad s^{-1} in 10 s. Calculate (*a*) angular acceleration and (*b*) angular displacement in 10 s.

[**Ans.** 10 rad s^{-2} ; 500 rad]

19. A grinder is in the form of a circular disc of mass 10 kg and diameter 0·4 m. Calculate the constant torque that has to be applied so that the disc acquires an angular velocity of 4 r/s in 5 sec. Calculate also the rate at which work is done by the torque at the end of 5 s.

[**Ans.** 1·006 Nm ; 25·28 W]

20. Calculate the velocity with which a body has to be projected vertically from the earth's surface so that it escapes the earth's gravitational influence. Radius of earth $=6·4\times10^{6}$ m and $g=9·80$ ms^{-2}.

[**Ans.** $11·2\times10^{3}$ ms^{-1}]

21. The mass of the Sun is 2×10^{30} kg. and it performs one rotation in 25·4 days. If it were suddenly to shrink to one half of its present size, what would be its new rate of rotation ? Assume that it rotates like a solid body and that its density is uniform throughout.

[**Hint** : M.I. of a solid sphere$=\frac{2}{5}MR^{2}$]

[**Ans.** The new rate of rotation is *four* times as large as the old one. One revolution takes 6·35 days.]

22. A small object of mass 0·5 kg is fastened to the edge of a circular disc of mass 2 kg and radius 10 cm. What is the total moment of Inertia of the system about the centre of the disc ?

[**Ans.** 0·015 kg m^{2}]

23. A light cord is wrapped around the rim of a wheel of radius 0·5 m and moment of Inertia 3·5 kg m^{2}, and a mass of 0·5 kg is hung from the cord. What is the angular acceleration of the wheel ? What is the angular velocity of the wheel when the mass has fallen through a distance of 6 m ? Frictional effects may be neglected.

[**Ans.** 0·70 rad s^{-2} · 4·0 rad s^{-1}]

24. If the rotation of the wheel in Problem 23 is opposed by a frictional torque of 0·5 Nm, what will be the angular velocity after the mass has fallen a distance of 6 m ? [**Ans.** 3·6 rad s^{-1}]

25. A small stone tied to an inextensible string of negligible mass is rotated in a circle of radius 2m in a vertical plane. Calculate

(*i*) the minimum speed of the stone at the highest point of the circle so that the string does not slacken.

(*ii*) the speed at the lowest point of the circle when it has the above minimum speed at its highest point. ($g=9·8$ ms^{-2})

[**Ans** : (*i*) 4·43 ms^{-1} (*ii*) 9·87 ms^{-1})

[*A.I.S.S.C. 1986*]

UNIT 4

KINETIC THEORY OF GASES

4·1 INTRODUCTION

The Greeks speculated whether or not matter could be divided indefinitely into smaller and smaller pieces. Should this be possible, they argued, then matter is continuous, but if not then matter must consist ultimately of very small entities now known as 'atoms'. This situation remained unresolved for many centuries as there was no evidence to support either of these possibilities. In fact, it was until the beginning of the nineteenth century that the atom became a precise concept based upon the laws of chemical combination and the kinetic theory of gases.

Early in the nineteenth century the quantitative study of chemistry revealed two general laws of chemical combination, the Law of Constant Composition and the Law of Multiple Proportions. These state, respectively, that a particular chemical compound always contains the same elements combined in the same proportions ; and that when one substance unites with another in more than one proportion, these different proportions bear a simple ratio to one another. These were interpreted by Dalton in 1803 to mean that compounds consist of molecules. *A molecule is the smallest part of a substance which can exist in a free state and retains all the properties of the parent substance.* These molecules are composed of atoms of various elements in definite proportions. There seems to be some doubt which came first, the theory or the experimental results, but there can be no doubt that one inspired and stimulated the other. Soon afterwards, in 1808, Gay-Lussac showed experimentally that simple ratios existed between the volumes of reacting gases. In 1811 Italian scientist Amadeo Avogadro combined Dalton's Atomic Theory with Gay-Lussac's observations and suggested that equal volumes of gases in the same conditions of pressure and temperature contain equal numbers of molecules. (Observations in the years since then have shown that each mole of gas contains $6{\cdot}02\times10^{23}$ molecules ; this number is known as Avogadro's number.) From this it follows that, since 2·0 g of hydrogen at STP (0°C and 76 cm of mercury pressure) occupy 22·4 litres, then 28 g of nitrogen or 32 g of oxygen also occupy 22·4 litres. In general, the mass of a gas which has a volume of 22·4 litres at STP, is equal to its molecular weight expressed in grams. *A mole* of any particular element or component is defined as the mass of the substance equal to the atomic or molecular weight expressed in grams.

These simple ideas led to the formulation of the *Atomic Theory of Matter* which in turn explained all chemical observations and theory during the next hundred years. Later in the century Mendeleev showed that if the elements were placed in order of atomic masses they displayed a periodicity of behaviour. The atomic theory was quite unable to explain this, which was a clear indication that the atom was not the simple indivisible unit initially conceived by Dalton. The full significance of the periodic table was not apparent until the development of our ideas of atomic structure in the present century.

As these ideas were taking shape in chemistry certain rather abstract ideas in Physics were beginning to emerge. In particular, experimental evidence began to accumulate which showed first that *heat was a form of energy*, and later that light, electricity, magnetism and Sound were also forms of energy. Physics appeared to be reduced to a study of the interactions of these various forms of energy with matter. It also became clear that in all natural processes energy is converted from one form into another and is never created or destroyed. This is the law of conservation of energy. Another conservation Law, concerning momentum, applies to both linear and angular momentum, and the study of atomic physics provides many elegant illustrations of this law.

Kinetic Theory of Matter is based upon the *two hypotheses : that matter is composed of molecules and atoms, and that heat is a form of energy and heat is manifest as the Kinetic Energy of the molecules in their random motion.*

On the basis of these hypotheses, many physical phemonena like the states of matter, elasticity ; evaporation, expansion, change of state, etc., have been explained satisfactorily. For example the three states of matter can be explained as follows :

The molecules of any substance are held together by *intermolecular forces.* These forces resist the breaking up of molecular aggregates due to internal thermal agitation. If the forces due to thermal agitation are weaker than the intermolecular forces, then the substance will remain a solid and retain both its shape and volume even when its temperature is considerably raised. If the forces due to thermal agitation are comparable to the intermolecular forces, then the molecules of the substance can slide past one another even at room temperature. Such substances are called liquids, and though they can retain their volume they take the shape of the container in which they are poured. If, on the other hand, the forces due to thermal agitation are stronger than the intermolecular forces, then the molecules of the substance fly apart and such substances are called gases. A gas can neither retain its volume nor its shape.

Evidence in support of the existence of molecules comes from such phenomena as diffusion , Evaporation , Brownian motion.

Diffusion. The phenomenon of diffusion furnishes a striking evidence of the molecular agitation of matter. Particles of one substance diffuse into another against gravity due to molecular motion. Hydrogen diffuses into carbon dioxide when a jar full of Hydrogen is held mouth to mouth over a jar full of carbon dioxide. despite the fact that hydrogen is lighter than carbon-di-oxide. Particles of colour diffuse into water when a piece of colour is placed gently at the bottom of a cylinder full of water. Similarly alcohol diffuses into water, and gold diffuses into lead.

Evaporation. The phenomenon of evaporation is intimately connected with the motion of the molecules. The molecules in a liquid are more or less free to move about. The molecules near the surface try to escape but are generally prevented from doing so on account of the attraction of other molecules. However, at any temperature, all the molecules do not move with the same velocity. Some are much faster than others. Such molecules, as are near the surface and move towards it,. will escape. Thus the process of evaporation consists in the escape of the fast moving molecules *i.e.*, molecules whose velocity is much above the average. This results in lowering the average velocity of the molecules which in turn means lowering of temperature *i.e.*, cooling—a well known fact.

Brownian Motion

Robert Brown, an English Botanist, first observed Brownian motion (named after him) in 1827. This is an evidence in support of existence of molecules in motion. This may be demonstrated by introducing cigarette smoke into a small hollow glass cube of about ten millimetre side placed on the stage of a microscope and illuminated by a strong horizontal beam of light from oneside. (Fig. 4·1) When viewed under high magnification, small bright specks of smoke particles may be seen. *These are observed to be continuously agitated in a random fashion.* The explanation is that with a heavy body the impacts of individual gas molecules on the surface are relatively too small to displace the body appreciably. Moreover the surface area is so large that the impulses delivered by the numerous molecules balance out. When, however, the size of the body is reduced, the impacts of the molecules all around it are less likely to be balanced and the lighter particle responds more readily to the resultant forces acting upon it.

Brownian motion in a liquid can be shown by mixing

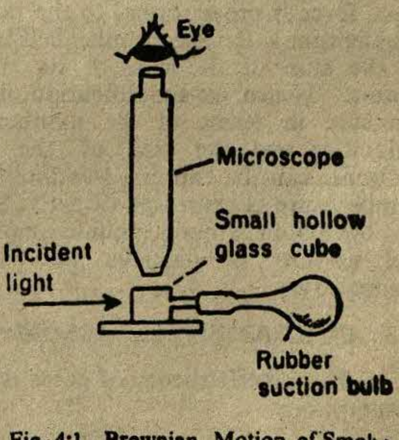


Fig. 4·1 Brownian Motion of Smoke Particles very dilute solutions of lead acetate and potassium carbonate in a rectangular tank of glass or perspex. When the tank is illuminated by light and viewed at right angles to the direction of illumination, each crystal platelet shows up as a bright speck when it comes into the position of reflection. The twinkling of these bright specks indicates that the crystals are being continuously agitated in much the same way as the smoke particles.

The study of the kinetic theory was first approached by applying it to matter in the gaseous state which is governed by simple and definite laws. Thus, Bernouilli, in 1730, was able to explain, for the first time, Boyle's Law by molecular motions, and for this reason he is considered as *the father of the Kinetic Theory*. From the middle of the 19th century, the kinetic theory of gases was greatly developed by Clausius, Maxwell, Boltzmann, Jeans, Vander Waals, Lorentz and Rayleigh, covering almost the entire field of different phenomena related with gases. Attempts have been made, in recent years, to extend the theory to liquid and solid states with some success, but the kinetic theory of liquids and solids is much more complicated and hence will not be treated in this book. We shall be, therefore, concerned only with the **Kinetic Theory of Gases**.

4·2 GAS MODEL

A model helps in explaining and understanding a large body of facts, both qualitatively and quantitatively. What should be the model of a gas which could explain the familiar laws like (*i*) gas laws (*ii*) Avogadro's Law (*iii*) Graham's Law of diffusion, etc.

The physical quantities which describe the behaviour of a gas are pressure, volume and temperature. Pressure is force per unit area, force is proportional to the rate of change of momentum and momentum is the mass multiplied by velocity. The mass of a body is the sum of the mass of its constituent particles, molecules or atoms. Hence, on simplification, it should be possible to express pressure in terms of the number of particles, their mass, their velocities and the area of the surface experiencing pressure. Volume can be expressed as the product of three lengths, and the temperature is expressed directly. So the model of a gas explaining its behaviour should include quantities, like the mass of particles, the number of particles and their velocities, volume and temperature.

4·3 FUNDAMENTAL ASSUMPTIONS

The kinetic theory of gases is based on the following basic assumptions.

(*i*)**Molecules** : Molecules of a gas are considered to be rigid, perfectly elastic, solid spheres, identical in all respects such as mass, form, etc. They are, however, of negligible size as compared with their distances apart.

***(ii)* Motion :** Molecules are in a state of incessant random motion, moving in all directions with all possible velocities. The gas is said to be in a state of molecular chaos.

***(iii)* Collisions:** Molecules in their motion collide with one another and with the walls of the containing vessel. At each collision their velocities are altered in direction and magnitude ; yet in the steady state the collisions do not affect the molecular density of the gas *i.e.*, the number of molecules per unit volume of the gas remains constant.

***(iv)* Forces:** The collisions are perfectly elastic and there are no forces of attraction or repulsion between the molecules, which means that the energy of the gas is all kinetic.

***(v)* Time :** The time spent in a collision is negligible as compared with that during which the molecules are moving independently.

***(vi)* Path.** Between collisions the molecules move in a straight line with uniform velocity. The distance between two collisions is called the *free path* of the molecules (Fig. 4·2). The average distance travelled by a molecule between successive collisions is known as the *mean free path*.

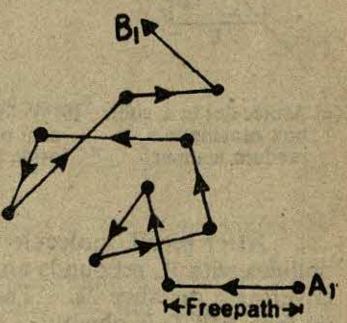


Fig. 4·2

These simplifying assumptions help to avoid mathematical complications and with these postulates the bulk properties of a gas can be explained. They also assure that the model developed will be applicable to *all the gases*. But, strictly speaking, none of these assumptions is true. They only set a limit beyond which the model developed is not applicable. For example, the model thus developed does not satisfactorily explain the behaviour of liquified gases.

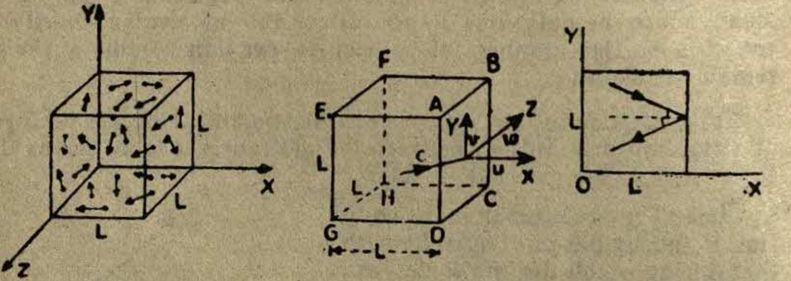
4·4 PRESSURE OF A GAS

Let a gas be contained in a cubical vessel side of L with perfectly elastic walls and let N represent the very large number of molecules in the vessel through which they move in a random manner [Fig. 4·3 (*a*)]. Imagine a single molecule moving with a velocity C. Let its mass be *m*. Its velocity can be resolved into three components *u*, *v* and *w* in the direction of the edges of the cube along X, Y and Z-axes [Fig. 4·3 (*b*)] such that

$$C^2 = u^2 + v^2 + w^2 \quad \text{(Pythagoras theorem, applied twice)} \quad \cdots(i)$$

If the molecule considered strikes the face ABCD of the cube, it will communicate a momentum to the face equal to *mu* since the component of velocity *u* which is effective, is the one normal to the face. Further since the collision is perfectly elastic, molecule will rebound from the face with the same speed u in the opposite direction. The change in momentum as a result of the collision is

$$mu-(-mu)=2mu$$



(*a*) Molecules in a cubic box moving in a random manner (*b*) Velocity component of a particle inside a cubic box (*c*) The collision of an ideal gas particle with a wall of the containing cubic box

Fig. 4·3

After recoil, molecule travels to the opposite face EFHG, collides with it, rebounds and travels back to the face ABCD, after covering a distance 2L. The time between the successive collisions on face ABCD is obviously given by the equation

$$u=\frac{2L}{t}$$

or $$\boxed{t=\frac{2L}{u}} \qquad ...(ii)$$

From this we get the number of collisions per second, which the molecule makes with ABCD as $\frac{u}{2L}$.

Hence the rate of change of momentum $=2mu\times\frac{u}{2L}$

$$=\frac{mu^2}{L}$$

Now, according to Newton's Second Law, the rate of change of momentum = impressed force. If F is the impressed force, then

$$\boxed{F=\frac{mu^2}{L}} \qquad ...(iii)$$

Since pressure is defined as force per unit area, the pressure exerted by the molecule on the face ABCD will be

$$=\frac{F}{L^2}=\frac{mu^2}{L^3}.$$

Since the vessel contains N molecules, then the pressure P_x exerted by them on the wall ABCD will be

$$P_x=\frac{m}{L^3}\left[u_1^2+u_2^2+u_3^3+\ldots\ldots u_N^2\right]$$

$$=\frac{m}{L^3}\,N\overline{u^2}$$

$$=\frac{m}{V}\,N\overline{u^2} \qquad \ldots(iv)$$

where $V=L^3$ is the volume of the cubical vessel.

and $$\overline{u^2}=\left(\frac{u_1^2+u_2^2+u_3^2+\ldots\ldots+u_N^2}{N}\right)$$

=average value of u^2 of all the N molecules.

Similarly, if P_y and P_z represent the pressures on the faces EFBA and FBCH which are perpendicular to the Y- and Z-axes respectively,

$$P_y=\frac{m}{V}\,N\overline{v^2}$$

and $$P_z=\frac{m}{V}\,N\overline{w^2}$$

Since the pressure exerted by a gas is the same in all directions, the average pressure P of the gas is given by

$$P=\frac{P_x+P_y+P_z}{3}$$

$$=\frac{m}{3V}N[\overline{u^2}+\overline{v^2}+\overline{w^2}]$$

or $$\boxed{P=\frac{mN}{3V}\overline{C^2}} \qquad \ldots(v)$$

where $$\overline{C^2}=\overline{u^2}+\overline{v^2}+\overline{w^2}$$

and is known as *average value of the velocity squared* which is given by the sum of the squares of its components.

If M is the total mass of the gas *i.e.*, $M=mN$, then

$$\boxed{P=\frac{M}{3V}\overline{C^2}} \qquad \ldots(vi)$$

Since $\frac{M}{V}=\rho$, the density of the gas,

$$\boxed{P=\tfrac{1}{3}\rho\,\overline{C^2}} \qquad \ldots(vii)$$

Root Mean Square Velocity

C_{rms}, the square root of $\overline{C^2}$ is called the root mean square velocity. *It is defined as the square root of the mean of the squares of the velocities of individual molecules*, which is evidently not same as the mean of the velocities of the different molecules.

$$\boxed{C_{rms}=\sqrt{\overline{C^2}}=\sqrt{\frac{3P}{\rho}}} \qquad \ldots(viii$$

For example

(*i*) In the case of *air* at N.T.P.

$$P=76\times 13{\cdot}6\times 981 \text{ dynes cm}^{-2}$$

$$\rho=0{\cdot}001293 \text{ gm cm}^{-3}$$

$$\therefore \quad C_{rms}=\sqrt{\frac{3\times 76\times 13{\cdot}6\times 981}{0{\cdot}001293}}$$

$$=4{\cdot}856\times 10^4 \text{ cm s}^{-1}$$

$$=0{\cdot}4856 \text{ km s}^{-1}$$

(*ii*) For *Hydrogen* at N.T.P.

$$\rho=0{\cdot}000089 \text{ gm cm}^{-3}$$

$$C_{rms}=\sqrt{\frac{3\times 76\times 13{\cdot}6\times 981}{0{\cdot}000089}}$$

$$=18{\cdot}4\times 10^4 \text{ cm s}^{-1}$$

$$=1{\cdot}84 \text{ km s}^{-1}$$

The kinetic theory of gases predicts the the velocity of the air molecules to be of the order of $0{\cdot}5$ km s^{-1}. This is much higher than the speed at which the smell of a scent appears to spread in a room. How do we explain it ? The guess is that a molecule undergoes multiple collisions back and forth tracing out a Zig-Zag path Fig. 4·2 and as a result, effective displacement per unit time is much less. For example, a molecule starting its journey from A_1 may end up at B_1 Fig. 4·2 after several collisions. For example in the case of bromine diffusing into air, the bromine gas molecule in travelling an effective distance of 0·1 m may have enroute 10^{12} collisions.

Relation between Pressure and Kinetic Energy

Equation (*vii*) can be put as

$$P=\frac{2}{3}\cdot\frac{1}{2}\rho\,\overline{C^2}$$

Here $\frac{1}{2}\rho\overline{C^2}$ represents the mean kinetic energy per unit volume (*e*) of the gas, so that

$$\boxed{P=\tfrac{2}{3}\,e} \qquad \text{...}(ix)$$

Thus the pressure of the gas is numerically equal to two-thirds of the mean kinetic energy of translation per unit volume.

4·5 INTERPRETATION OF TEMPERATURE

Let us consider one-gram molecule of gas having volume V at absolute temperature T. Let it have N molecules. Then N is called the *Avogadro number*. Let *m* be the mass of each molecule. According to equation (*v*) derived above

$$P=\frac{mN}{3V}\,\overline{C^2}$$

or

$$PV=\tfrac{1}{3}\,mN\,\overline{C^2}$$

$$=\tfrac{1}{3}\,M\,\overline{C^2}$$ where $M=mN$ is the molecular weight of the gas.

Using the gas law *i.e.* perfect gas equation viz ;

$PV=RT$ where $R=$Gas constant$=8·314$ J mole^{-1}K^{-1}

we get

$$\tfrac{1}{3}M\overline{C^2}=RT$$

$$\boxed{\tfrac{1}{2}M\overline{C^2}=\frac{3}{2}\,RT} \qquad \text{...}(x)$$

Dividing both sides of equation (*x*) by N, we have

$$\frac{1}{2}\,\frac{M}{N}\,\overline{C^2}=\frac{3}{2}\,\frac{R}{N}\,T \qquad \text{...}(xi)$$

But $\frac{M}{N}=m$

and $\frac{R}{N}=k$, *Boltzmann's constant* (*i.e.* value of gas constant for one molecule) and its value is $1{\cdot}3804\times10^{-23}$ J K^{-1}

Thus equation (*xi*) becomes

$$\boxed{\tfrac{1}{2}m\overline{C}^2=\frac{3}{2}\,kT} \qquad (xii)$$

If E represents the total mean kinetic energy of translation then

$$E=\tfrac{1}{2}M\overline{C}^2=\tfrac{1}{2}mN\overline{C}^2$$

and hence equation (*xii*) can be written as

$$\boxed{E=\frac{3}{2}\,NkT} \qquad ...(xiii)$$

or the mean kinetic energy of translation of one molecule will be

$$\boxed{\frac{E}{N}=\frac{3}{2}\,kT} \qquad ..(xiv)$$

Relations (*x*) and (*xiv*) clearly show that mean kinetic energy per molecule of a perfect gas is proportional to its absolute temperature. *Thus the temperature of a gas is a measure of the mean kinetic energy of translational motion per molecule of the gas which is referred to as the kinetic interpretation of temperature.* Whenever a gas gains energy, its temperature rises. If the absolute temperature T is zero then $\overline{C}^2=0$ or $C=0$. Hence the *absolute zero of temperature is that temperature at which the molecular translational velocities of a gas are reduced to zero.*

4·6 DEDUCTION OF THE LAWS

By making certain assumptions we have developed the kinetic theory of gases which includes quantities like the mass, velocity, number, temperature etc. of the molecules. Let us now examine how far this theory explains the various laws which describe the behaviour of gases.

(1) ***Boyles' law.*** **According to equation (*v*)**

$$P=\frac{1}{3}\cdot\frac{mN}{V}\overline{C^2}$$

or
$$PV=\tfrac{1}{3}mN\,\overline{C^2}$$

Since mN, being the mass of the gas, is constant,

$$PV\propto\overline{C^2}$$

Thus PV is constant if C is constant. Now if the temperature of the gas remains constant, the total energy of the gas molecules and hence their kinetic energy is also constant. Since the average kinetic energy per molecule of a gas is given by $\frac{1}{2}m\overline{C^2}$, the kinetic energy is constant, if $\overline{C^2}$ is constant. Therefore it follows that PV= *constant provided temperature remains constant* which is *Boyle's law.*

(2) *Charles' law.* (Relationship between V and T at constant P).

According to equation (*v*)

$$P=\tfrac{1}{3}\frac{m\times N}{V}\overline{C^2}=\tfrac{1}{3}\frac{M}{V}\overline{C^2}$$

or
$$V=\frac{M}{3P}\overline{C^2}$$

Now for a given mass of an enclosed gas at constant pressure P,

$$V\propto\overline{C^2}$$

But the mean square velocity of a gas is proportional to its absolute temperature or $\overline{C^2}\propto T$

$$\therefore\qquad V\propto T$$

It means that the *volume of a given mass of an enclosed gas at constant pressure is proportional to its absolute temperature* which is *Charles' law.*

(3) *Law of Pressures.* (Relationship between P and T at constant V).

Again,
$$P=\tfrac{1}{3}\frac{mN}{V}\overline{C^2}=\tfrac{1}{3}\frac{M}{V}\overline{C^2}$$

For a given mass of a gas at constant volume,

$$P\propto\overline{C^2}$$

But as $C^2\propto T$, we have

$$P\propto T$$

Or '*the pressure of a given mass of a gas at constant volume is proportional to its absolute temperature* which is referred to as *the law of pressures.*

(4) *Avogadro's law.* It states that '*equal volumes of all gases under similar conditions of temperature and pressure contain the same number of molecules.*

Let us consider *equal* volumes V of two different gases at the same temperature and pressure. Let N_a be the number of molecules each of mass m_a in one gas and N_b molecules each of mass m_b in the other. Let $\overline{C_1^2}$ and $\overline{C_2^2}$ be the mean square velocities of the molecules in the two gases.

Since the temperature of the two gases is the same, the average kinetic energy of translation per molecule is the same *i.e.*

$$\tfrac{1}{2}m_a\overline{C_1}^2=\tfrac{1}{2}m_bC_2^2$$

or
$$m_a\overline{C_1}^2=m_b\overline{C_2}^2 \qquad ...(xv)$$

Further, if P is the pressure of each gas, we have

$$PV=\tfrac{1}{3}m_a N_a \overline{C_1}^2$$

and also
$$PV=\tfrac{1}{3}m_bN_b \overline{C_2^2}$$

$$\therefore \qquad m_aN_a\overline{C_1}^2=m_bN_b\overline{C_2^2} \qquad ...(xvi)$$

From (*xv*) and (*xvi*), we have

$$\boxed{N_a=N_b}$$

which is Avogadro's Law.

(5) *Graham's Law of Diffusion.* It states that '*for any specified temperature and pressure, the rates of diffusion of gases are inversely proportional to the square roots of their densities.*

Consider two gases of densities ρ_a and ρ_b diffusing into each other. When their rates of diffusion become steady, the pressure exerted by either gas is the same. If C_{arms} and C_{brms} are the root mean square velocities of the two gases and P the pressure, we have [from equation (*vii*)]

$$P=\tfrac{1}{3}\rho_a\overline{C_a^2}_{rms}$$

or
$$C_{arms}=\sqrt{\frac{3P}{\rho_a}}$$

Similarly
$$C_{brms}=\sqrt{\frac{3P}{\rho_b}}$$

Dividing the two, we get $\frac{C_{a rms}}{C_{b rms}}=\sqrt{\frac{\rho_b}{\rho_a}}$ ...(*xvii*)

Now the rate of diffusion of a gas into another gas is evidently proportional to the average velocity of its molecules. But, as shown by Maxwell, the average velocity is proportional to the root mean square velocity of the molecules. Therefore, the rate of diffusion of a gas is proportional to the root mean square velocity of its molecules. Hence if R_a and R_b are the rates of diffusion of the above two gases, we have

$$\frac{R_a}{R_b}=\frac{C_{a rms}}{C_{b rms}}$$

or

$$\frac{R_a}{R_b}=\sqrt{\frac{\rho_b}{\rho_a}} \qquad ...(xviii)$$

Thus the rate of diffusion of a gas is inversely proportional to the square root of its density which is Graham's Law of diffusion.

(6) **Dalton's Law of Partial Pressures**. If a number of gases having densities ρ_1, ρ_2, ρ_3...and root mean square velocities C_{1rms}, C_{2rms}, C_{3rms}...are mixed up in the same volume, the resultant pressure P, considering each set of molecules is given by

$$P=\frac{1}{3}\rho_1\overline{C_1^2}+\frac{1}{3}\rho_2\overline{C_2^2}+\frac{1}{3}\rho_3\overline{C_3^2}+...$$

where $\overline{C_1^2}$, $\overline{C_2^2}$, $\overline{C_3^2}$,...are mean square velocities of the molecules.

Here it is assumed that the molecules do not interact chemically and the whole mass of the mixed gases is at the same temperature and each gas fills the whole volume V of the container.

Putting $P_1=\frac{1}{3}\rho_1\overline{C_1^2}$; $P_2=\frac{1}{3}\rho_2\overline{C_2^2}$; $P_3=\frac{1}{3}\rho_3\overline{C_3^2}$, ...

We get $$\boxed{P=P_1+P_2+P_3+...} \qquad ...(xix)$$

Thus the total pressure exerted by a mixture of gases which do not interact chemically is equal to the sum of their partial pressures.

Proof :—

Consider Two gases A and B.

P_A and P_B are the partial pressures.

Total pressure$=$P

To prove

$$P=P_A+P_B$$

$$P_A=\frac{1}{3}\frac{M_A}{V}\overline{C_A^2}$$

$$P_B=\frac{1}{3}\frac{M_B}{V}\overline{C_B^2}$$

$$E_A=\frac{1}{2}M_A\overline{C_A^2} \qquad E_B=\frac{1}{2}M_B\overline{C_B^2}$$

Let E_T be the total energy of the mixture

$$E_T=\frac{1}{2}[M_A\overline{C_A^2}+M_B\overline{C_B^2}]$$

But $$E_T = 3/2\,RT = 3/2\,PV$$

$$3/2\,PV = \tfrac{1}{2}\,[M_A\overline{C_A^2} + M_B\overline{C_B^2}]$$

$$P = \frac{1}{2} \times \frac{2}{3} \times \frac{1}{V}\,[M_A\overline{C_A^2} + M_B\overline{C_B^2}]$$

$$P = \tfrac{1}{3}\frac{M_A}{V}\overline{C_A^2} + \tfrac{1}{3}\frac{M_B}{V}\overline{C_B^2}$$

$$P = P_A + P_B$$

(7) **Gas Equation.** As derived above, Average Kinetic Energy of the molecules $(E) = \frac{3}{2}k\,TN$ where k is Boltzmann's constant and N is the total number of molecules of the gas.

$$P = \tfrac{1}{3}\frac{mN\overline{C^2}}{V}$$

or $$PV = \tfrac{1}{3}mN\overline{C^2} = \frac{2}{3}(\tfrac{1}{2}mN\overline{C^2}) = \frac{2}{3}E$$

Consequently, $PV = NkT$

If one mole of gas is considered then

$N = N_0$ = Avogadro's number and hence

$$\boxed{PV = N_0kT = RT}$$

where R is universal gas constant and has the value $R = 8.314$ J mole^{-1} K^{-1}. If there are n moles of gas present, the gas equation may be written

$$\boxed{PV = nRT}$$

4.7 SPECIFIC HEATS OF A GAS

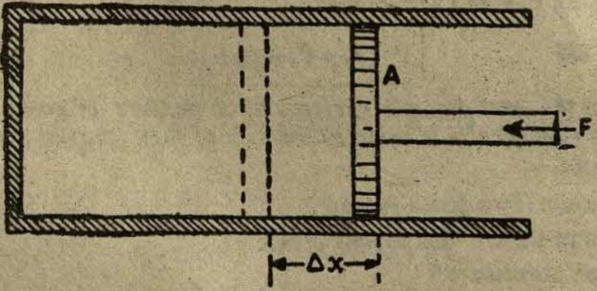


Fig. 4.4

For our very simple model of a gas the internal energy is simply the total kinetic energy, since there are no interactions at all between the molecules. Hence, by the equation (*xiii*), we have for the internal energy the expression

$$E = \frac{3}{2}\,NkT = \frac{3}{2}\,nRT \text{ if there are } n \text{ moles of a gas.}$$

Let us now consider the rise in temperature that occurs when a container of gas is heated. To make the situation as general as possible, let the container be fitted with a movable piston that can slide without friction, as shown in Fig. 4.4. Energy can be communicated to the gas in two distinct ways : mechanical work can be done on the gas by moving the piston in or the gas can be heated. The equation governing both of these process is the *first law of thermodynamics*,

$$\boxed{Q+W=E_2-E_1}$$

where Q is the heat supplied to the system, W is the mechanical work done on the system, E_2-E_1 is the difference of the internal energy of the system. If the pressure outside the piston is taken to be zero, the work done on the gas when the piston moves in a small distance $\triangle x$ is $F.\triangle x=PA\triangle x$, where A is the cross sectional area of the piston and P is the pressure of the gas. The product $A\triangle x$ is the magnitude of the change in volume, $\triangle V$, of the gas. Since the change in the volume is negative, corresponding to a comparison, the work done on the gas is $W=-P\triangle V$. This is a positive quantity. The work done on the gas would be negative if the change in volume were positive, corresponding to an expansion. The first law of thermodynamics may be recast in the form

$$\boxed{Q=(E_2-E_1)+P\triangle V} \qquad \ldots(xx)$$

$$(\because \quad W=-P\triangle V)$$

The amount of heat required to raise the temperature of the gas will depend on the amount of work done. *There are an infinite number of ways of intermingling the heat term and the work term in the first law to produce the same change in internal energy and so there are infinite number of specific heats.* Two of these of special simplicity are chosen. The first, where no work is done at all and so the volume of the gas does not change, is the **specific heat at constant volume** and is given the symbol c_v ; the second, where the pressure of the gas is kept constant by allowing it to expand during the transfer of heat, is the **specific heat at constant pressure** and is given the symbol c_p. The corresponding **molar heats** are C_v and C_p *i.e.* $C_p=Mc_p$ and $C_v=Mc_v$ where M is the molecular weight of the gas.

Specific heat of a gas at constant volume (C_v) is defined as the amount of heat required to raise the temperature of one gm-molecule of the gas through 1 K when its volume is kept constant.

Thus when the gas in the cylinder is heated at constant volume, there is no work term and equation (*xx*) is simply

$$\boxed{Q_v=E_2-F_1=nC_v(T_2-T_1)}$$

where Q_v = heat supplied to the system at constant volume

(T_2-T_1) = Rise in temperature of the *n* moles of gas in the cylinder.

Specific heat of a gas at constant pressure (C_p) *is defined as the amount of heat required to raise the temperature of one gm-molecule of the gas through 1 K when its pressure is kept constant.*

Thus when the gas in the cylinder is heated at constant pressure, we assume the same rise of temperature (T_2-T_1) as in the constant volume case, and so heat supplied at constant pressure, $Q_p=nC_p(T_2-T_1)$ by the definition of the molar heat at constant pressure. In this case, since the pressure would rise if the volume did not increase by a compensating amount, the piston must be allowed to move outwards. The work term $P\triangle V$ is now simply $P(V_2-V_1)$ since the pressure is constant. (V_2-V_1) is the increase in volume. So the first law of thermodynamics gives

$$\boxed{Q_p=(E_2-E_1)+P(V_2-V_1)=nC_p(T_2-T_1)} \qquad ...(xxii)$$

The change of internal energy is the same as it was in the constant volume process, since the initial and the final temperatures are the same in the two cases. Substituting the value of (E_2-E_1) from equation (xxi) into $(xxii)$ gives

$$nC_v(T_2-T_1)+P(V_2-V_1)=nC_p(T_2-T_1) \qquad ...(xxiii)$$

But according to perfect gas equation

$PV_1=nRT_1$ where R is gas constant and is measured in $J\ mole^{-1}\ K^{-1}$

and $PV_2=nRT_2$

$$\therefore \quad P(V_2-V_1)=nR(T_2-T_1)$$

Hence equation $(xxiii)$ reduces to

$$nC_v(T_2-T_1)+nR(T_2-T_1)=nC_p(T_2-T_1)$$

or

$$\boxed{C_p-C_v=R} \qquad ...(xxiv)$$

Converting work units into heat units, we get

$$\boxed{C_p-C_v=(R/J)\ \text{cal/deg.}} \qquad ...(xxv)$$

where J is *Joules' Mechanical equivalent of heat i.e.* work done in joules to produce one calorie of heat and its value is

4·18 × 10⁷ ergs/mole K or *4·18 joules/calorie*

Since $E=\frac{3nRT}{2}$, we get from eq. (*xxi*)

$$E_2-E_1=\frac{3}{2}\,nR\,(T_2-T_1)=nC_v(T_2-T_1)$$

and consequently for a monoatomic gas

$$\boxed{C_v=3/2\ R} \qquad \text{...(xxvi)}$$

From equations (*xxiv*) and (*xxvi*), the ratio

$$\boxed{\frac{C_p}{C_v}=\frac{5}{3}} \qquad \text{...(xxvii)}$$

This ratio is usually denoted by the symbol γ.

And $$\boxed{C_p=5/2\ R} \qquad \text{...(xxviii)}$$

for a monoatomic gas.

From the above discussion it is clear that if a gas is heated at constant volume, no work is done against external pressure and the heat supplied is spent to cause a rise in temperature *i.e.* increase the internal energy of the gas. But when the gas is heated at constant pressure, its volume increases so that heat is required not only to heat the gas through the same temperature, but also to work against external pressure. Thus more heat is required when the gas is heated at constant pressure than when heated at constant volume. This means the specific heat of a gas at constant pressure is greater than the specific heat of a gas at constant volume by the thermal equivalent of the work done by the gas in expanding against external pressure.

4·8 DEGREES OF FREEDOM

The number of degrees of freedom of an object is the number of facts which must be specified in order to describe completely its state or position. For instance, the position of a point in space is determined by the three axes X, Y and Z of the Cartesian coordinate system *i.e.*, three coordinates are required (x, y, z in Cartesian ; r, θ, ϕ in polar coordinates). Three is the smallest number of coordinates which can define the position of the point in space. Similarly two points in space are specified by *six* coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2). Applying this to a dynamical system where a body is in motion, if the body/

is a point, it can have at most three degrees of translational motion. If the body were to be with a finite size and rigid, it can not only move, but also rotate about an axis passing through it. Since this rotatory motion can be resolved, like the translatory motion, into components about any three mutually perpendicular axes through a point fixed in it, a rigid body will have *three more* degrees of freedom of rotational motion. Hence to completely describe the state of motion of a rigid body six coordinates are required. **The degrees of freedom may, therefore, be defined as the total number of coordinates required to specify completely the position and configuration of any dynamical system.**

We have seen above that two points in space are completely defined by six coordinates. But if there is some definite relation between the two points *i.e.*

$$d^2=(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2$$

the number of coordinates necessary for a complete description is reduced by one, so that a system of these two points is said to have only *five* degrees of freedom, the sixth being calculated from the above equation. Including this aspect, the *number of degrees of freedom is actually equal to the total number of coordinates required to define the positions of the elements of the system minus the number of independent relations existing between the elements.* Hence if the number of atoms in a molecule of a gas is x, the number of degrees of freedom cannot exceed $3x$; for a **monoatomic gas**, $x=1$. so that its degrees of freedom will be 3 ; for a **diatomic gas** having two atoms at a fixed distance apart, the number of degrees of freedom will be $(3\times 2-1)=5$.

If we consider a diatomic molecule such as oxygen, it has a dumb-bell configuration as shown in Fig. 4·5 and can have

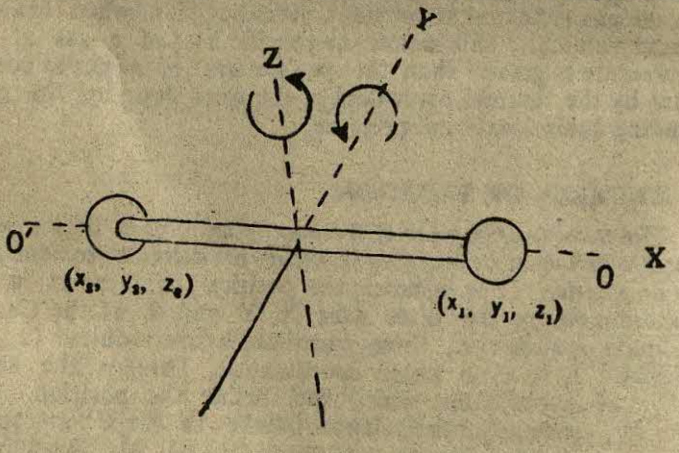


Fig. 4·5 A diatomic molecule, showing possible relations.

other motions besides simple translation. Thus it can rotate about any of three perpendicular axes through the centre of the molecule, one axis being aligned with, and the other two being perpendicular to, the line of centres OO′. *The moment of inertia about this line of centres is so small that this particular rotation is ineffective,* so we are left with two possible rotations or two extra degrees of freedom and hence in all 5 degrees of freedom (three translational and two rotational).

For a triatomic gas, we may have two different arrangements of the atoms : either a central atom with the two others placed on either side of it or the three atoms at the apices of a triangle. The number of degrees of freedom in the first case is equal to $(3\times3-2)$ or 7, and in the second case $(3\times3-3)$ or 6.

In this manner one may calculate the number of degrees of freedom of any polyatomic molecule ; the number of degrees of freedom will increase, in general, with the complexity, unless other factors interfere. For example, the atoms in a diatomic molecule *may also vibrate* along the line joining them, thus, having, one more degree of freedom corresponding to *vibratory motion* only when temperature is very high. In this situation the diatomic molecule come to have six degrees of freedom—three corresponding to motion of translation, two due to motion of rotation and one due to vibratory motion.

4·9 LAW OF EQUIPARTITION OF ENERGY

All relations involving the molar heats of a gas can be checked by experiment. It is found that the noble gases such as argon, helium, neon, krypton (monoatomic gases) fit the simple theory very well indeed at all temperatures *i.e.*, $\gamma=C_p/C_v=5/3=1{\cdot}67$ which is in excellent agreement with the experimental values ; but that other gases give very different results for the molar heats at high temperatures. Thus for oxygen at room temperatures, $C_v=2{\cdot}54$ R instead of the theoretical value 1·5 R and $\gamma=1{\cdot}39$ rather than 1·67 as the above theory predicts. In other words, our simple model has broken down. In order to predict the ratio of specific heats for diatomic gases a further principle has to be used, namely the *law of the equipartition of energy* due to Maxwell.

As derived in the previous section, the average kinetic energy per molecule of a gas is given by

$$1/2\ m\overline{C^2}=3/2\ kT \qquad \text{[see equation } (xii)\text{]}$$

where m is the mass of the molecule, $C_{rms}=\sqrt{\overline{C^2}}$, the root mean square velocity, k the Boltzmann's constant and T is the absolute temperature of the gas. In deriving this relationship, the molecules were assumed to be mere point masses having no dimensions. In such cases, each molecule has only three degrees of freedom corresponding to its translatory motion. Hence the energy associated with three degrees of freedom namely

$\frac{1}{2}m(\overline{u^2}+\overline{v^2}+\overline{\omega^2})=3/2\,kT$. Since the density of the gas throughout its volume in the vessel is the same and the molecule has no preference to move along any particular direction, the average kinetic energy of the molecule along each of the three axes must be the same *i.e.*, $\frac{1}{2}m\overline{u^2}=\frac{1}{2}m\overline{v^2}=\frac{1}{2}m\overline{\omega^2}$. Hence the average kinetic energy per degree of freedom per molecule$=\frac{1}{3}\times 3/2kT=\frac{1}{2}kT$.

This average kinetic energy is associated with each degree of freedom and holds good whether the degree of freedom are due to the motion of translation or rotation or vibration. This result is known as **Law of equipartition of Energy** and is stated as follows :

"*For any dynamical system in thermal equilibrium the total energy is divided equally among all the degrees of freedom and the energy associated with one molecule per degree freedom* $=\frac{1}{2}kT$ *and hence a constant.*"

The mean kinetic energy per gram molecule per degree of freedom$=\frac{1}{2}\,kT\times N=\frac{1}{2}RT$ where N is Avogadro's number and R the gas constant for a gram molecule.

Now if we consider a diatomic molecule which has five degrees of freedom, the kinetic energy associated with it according to law of equipartition of energy will be $\frac{5}{2}\,kT$ with each molecule which leads to the value 2·5 R for the molar heat at constant volume, in good agreement with the experimental result and $\gamma=\frac{C_p}{C_v}=\frac{7}{5}=1{\cdot}4$ which again is in excellent agreement with the experimental values for oxygen and hydrogen.

From the above discussion in this chapter, we find that the kinetic model of gases in its simplified form has explained the behaviour of gases with in a reasonable range of temperature and pressure. However, it gives no idea about the velocity distribution *i.e.* the number of molecules moving with one velocity, with another velocity and so on because not all the molecules will be travelling with the same speed ; since collisions among them would soon destroy that equality. Besides this, the model will be applicable only to perfect gases. The model therefore needs improvement regarding the size of the molecules, the forces between them etc. These consideration lead to Van der Waal's equation of state and Maxwells' Boltzmann's distribution Law, both of which you will study in higher classes.

Example 4·1. If three molecules have velocities 0·5, 1 and 2 kms^{-1} respectively, calculate the relation of the root mean square speed and the average speed.

Solution.

Since root mean square velocity is the square root of the mean of the squares of the velocities of individual molecules

$$\therefore \quad C_{rms}=\sqrt{\frac{(0{\cdot}5)^2+(1)^2+(2)^2}{3}}$$

$$=1{\cdot}32 \text{ kms}^{-1}$$

$$\text{Average Velocity } \overline{C}=\frac{0{\cdot}5+1+2}{3}=1{\cdot}17 \text{ kms}^{-1}$$

$$\therefore \quad \frac{C_{rms}}{\overline{C}}=\frac{1{\cdot}32}{1.17}=\frac{33}{29}$$

Example 4·2. (*a*) Calculate (*i*) root mean square speed and (*ii*) the mean Kinetic Energy of one gram molecule of hydrogen at STP (given that the density of hydrogen is 0·09 kg m^{-3}).

(*b*) Given that the mass of a molecule of hydrogen is $3{\cdot}34\times10^{-27}$ kg, calculate Avogadro's number.

(*c*) Calculate Boltzmann's constant.

Solution.

(*a*) (*i*) Since $P=\frac{1}{3}\rho\,\overline{C^2}$

$$\therefore \quad C_{rms}=\sqrt{\overline{C^2}}=\sqrt{\frac{3P}{\rho}}$$

$$=\sqrt{\frac{3\times0{\cdot}76\times13{\cdot}6\times10^3\times9{\cdot}8}{0{\cdot}09}}$$

$$=1838 \text{ ms}^{-1}$$

$$=1{\cdot}838 \text{ kms}^{-1}$$

(*ii*) Kinetic Energy $=\frac{1}{2}\,m\,\overline{C^2}$

Now, Gram molecule of hydrogen, $m=2{\cdot}016$ g

$=0{\cdot}002016$ kg

∴ Kinetic Energy $=\frac{1}{2}\times0{\cdot}002016\times(1838)^2$

$=3386$ joules. $\quad [\because \ \overline{C^2}=(C_{rms})^2]$

(*b*) Mass of one molecule of hydrogen $=3{\cdot}34\times10^{-27}$ kg

Gram molecule of hydrogen $=0{\cdot}002016$ kg

∴ Number of molecules in 1 gm molecule of hydrogen

$$=\frac{0{\cdot}002016}{3{\cdot}34\times10^{-27}}$$

$$=6{\cdot}04\times10^{23} \text{ molecules}$$

or Avogadro's Number $(N)=6{\cdot}04\times10^{23}$

(*c*) Gas constant, $R=8{\cdot}3\ \text{J mole}^{-1}\ \text{K}^{-1}$

Avogadro's number $(N)=6{\cdot}04\times10^{23}$

$$\text{Boltzmann's constant } k=\frac{R}{N}=\frac{8{\cdot}3}{6{\cdot}04\times10^{23}}$$

$$=1{\cdot}37\times10^{-23}\ \text{J mole}^{-1}\text{K}^{-1}$$

Example 4·3. Using the ideal gas equation, determine the value of R (one gm molecule of a gas at S.T.P. occupies 22·4 litres).

Solution.

Since $PV=RT$

$\therefore$ $R=\frac{PV}{T}$

$$=\frac{0{\cdot}76\times(13{\cdot}6\times10^{3})\times9{\cdot}8\times(22{\cdot}4\times10^{-3})}{273}$$

$$=8{\cdot}269\ \text{J mole}^{-1}\ \text{K}^{-1}$$

Example 4·4. Calculate C_p and C_v for Argon (given $R=8{\cdot}3$ joules/mole/ K).

Solution.

The mean kinetic energy per gm molecule per degree of freedom$=\frac{1}{2}RT$

Since Argon gas is monoatomic and it has *three* degrees of freedom

$\therefore$ Mean kinetic energy per gm-molecule of argon $=\frac{3}{2}RT$

Since heat has been defined as the K.E. of the vibrating molecules of the body, the amount of heat required to raise the temperature of 1 gm-molecule of gas through 1 K keeping its volume constant

$$C_v=\frac{3}{2}R\ (T+1)-\frac{3}{2}RT \text{ (\textit{i.e.} change in internal energy)}$$

or $C_v=\frac{3}{2}k$

Expressed in heat units.

$$C_v=\frac{3}{2}\frac{R}{J}=\frac{3}{2}\times\frac{8{\cdot}3}{4{\cdot}18}$$

where $J=4{\cdot}18\ \text{J cal}^{-1}$, Joule's Mechanical equivalent of heat.

$$=2{\cdot}98 \text{ cal mole}^{-1}\ \text{K}^{-1}$$

Now $$C_p=C_v+\frac{R}{J}=\frac{3}{2}\frac{R}{J}+\frac{R}{J}=\frac{5}{2}\frac{R}{J}$$

$$=\frac{5}{2}\times\frac{8{\cdot}3}{4{\cdot}18}=4{\cdot}96 \text{ cal mole}^{-1}\ \text{K}^{-1}$$

Example 4·5. One mole of helium at 300 K is mixed with three moles of oxygen at 400 K. If the mixing takes place at constant pressure, and if there is no exchange of heat with the environment, calculate the final temperature.

Solution.

Since the molar heats of monoatomic and diatomic gases at constant pressure are $\frac{5R}{2}$ and $\frac{7R}{2}$ respectively,

we have, $$1\times\frac{5}{2}R\times(T-300)=3\times\frac{7}{2}R\times(400-T)$$

or $$26T=9900 \text{ K}$$

$\therefore$ $$T=381 \text{ K}$$

SUMMARY

1. In this chapter we have seen how atomic and molecular hypotheses have led to the kinetic theory of gases, which in turn explains a wide range of phenomena, especially in the field of heat.
2. In particular, it has shown that heat itself is a manifestation of kinetic energy of molecules.
3. According to *Boyles' Law*, the product of the pressure and volume of a gas is constant at constant temperature. According to *Charles' Law*, the volume of a gas is directly proportional to the absolute temperature at constant pressure.
4. A 'mole' of gas always contains the same number of molecules.
5. The general gas law states that the product of pressure and volume of any gas divided by the product of the number of moles and the absolute temperature is constant for all temperatures $\left(i.e.\ R=\frac{PV}{nT}\right)$ well above the liquefaction temperature.
6. The concept of the ideal gas explains macroscopic characteristics of gases in microscopic terms.
7. Speeds of molecules or atoms in a gas are distributed in characteristic fashion.
8. Whenever gases are mixed, each one exerts a pressure in the mixture equal to that it would produce if the amount in the mixture were present alone in the container.

QUESTIONS

1. Describe kinetic theory of matter. How is it able to explain the phenomenon of (*i*) Evaporation (*ii*) Brownian Movement.

2. Evaporation is said to be a cooling process, that is, the liquid remaining is cooler. Explain in terms of molecular motion.

3. How does the kinetic theory explain the fact that gases exert pressure on all surfaces with which they come into contact ? [*A.I.S.S.C. 1986*]

4. State the Basic assumptions of the kinetic theory of gases and derive an expression for the pressure exerted by an ideal gas on the basis of the Kinetic Theory.

5. What effect do elastic collisions between gas molecules have upon our derivation of the gas law ?

6. When air in a bicycle pump is compressed, it heats. Why ?

7. A container of gas is placed into a 'gravity free' place in other space. Since the molecules of the gas exert a pressure and hence a force on the walls, why does not the container accelerate steadily ?

8. What is the interpretation of temperature according to the kinetic theory of gases ?

9. Derive the following gas laws for an ideal gas on the basis of the kinetic theory of gases :
   (*a*) Boyles' Law
   (*b*) Charles' Law
   (*c*) Law of Pressures
   (*d*) Avogadro's Law
   (*e*) Graham's Law of Diffusion
   (*f*) Dalton's Law of Partial Pressures.

10. Deduce an ideal gas equation on the basis of the kinetic theory of gases.

11. Briefly explain why there is practically no atmosphere on the surface of the moon ?

12. "The Brownian motion of a colloidal particle is due to the inequality of the impact of the surrounding molecules of the medium." Explain.

13. Explain what do you mean by degrees of freedom. Discuss the degrees of freedom of
    (*i*) monoatomic gas, and
    (*ii*) a diatomic gas

14. State and discuss the Law of Equipartition of Energy due to Maxwell.

15. Define specific heat of a substance. Explain why a gas may have a large range of specific heats. Under what conditions the specific heat of a gas is (*i*) zero (*ii*) infinity.

16. Define molar specific heats of a gas and derive the relation.

$$C_p - C_v = \frac{R}{J}$$ where the symbols have their usual meanings.

17. Explain why the specific heat of a gas at constant pressure is greater than the specific heat of a gas at constant volume ?

PROBLEMS

1. Calculate the root mean square velocity of nitrogen molecules at S T.P. The atomic weignt of nitrogen is 14 and the molecule is diatomic.

[**Ans** : $4 \cdot 9 \times 10^2$ ms^{-1}]

2. Calculate the root mean square velocity of helium molecules at a temperature of -50°C. The atomic weight of helium is 4 and it is monoatomic.

[**Ans** : $11 \cdot 7 \times 10^2$ ms^{-1}]

3. At what temperature would argon have to be raised so that the molecules would have same root mean square velocity as nitrogen molecules at 0°C ? Argon is monoatomic and its atomic mass is 40. The atomic weight of nitrogen is 14 and the molecule is diatomic. [**Ans** : 117°C]

4. If the root mean square speed of hydrogen molecules at 0°C is 1840 ms^{-1}, what is the *r.m.s.* speed at 100°C ?

[Ans : 2150·96 ms^{-1}]

5. A container of volume 10 litres holds a gas at a pressure of 2 atm. Calculate the total kinetic energy of all the particles of the gas.

[**Ans** : 3038·78 J]

6. If the *r.m.s.* speed of the molecules of a gas is 400 ms^{-1} when the pressure is 700 mm of Hg, what is the density of the gas ?

[**Ans** : 1·74 kg m^{-3}]

7. Calculate the total translational kinetic energy of 3 mol of an ideal gas at 227°C.

[**Ans** : $1 \cdot 87 \times 10^4$ J]

8. Two moles of a monoatomic gas at 300°C are mixed with 1 mole of a diatomic gas at 100°C. Calculate the final temperature of the mixture, assuming that the mixing takes place at

constant volume and that there is no exchange of heat with the environment.

[**Ans** : 209°C]

9. A Vessel contains 12 g of methane (CH_4) and 55 g of carbon dioxide (CO_2) under a total pressure of 2 atm. and at a temperature of 27°C. What are the partial pressures of the two gases and what is the volume of the vessel ? Take atomic masses of H, C and O as 1, 12 and 16.

[**Ans** : 0·75 atm ; 1·25 atm ; 25 litre]

10. Calculate the value of the universal gas constant for one gm-molecule of gas.

[**Ans** : 8·31 J mole^{-1} K^{-1}]

11. Calculate the value of the gas constant for one gram of oxygen ; 22·4 litres of oxygen at N.T.P. weigh 32 g.

[Ans : 26×10^{-2} J/g /K]

12. Calculate the specific heat at constant volume for air, given that $C_p=0{\cdot}23$, density of air at 27°C and standard pressure of atmosphere = 1·18 g / c.c. J = 4·2 J/calorie.

[Ans : 0·162]

13. Under normal conditions the average distance a hydrogen molecule travels between collisions is $1{\cdot}83\times10^{-10}$ km and the speed of the molecule is 1·830 kms^{-1}. Calculate the time between collisions and the frequency of the collisions.

[**Ans** : 10^{-10}s and 10^{10} collisions s^{-1}]

14. **Find the** root mean square speed of argon molecule at 60°C, **given that** the rms speed of oxygen molecule is 920 ms^{-1} at **1092 K.** Molecular weights of oxygen and argon are 32 and 40 respectively.

[*A.I.S.S.C. 1986*]

[**Hints** : $c_{rms}=\sqrt{\frac{3RT}{M}}$]

[**Ans.** : 454·44 ms^{-1}]

Unit 5

THERMODYNAMICS

5.1 INTRODUCTION

Energy may exist in different forms of which heat is one. Like all other forms of energy, heat energy can also be converted intoother forms of energy. Literally, thermodynamics should mean a branch of Physics dealing with the inter-relations of heat and mechanical energy. It has been seen, however, that it bears a much wider scope. The principles of thermodynamics have been successfully applied not only to all branches of Physics but also to branches of Chemistry like chemical equilibria, electrochemistry etc.

Applied thermodynamics or technical thermodynamics means those principles of heat and mechanical energy which have been applied to technical uses. In the present day world it embraces a huge field of learning. Of the multifarious technical applications of thermodynamics heat engines such as steam engines, steam turbines, internal combustion engines etc., stand out more prominently than others. In these engines heat energy is employed to develop mechanical energy. The reverse of the heat engine is a refrigerating machine. In the frigidaire type, otherwise called the mechanical type of refrigerators, heat is transferred from a body at low temperatures to surroundings which are at higher temperatures by the expenditure of mechanical energy. That is, it is a heat pump. Obviously, both these types of machines concern with transformation of heat energy and mechanical energy and as such belong to the study of applied thermodynamics.

The different phenomena in which heat plays a role may be studied under the two heads : *energy relations* and the *manner of* energy transformations. The first law of thermodynamics and the second law of thermodynamics are the two important laws which apply respectively to these investigations.

5.2 CONCEPT OF TEMPERATURE (ZEROTH LAW AND THERMAL EQUILIBRIUM)

In common language we understand what we mean by low and high temperatures. Ice is cold and steam is hot. We say steam is at a higher temperature than ice. By using the sensation of touch, we can compare two temperatures qualitatively. But this method can prove deceptive sometimes. The response of our senses depends upon their previous state. Another drawback is that qualitative analysis does not help us much. We have to answer 'how much'? Therefore, we need some operational definition of temperature.

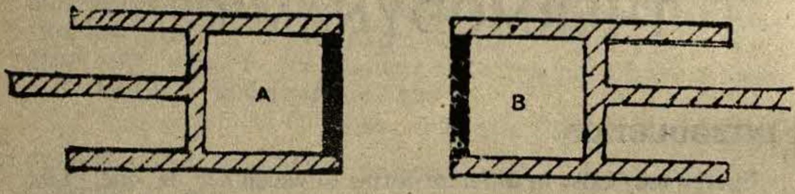


Fig. 5.1 Thermal contact between two systems

Let us take two bodies A and B, as in Fig. (5.1). To simplify our discussion we can take two gases. Let us bring them into thermal contact with the help of a diathermic wall. If A is hotter than B then, pressure remaining constant, the volume of A will decrease and the volume of B will increase ; thus changing the thermodynamic coordinates of both the systems A and B. We may say that the two systems are not in 'thermal equilibrium'. After sometime, the volumes cease to change with time, and the systems achieve thermal equilibrium. We say that now both the systems are at the same temperature. *Thus temperature is the property which tells us whether a system will be in thermal equilibrium with another system or not when brought into contact through a diathermic wall.* If the thermodynamic coordinates (say pressure and volume in the case of gases) of two bodies after thermal contact remain the same as they were before this, we say the two bodies were at the *same temperature.*

It is also our experience that if two systems A and B in thermal equilibrium with another system C (say a thermometer) separately are brought into contact by a diathermic wall, their coordinates will not change. This means they (*i.e.*, A and B) will also be in thermal equilibrium with each other. This experimental law is often known as *Zeroth Law of Thermodynamics* and may be stated.

"*If two bodies A and B are each separately in thermal equilibrium with body C, then A and B are in thermal equilibrium with each other.*"

This law is the basis of the existence of the concept of temperature. The law says that there exists a useful quantity called 'temperature' and if it was not true, taking thermometer readings would be a pointless exercise.

5.3. HEAT ENERGY

If the law of conservation of energy is to hold good, then 'something' (called heat) which is shared between two bodies at unequal temperatures is also energy. For instance if we drop a hot iron ball in a vessel containing cold water, the water warms up till the temperature of the two becomes the same. This change in temperature is due to the fact that energy flows from the hot body to cold body. *This flow of energy from one body to the other, which takes place by virtue of their temperature difference alone, is called heat energy or heat.*" The hot iron ball and the cold water, if considered separately, do not have any heat energy though they do have energy. Heat energy becomes evident only when energy flows from one to the other. When the two attain a common temperature, the quantity of heat energy transferred will be zero and the two systems will be in thermal equilibrium. An *adiabatic* process is one in which there is no transfer of heat energy.

Conventionally, the quantity of heat energy transferred to a body is said to be positive (+Q), and that transferred from a body is said to be negative (−Q).

In the early eighteenth century heat was called 'caloric fluid'. People used to measure heat in terms of an arbitrary unit known as *Calorie*. It is usually defined as the amount of heat needed to raise the temperature of 1 g of water through 1°C *i.e.*, from 14·5 to 15·5°C. It was further believed that the caloric fluid is conserved *i.e.*, the loss in the caloric fluid of the hotter body is equal to the gain in the caloric fluid of the colder body. This concept can be easily disproved. If a girder is cut with a saw and water is continuously poured at the place of cutting to keep it cool, the water continuously gains in the caloric fluid while the girder remains in the same state. One can conjecture that the continuous dissipation of work done on the saw may be directly responsible for heating. It was shown by Rumford and Joule that heat is a form of energy. It should be measured in the same unit as that of energy. It has recently been internationally agreed to express all energy changes in the energy unit, the ***JOULE* (*J*)**. A *joule* is defined as the work done when a force of one newton acts through one metre. It is found that

1 calorie = 4·18 joules

5.4 WORK DONE

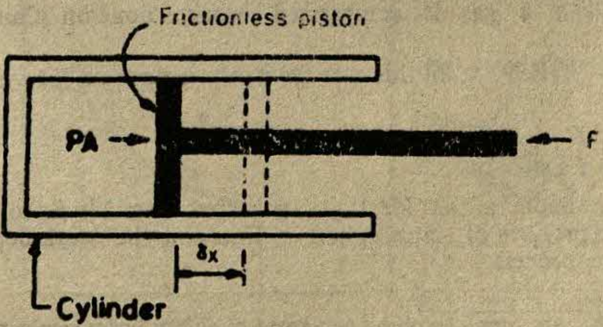


Fig. 5.2

Consider a mass of gas enclosed in a cylinder by a frictionless piston of cross-section area A which is in equilibrium under the action of an external force F acting to the left and a force due to the pressure P of the gas acting to the right (Fig 5.2). We have

$$F = P.A.$$

Let the gas expand quasi-statically. By the term quasi-static we mean that the gas should change its state *so slowly* that the temperature throughout the gas should remain uniform. This is because we are assigning one temperature to the whole system. Let the piston move outwards through a small distance δx which is so small that P remains practically constant during the expansion The external work done δw by the gas against F will be

$$\begin{aligned}\delta W &= F.\ \delta x\\ &= P.\ A.\ \delta x\\ &= P.\ \delta V\end{aligned}$$

where δV ($= A\ \delta x$) is the increase in the volume of the gas. The work done by the gas in this small expansion equals its pressure multiplied by the increase of volume.

The total work done W by the gas in a finite expansion from V_1 to V_2 is given by

$$W = \int dW = \int_{V_1}^{V_2} P dV$$

To evaluate this integral and obtain a value for W requires the relation between P and V to be known. If P is constant during the expansion from V_1 to V_2, then

$$W = P(V_2 - V_1)$$

is this work done a positive or a negative quantity ? In mechanics work done on a system is taken as positive, while here it is just the opposite *i.e., work done by a system is taken to be positive.* Here, we concentrate on the forces exerted by the *system on the **surroundings. Thus in the case we have discussed just now, force and displacement are in the same direction and, hence, work is done by the system, and it is taken as positive.

When a gas is compressed, work is done on it and is also given by $\int_{V_1}^{V_2} PdV$. In this case the work is done on the system and conventionally it will be negative.

Example 5.1 :

On mole of an ideal gas is heated from 273 K to 546 K at constant pressure of 1 atmosphere. Compute the work done by the gas in the process.

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* system means an object or a group of objects under consideration.

** Surroundings means everything external to the system.

Solution :

Initial volume of the gas $V_1 = 22{\cdot}4$ litres. Final volume can be obtained by using the gas equation

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\text{or } V_2 = \frac{P_1 V_1}{T_1} \cdot \frac{T_2}{P_2}$$

Here $T_1 = 273\text{K}$

$T_2 = 546\text{K}$

$$P = P_1 = P_2 = 1 \text{ atmosphere}$$
$$= 76 \times 13{\cdot}6 \times 980$$
$$= 1{\cdot}013 \times 10^6 \text{ dynes cm}^{-2}$$

$$V_2 = \frac{1 \text{ atm} \times 22{\cdot}4 \text{ litres} \times 546}{1 \text{ atm} \times 273}$$
$$= 44{\cdot}8 \text{ litres}$$

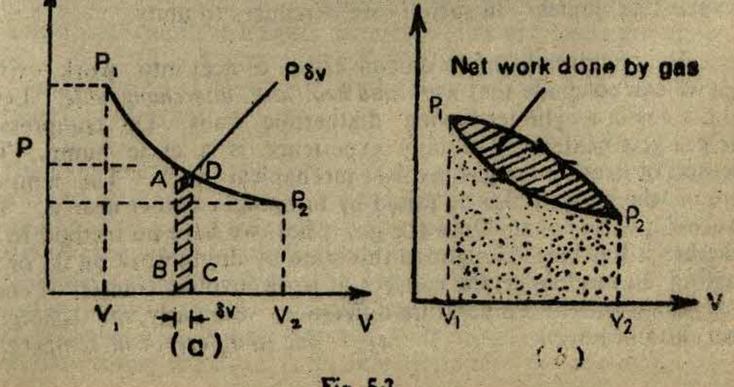
$$\text{Work done} = W = \int_{V_1}^{V_2} P dV = P(V_2 - V_1)$$
$$= 1.013 \times 10^6 (44{\cdot}8 - 22{\cdot}4) \times 10^3 \text{ ergs}$$
$$= \frac{1.013 \times 10^9 \times 22.4}{4{\cdot}18 \times 10^7} \text{cal}$$
$$= \mathbf{542{\cdot}8} \text{ cal.}$$

+ve sign indicates that the work is done by the system.

5.5 INDICATOR DIAGRAM

Indicator diagram is a graph showing how the pressure P of a gas varies with its volume V during a change. The work done in any particular case can be derived from it.

In Fig 5.3 (a) if the pressure P=AB at the start of a very small expansion δv=BC, the work done P δv is represented to a good



(a) (b)

Fig. 5.3

approximation by the area of the shaded strip ABCD. The total work done by the gas in a large expansion from V_1 to V_2 is therefore the sum of the areas of all such strips, that is, area $P_1P_2V_2V_1$ and clearly depends on the shape of the P-V graph. If the graph had represented a compression of the gas from V_2 to V_1, the work done on it would be represented by the same area. When the gas suffers changes which eventually return it to its initial state, it is said to have undergone a *cycle* of operations and the indicator diagram is a closed loop like that in Fig 5.3 (b). The net work done by the gas in this case is represented by the shaded area.

The indicator diagram is useful in calculating the work done in the process of expansion or compression of a gas where the relationship between P and V is not known. It plays a very important role in the theory of a heat engine.

5.6 WORK AND HEAT

The exact relation that existed between mechanical work done and the amount of heat produced was found by Dr. Joule of Manchester in 1848. He found that if a certain amount of work is wholly converted into heat then the heat produced, H, is directly proportional to the work done, W, *i.e.*,

$$W \propto H$$

$$\text{or} \quad \boxed{W = JH}$$

where J is a constant of proportionality and is known as *Joule's mechanical equivalent of heat*. It is defined as the amount of work done for producing unit quantity of heat.

In SI units work W is in joules and heat H is in kilocalories (heat needed to raise one kg of water from 14·5° to 15·5°C). Thus units of J will be joules/k cal. The value of J is 4184 joules/k cal, or 4·184 J/cal. Now-a-days the units used for both H and W are the same *i.e.* joules. In such a case, J reduces to unity.

Heat engines function on conversion of heat into work. From this we can conclude that *work and heat are interchangeable.* Let us take a gas in a cylinder having diathermic walls. On compression the gas gets heated. The daily experience is a cycle pump. This method of heating the gas involves mechanical work. The temperature of the gas can also be raised by bringing a burner near it. This process involves heat. Once the gas is hot, we have no method to tell whether it has been brought to this state by doing work on it or by heating. In mechanics, the concept of work involves transfer of energy other than due to temperature difference. Similarly, *the concept of heat also involves transfer of energy due to difference in temperature.*

The original method used by Dr. Joule to determine the relation between heat and work is as follows :

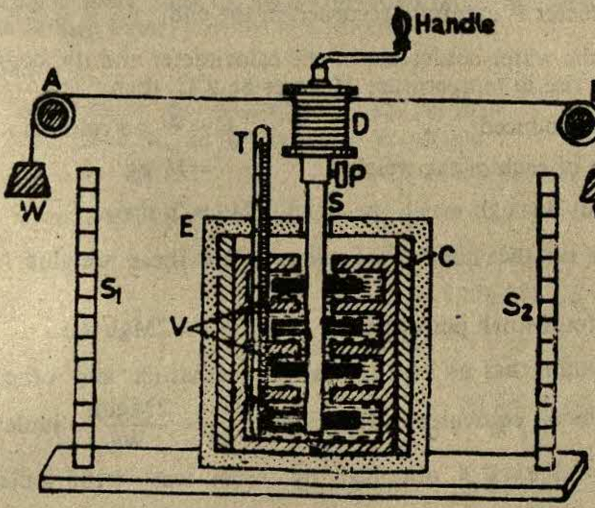


Fig. 5.4

The apparatus consists essentially of a copper calorimeter C to the sides of which are soldered the vanes V. The calorimeter is provided with a water tight lid through the centre of which is introduced a spindle S which carries a paddle wheel having a set of paddles. The paddles can rotate in between the corresponding openings in the fixed vanes. The vanes prevent the rotation of water moving along with the revolving paddles. The spindle is attached by a removable pin P to the drum D.

Two cords are wounded round the drum in the same direction in such a way that they leave the cylinder at the opposite extremities of a diameter. These cords pass over frictionless pulleys A and B and to the free ends are attached two equal loads W, W. The vertical scales S_1, S_2 help to measure the heights through which the loads are allowed to fall. A sensitive thermometer T is placed as shown to read the rise in temperature. The whole arrangement is well insulated to avoid radiation of heat.

The calorimeter contains a known weight of water whose temperature is read by a very sensitive thermometer. When the weights are allowed to fall the spindle turns and the paddles produce a churning motion of the water inside the calorimeter. The potential energy of the weights is converted into the kinetic energy of water. The motion of water is prevented by vanes and thus the kinetic energy of water is converted into heat energy. Just as the weights are about to reach the ground the drum is quickly detached by removing the pin P and the weights are raised to their previous positions. The pin is replaced and the weights are allowed to fall as before. The operation is repeated several times and the final temperature of water in the calorimeter is carefully recorded in the end.

Let the water equivalent of the calorimeter and its contents be W and the rise in temperature of water be θ°C, then

Heat produced $H = W \times \theta$ calories

Mass of each of the weights $= M$ kg

Height through which the weights fall= h metre

Let n be the number of times that these weights descend, then

The total work performed $W = 2Mgh \times n$

Assuming that no heat is lost by radiation and other causes, the mechanical equivalent of heat $J = \frac{W}{H} = \frac{2Mghn}{W\theta}$ joules/calorie

The value of J obtained by Joule was 4·185 joules/calorie

Sources of error : (*i*) *Loss of heat due to radiation* : The temperature during the experiment rises ; hence a part of heat is lost due to radiation. A correction, therefore, must be applied to the final temperature of the contents of the calorimeter. This correction may be determined by noting the fall of temperature in half of the time during which the experiment was performed and then adding this to the final temperature.

(*ii*) *Loss of energy due to friction of pulleys* : Part of the work done is utilized in overcoming the friction between the various parts and due to the friction at the pulleys outside the calorimeter. This correction is applied by noting the amount of work required to run the paddle wheel when there is no liquid inside the calorimeter.

(*iii*) *The Kinetic Energy of the falling weights*: When the weights fall the potential energy is converted into kinetic energy of water which is utilized in raising the temperature. Also part of the energy is converted into the kinetic energy of the weight which acquire a certain velocity in falling to the ground. Hence this correction must be applied.

If V ms^{-1} is the velocity of each weight just before it reaches the ground, then the kinetic energy attained by the weights before striking the ground $= 2 \times \frac{1}{2}MV^2 = MV^2$ joules.

Hence net potential energy utilised in rotating the paddles in each fall $= (2Mgh - MV^2)$

The work done in n falls

$W = (2Mgh - MV^2) \times n$ joules.

Example 5.2 :

Water at Bhakra Dam falls through a height of 200 m. Assuming that all the energy due to fall is converted into heat and is retained by the falling water, find the rise in temperature.

Solution :

Let M kg be the mass of water falling then work done by the falling water $W = Mgh$

$$= M \times 9.8 \times 200 \text{ J}$$

If θ is the rise of temperature, then

$$\text{Heat produced } H = M\theta \text{ kilocal}$$

$$= 1000\, M\theta \text{ cal}$$

$$\text{But } H = \frac{W}{J}$$

$$1000M\theta = \frac{M \times 9{\cdot}8 \times 200}{4{\cdot}2}$$

$$\text{or } \theta = \frac{9{\cdot}8 \times 200}{4{\cdot}2 \times 1000} = 0{\cdot}467^\circ\text{C}$$

5.7 INTERNAL ENERGY

A body always has certain amount of energy within itself. We call this energy as the *internal energy*. It is usually represented by the symbol U. In general, the internal energy of a gas or a liquid or a solid consists of two components :

(*i*) Kinetic energy due to the translational, rotational or vibrational motion of the molecules, all of which depend only on the temperature. (In a monoatomic gas there is only translational motion)

(*ii*) Potential energy due to the intermolecular forces; this depends on the separation of the molecules *i.e.*, the volume.

In an ideal gas only the kinetic component is present (why ?) and the kinetic theory shows that the translational part of it equals $\frac{3RT}{2}$ per mole. For real gases, both components are present with the kinetic form predominating and their internal energy depends on the temperature and the volume of the gas. In a solid kinetic energy and potential energy are present in roughly equal amounts.

When a thermodynamic system changes from one state to another, its internal energy also changes. The changes in the internal energy when the system changes from one state to another depends only on the initial and final states of the system. For instance consider a gas enclosed in a cylinder with a moving piston. Let the piston be pushed by doing some work. The gas gets compressed and its temperature rises. If the walls of the system are thermally insulated no heat will be transferred *i.e.*, the process will be *adiabatic*. In this process the mechanical work done on the system increases the internal energy of the system from U_1 to U_2 such that

$$-W=U_2-U_1 \text{ where } W= \text{work done on the system.}$$

In conclusion, therefore, we can say that *each definite state of a body corresponds to a fixed value of the internal energy. An isolated body has a constant internal energy. A change in the internal energy can occur only if any transfer of energy between the body and its surroundings is allowed.* This can take place by (*a*) performance of work, (*b*) by heat transfer.

5.8 FIRST LAW OF THERMODYNAMICS

Let us consider a system consisting of a gas enclosed in a cylinder fitted with a movable piston. Let all the walls of the cylinder except the bottom be thermally insulated. Let the bottom be brought into contact with a hot body or a burner. When heat energy is added to the gas, it causes (i) a rise in temperature of the gas and (ii) movement of the piston. Rise in temperature of the gas means that a part of the heat energy has been converted into internal energy of the gas and the movement of the piston shows that a part of the heat energy has been used in doing an external work.

If δQ is the heat supplied to the mass of gas and if δW is the external work done by it then the increase of internal energy δU equals $(\delta Q-\delta W)$ if energy is conserved. Hence

$$\boxed{\delta Q=\delta U+\delta W} \qquad \ldots(5\text{-}1)$$

If δQ is infinitesimally small, so are δU and δW. In such a case equation (5·1) may be mathematically written as

$$\boxed{dQ=dU+dW} \qquad \ldots(5\text{-}2)$$

This equation is taken as the *first law of Thermodynamics*. It can be stated in words as follows :—

"If heat is supplied to a system which is capable of doing work, then the quantity of heat absorbed by the system will be equal to the sum of the increase in the internal energy of the system and the external work done by it."

Here dQ is taken as *positive* if heat is supplied to the gas and *negative* if heat is transferred from it. dW is *positive* when external work is done by the gas (expansion) and negative if work is done on it (compression).

It must be noted that

(*a*) the first law of Thermodynamics is not a separate law but is a special case of the general law known as the Universal

Law of Conservation of Energy. It may be called the law of conservation of energy applied to the case of the mutual transformation of heat energy and mechanical energy.

(*b*) The first law of Thermodynamics applies to all states of matter, solid, liquid and gas.

(*c*) dU may be any type of internal energy (translational, rotational K.E.; Binding energy) stored in the system and dW may be any type of work done by the system.

(*d*) dU is a characteristic of a system whereas dQ and dW depend on the manner in which the system is changed from one state to another and hence they are not the characteristic of the state of the system.

It may be interesting to point out that through the Zeroth Law of Thermodynamics we have introduced a thermodynamic quantity called *temperature*, and through this (first) law of thermodynamics we have brought out the existence of a thermodynamic function ***internal energy***.

5·9. APPLICATIONS OF THE FIRST LAW OF THERMODYNAMICS

1. *Latent Heat* :

When the temperature of a body (solid or liquid) is steadily raised, the body may suddenly change from solid to liquid state (*fusion*) or from liquid to gaseous state (*evaporation*). In such discontinuous changes the energy also changes discontinuously. This is the phenomena of Latent heat.

Let us consider a boiling process. We know that on adding heat a substance changes its phase from liquid to vapour at constant temperature and pressure. If V_1 is the volume of the liquid and V_2 be the volume of the vapour, then work done (dW) in expanding from V_1 to V_2 at constant temperature and pressure P will be given by

$$dW = P(V_2 - V_1)$$

Let the Latent heat of vaporization be L. It represents the heat needed per unit mass to change from liquid to vapour at constant pressure and temperature. The heat absorbed (dQ) by the liquid during boiling process will be given by

dQ = mL where m is the mass of the substance undergoing phase change.

By applying first law of thermodynamics to the process, we get

$$dQ = dU + dW$$

$$\text{or} \quad \boxed{mL = (U_2 - U_1) + P(V_2 - V_1)}$$

where U_1 is the internal energy in the liquid phase and U_2 is the internal energy in the vapour phase.

Thus knowing m, L, P, V_1 and V_2, we can easily calculate gain in internal energy $dU=(U_2-U_1)$.

Similarly, we can apply the first law of thermodynamics to obtain an expression for the change in the internal energy during the melting process. Often during melting a substance does negligible work on its surroundings as the change in volume is usually very small. Thus

$$dQ=dU \qquad (\because dW=O)$$

$$\text{or} \qquad \boxed{mL \quad = \quad dU}$$

2. *Two specific heats of a gas* :

We have already discussed this in the UNIT-4 on kinetic theory of gases ; but we will now see how the relation $C_p-C_v=R$ can be obtained by applying first law of thermodynamics.

Consider one mole of an ideal gas. If its temperature is raised by an amount dT at constant volume then the heat transferred is $C_v\ dT$ where C_v is the molar specific heat at constant volume. Since there is no change in its volume the work done in the process will be zero.

By applying the first law of Thermodynamics

$dQ=dU+dW$, we have

$$\boxed{C_v\ dT=dU} \qquad \ldots(i)$$

$$(\because dW=PdV=O)$$

Now, let the gas be heated at constant pressure P. The heat required to change its temperature by the same amount dT will now be $C_p\ dT$ where C_p is the molar specific heat at constant pressure. The work done in the expansion will be P.dV where dV is the change in volume of the gas. By applying the first law, we have

$$\boxed{C_p.dT = dU + PdV} \qquad \ldots(ii)$$

Let us assume that the change in the internal energy (dU) of an ideal gas is entirely due to change in its kinetic energy and since change in kinetic energy of an ideal gas depends only on the change in the temperature (Kinetic Theory of Gases) so change in temperature dT remaining the same, the change in internal energy dU will also be the same in both the cases.

Subtracting (*i*) from (*ii*)

$$C_p dT-C_v\ dT=PdV \qquad \ldots(iii)$$

Since for an ideal gas

$$PV=RT \qquad ...(iv)$$

Differentiating both sides of eq. (iv) we have

$$PdV=RdT \qquad ...(v)$$

From (iii) and (v) we have

$$C_p dT-C_v dT=RdT$$

or

$$\boxed{C_p-C_v=R} \qquad ...(vi)$$

Here C_p, C_v and R are in the *same units.* You must be remembering that if we take 1 gm of a gas, then C_p and C_v are specific heats of a gas and R is the gas constant and *not* the universal gas constant.

5.10 CONVERSION OF HEAT INTO MECHANICAL WORK—(HEAT ENGINE)

It is now within the experience of every body that mechanical work can be converted into heat and heat also into work. It is comparatively easy to get some mechanical work out of a process expanding some heat energy. This does not mean, however, that the problem is as easy to produce a device for *continuous* production of mechanical work from a heat supply. Moreover, what portion of the heat supplied is convertible into work is a big question and this must by all means be as important a criterion for the device as the construction of the device itself. Therefore any approach to the subject involves two questions : namely the question of the principle of construction and the question of thermal efficiency of the device. *Any device which can continuously convert heat into work efficiently is called a heat engine.* Such devices or machines today are numerous. The most important among them are the reciprocating steam engines, the steam turbines, the gas turbines, the internal combustion engines, the jet engines etc. The **refrigerator** is a reversed heat engine. The heat engines are often referred to as *prime-movers* because they develop their motive power (heat developed by combustion of fuels) and supply the same to dynamos.

The *essential features* of a heat engine, leaving aside the mechanical details, should be as follows :

(*i*) There must be a *hot reservoir* or *source* from which heat will be supplied at a high temperature.

(*ii*) There must be a *cold reservoir* or *sink* or *refrigerator*, at a temperature lower than that of the source, called the receiver which will take back the portion of heat not converted into work.

(*iii*) There must be a medium, called the *working substance,* which will take in heat from the source, convert a part of it into mechanical work suffering a lowering of temperature thereby and reject the balance of heat to the sink

The magnitude of the source and the sink is so large that their temperature remain unchanged because of any heat supplied to them or removed from them.

Principle of operation : The working substance shall operate through a cyclic process of changes which can be repeated indefinitely A definite amount of external work will be done in each cycle of operation. The working substance comes back to the same initial state at the end of each cycle and will, thus, ensure a ***continuous*** production of mechanical work at the same rate. The indicator diagram of the system is a closed curve and not a line.

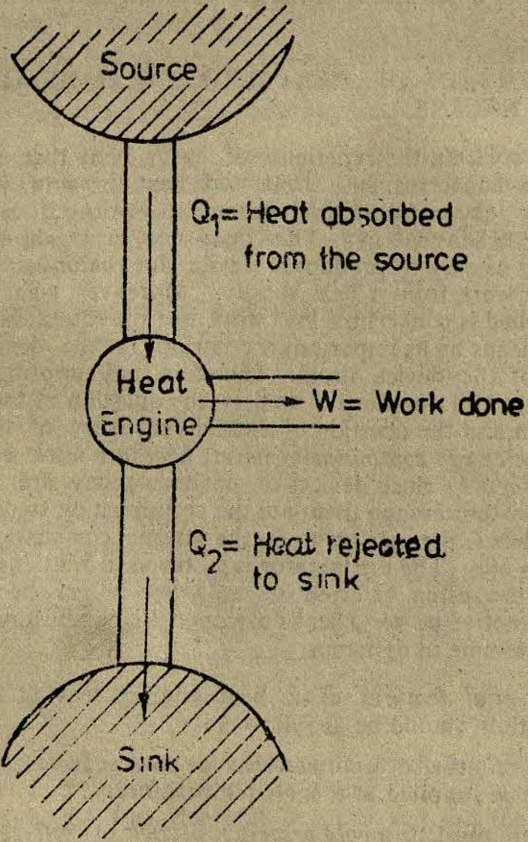

Source
Q_1= Heat absorbed from the source
Heat Engine
W = Work done
Q_2= Heat rejected to sink
Sink

Fig. 5·5

All the properties of the system remain unchanged during the process.

Thermal Efficiency of a heat engine :

Thermal efficiency (η)

$$= \frac{\text{Heat equivalent of work done during a time}}{\text{Heat input during the same time}}$$

$$= \frac{W}{Q}$$

For an ideal engine, if Q_1=heat absorbed from the source during a cycle

and Q_2=heat rejected to the sink in that cycle

then heat converted into work $= Q_1 - Q_2 = W$ according to first law of Thermodynamics. Here change in internal energy is zero as the system returns to its initial state after completing the cycle.

There, the efficiency (η) $= \frac{Q_1 - Q_2}{Q_1}$

$$\text{or} \quad \boxed{\eta = 1 - \frac{Q_2}{Q_1}} \quad \cdots \quad (5\text{-}3)$$

So, for the given value of Q_1, the smaller the value of Q_2, the higher is the thermal efficiency. In other words if work done by an engine is larger then its thermal efficiency is higher. Moreover it is clear from the eq. (5-3) that the value of η is always less than unity.

5.11 SECOND LAW OF THERMODYNAMICS

The first law of Thermodynamics enunciates that heat and mechanical energy (work) are both forms of energy and one of them can be transformed into the other form. Further, it enjoins an equivalence between them (W=JH) when the transformation takes place. It simply requires that when work is produced by heat, the heat equivalent of work done must come from somewhere. ***But it is silent about wherefrom and how this heat will come.*** **That is it lacks the sense of direction.** Moreover, it does not suggest whether the heat equivalent of the work done is the whole of the heat coming from somewhere or a part of it or, if a part of it, which part it is. Mechanical work can be fully convetted into heat. The question arises whether heat can be wholly converted into mechanical work or not. The answer to these queries is given by the new principle known as *Second Law of Thermodynamics.* In other words the Second Law of Thermodynamics deals with a question which is not at all covered by the First Law. A few illustrations will clear the point.

(*i*) When two bodies A & B, exchange heat, the first law tells us that the heat lost by one body is equal to the heat gained by the other. ***But it does not tell us in which direction this heat will flow.*** Only experience tells us that heat will pass from the hotter body to the colder one spontaneously ; but in speaking of hotter and colder bodies, we are making use of a new physical concept (temperature) which is not at all included within the scope of the first law.

(ii) Let us consider a thermodynamical process illustrated by the equation

$$2H_2+O_2=2H_2O+U \text{ joules}$$

The equation tells us that if 2 mols of H_2 gas combine with one mol. of O_2, 2 mols. of H_2O vapour are formed and U joules of heat is evolved, or *vice versa*, when 2 mols. of H_2O vapour are decomposed completely into H_2 and O_2, U joules of heat must be absorbed. This result is obtained as a matter of experience from calorimetric experiments, and we interpret the result according to the First Law of Thermodynamics by saying that in the combination of two molecules of hydrogen with 1 molecule of oxygen to form 2 molecules of H_2O, the diminution in energy amounts to U joules. ***But the first law cannot tell us in which direction the reaction will take place.*** If we have a mixture consisting of H_2, O_2 and H_2O vapour in arbitrary proportions at some definite temperature and pressure, the first law cannot tell us whether some H_2 and O_2 will combine to form H_2O or some H_2O will dissociate into H_2 and O_2.

(iii) Consider a cricket ball rolling on the cricket ground. When left to itself it comes to rest after a certain interval of time due to friction. We interpret this according to the First Law of Thermodynamics by saying that the Kinetic energy of the cricket ball is converted into heat energy but the first law utterly fails to answer as to why this process does not happen in the opposite direction. Nobody has seen a stationary cricket ball in the cricket ground suddenly gaining heat energy and start rolling after cooling the surroundings.

(iv) In a modern power station steam enters the turbines at about 830 K but, even so, the efficiency attained is just over 30 percent which means that about two-thirds of the heat absorbed is rejected to the atmosphere usually via the cooling towers.

(v) In a heat engine, heat is continuously absorbed from the reservoir, and rejected into the sink. Thus the temperature of the reservoir or source goes on falling and that of the sink goes on rising. Suppose that heat is rejected into the surrounding bodies. In order to get a continuous supply of work, the temperature of the source must be maintained higher than that of the bodies surrounding it. If this is not done, the source will cool to the temperature of the surroundings, and the engine will stop working.

These observations were generalised by Clausius and Kelvin & Planck into well known statements of Second Law of Thermodynamics.

Clausius Statement of Second Law of Thermodynamics :—

> "*It is impossible for a self-acting machine when it operates in a cycle unaided by an external agency, to convey heat from a body at a low, to one at a higher temperature*" or "*heat cannot of itself i.e., without the performance of work by some agency pass from a colder to a warmer body.*"

The second law as stated above implies that heat is not directly transferred from a cold body to a hot body ; an intermediary substance absorbs heat from the cold body and rejects it into the hot body. Take the case of a refrigerator (ammonia ice plant). Heat is absorbed from the brine solution at a lower temperature and rejected into water at a higher temperature. This does not happen of its own accord. An external agency (the pump) has to do work in order to achieve this.

Kelvin - Planck Statement of Second Law of Thermodynamics :—

> "*It is impossible to construct an engine, operating in a cycle, which will produce no effect other than extracting heat from a reservoir and performing an equivalent amount of work.*"

The law as stated above implies that the working substance working in a cycle cannot convert all of the heat extracted into work. It has to reject some amount of heat to the sink. So in order to convert heat into work it is essential to have both a source and a sink. Since all of heat extracted is never converted into work no matter how good the engine design may be it follows that the efficiency of an engine is never unity.

It should be noted that both the above statements of the Second Law of Thermodynamics are equivalent to one another. They are one and the same thing and one can be derived from the other. They cannot be proved directly but their consequences can. In both the statements an important point to note is the phrase "*when it operates in a cycle.*" Many of the so-called violations of the second law are based on ignoring this important point. The Second Law of Thermodynamics has been applied to a wide range of phenomena.

5.12. REFRIGERATORS AND HEAT PUMPS

The action of heat pumps and refrigerators is the reverse of that occurring in a heat engine. In these devices work (W) is done on the system which enables it to transfer heat (Q_2) from a low temperature reservoir (sink) at T_2 to a higher temperature reservoir at T_1 (Fig. 5.6). The function of a heat pump is to supply as much heat as possible to the hot body, *i.e.*, to make Q_1 large. *Its co-efficient of performance* is measured by $\frac{Q_1}{W}$ (which is greater than 1). A refrigerator, on the other hand, is designed to remove heat from the cold body *i.e.*, to make Q_2 large. Its coefficient of performance is given by

$$\frac{Q_2}{W}$$

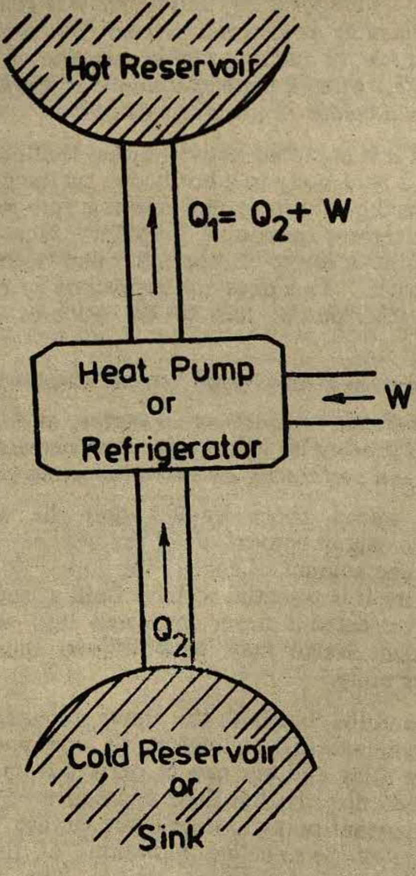


Fig. 5.6

Let the quantity of heat absorbed by the refrigerant in a refrigerator at a lower temperature be Q_2. The work done on the refrigerant be W and let the heat rejected by the refrigerant be Q_1. The substance is working in a cycle such that there is no change in its internal energy. Applying First Law of Thermodynamics

$$dQ = dU + dW \text{ we have}$$

$$Q_2 - Q_1 = -W$$

$$\boxed{Q_1 = Q_2 + W}$$

This equation implies that it is always necessary to do work on the substance in order to transfer heat from a sink to the source. In household refrigerators the work is done by an electric motor. The working substance commonly used in it is **freon** (CCl_2F_2). It absorbs heat from materials kept inside the refrigerator and rejects it to the surrounding air which is at a higher temperature.

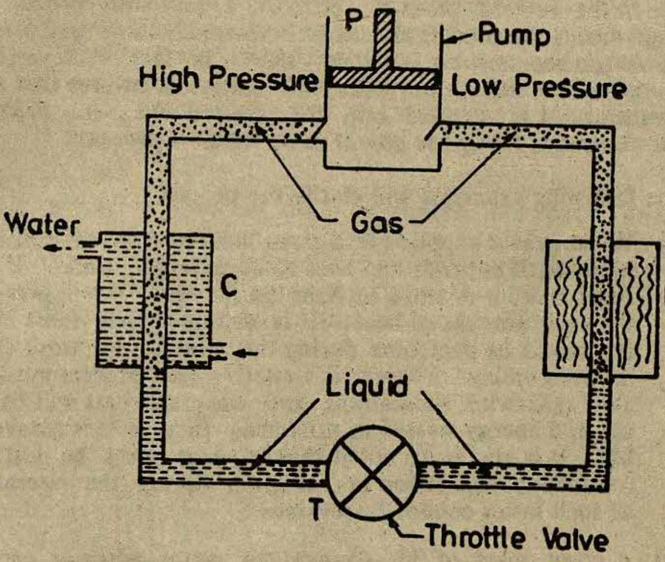


Fig. 5.7 Principles of a refrigerator

The principle of a refrigerator is shown in Fig (5.7). The gas is compressed to a high pressure with the help of a compressor P. It is cooled down by water circulating through a spiral pipe at C (in home refrigerators cooling is done by the atmospheric air). The gas then gets liquified. This high pressure liquid passes through a throttle valve T towards a low pressure side. It comes out as a mixture of liquid and vapour and its temperature gets lowered. At D this liquid-vapour mixture takes heat from its surrounding material (thus cooling this stuff), and the remaining liquid also turns into vapour. This vapour is finally sucked by the pump P.

5.13 REVERSIBLE AND IRREVERSIBLE PROCESSES

Reversible Process :

A reversible process is one which can be retraced in the opposite direction so that the system and the surroundings pass through exactly the same states in all respects as in the direct process. **After the conclusion of a process, the system and the surroundings are restored to their initial state without producing any change in either of them. Further, the *thermal, chemical* and *mechanical* effects at each stage should be exactly reversed *i.e.*, the amounts of heat received and of**

work done in each step should be the same as in the direct process, but with opposite signs. That is, where heat is absorbed in the direct process it should be given out in the reverse process and vice versa, and where work is done by the working substance in the direct process, an equal amount of work should be done on the working substance in the reverse process. In short a reversible process at every stage during the process should be in *thermodynamic equilibrium*. A reversible process must be *infinitely slow*. In fact a reversible process implies that there is no wastage of energy. It ensures that any energy transformed is directed into the desired channel. *Friction and other dissipative forces are absent in a reversible process.*

The following examples will clarify the process :

(*a*) When heat is supplied to a given mass of a gas at constant pressure, it expands and does some external work. If the same amount of work be done on the gas, it will give out the same amount of heat. It is assumed that there is no friction to be overcome during the process as work done in overcoming friction is wasted. The process must be slow otherwise oscillations and eddy currents will be set up, and energy wasted in producing them is not recoverable. It is also to be noted that no heat must be lost by conduction, convection or radiation during the operation as such losses cannot be reversed.

(*b*) A given mass of ice changes to water when a certain amount of heat is absorbed by it, and the same mass of water changes to ice when the same quantity of heat is removed from it.

(*c*) Evaporation is reversible.

(*d*) All isothermal and adiabatic changes are reversible when *performed very slowly*. When a gas is compressed isothermally its volume decreases, and on releasing the pressure the gas regains original volume, if there is no friction.

(*e*) The reactions in some electrical cells can also be approximately reversible.

(*f*) The transfer of heat from one body to another is reversible only when the two bodies are at the same temperature, or have *infinitesimal* temperature difference. In case of two bodies at *finite* different temperatures, the transfer of heat occurring by conduction or radiation cannot be reversed.

(*g*) Take the case of a spring balance. When the spring is *very slowly* stretched, work is done upon it. If, on the other hand, it is allowed to contract slowly by the same amount, the same amount of work is done by the spring, for the work done in increasing the length of the spring by δl is equal to $F.\delta l$ where F is the force. Both the quantities F and δl depend upon the state of spring at the instant and will have the same value whether the spring is expanding or contracting. Thus the work done upon the spring in stretching will be equal to the work done by the spring while contracting during the reverse process and the process is reversible. *It is essential, however, that the stretching must be produced or reduced gradually by the application of an external force which should differ infinitesimally at every instant from the stress developed in the spring*, otherwise a part of the work will be spent in setting up vibrations of the spring and the process will not be reversible.

A reversible process may be represented by a line on the indicator diagram as shown in Fig (5.8) as it can be performed backward so that all the changes occurring in any part of the direct process are exactly reversed in the corresponding part of the reversed process, and no other changes are left in the external bodies.

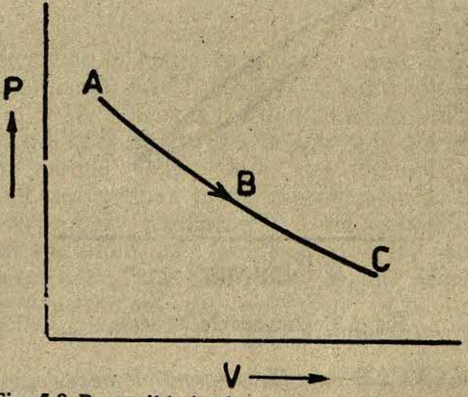


Fig. 5.8 Reversible isothermal expansion of a gas

In practice the *perfect reversible change does not exist* but the idea is a useful theoretical standard for judging real changes. All attempts in practice are aimed at approaching this ideal condition of reversibility though it may not be actually reached. The maximum efficiency of a heat engine under given conditions is worked out on assuming the operating cycle to be reversible.

An Irreversible Process

An irreversible process is such that it cannot be retraced in the opposite direction by reversing the controlling factors.

Examples :

(*i*) Sudden unbalanced expansion of a gas, either isothermal or adiabatic.

(*ii*) Heat produced by friction.

(*iii*) Heat generated when a current flows through an electrical resistance in one or the opposite direction.

(*iv*) Exchange of heat between bodies at different temperatures by conduction or radiation.

(*v*) Diffusion of liquids or gases etc.

(*vi*) All chemical reactions are irreversible.

Examples : (*i*) exhibit internal mechanical irreversibility ; (*iii*) and (*iv*) exhibit thermal irreversibility and (*v*) exhibit chemical irreversibility.

An irreversible process can be represented on the indicato diagram as shown in Fig. (5.9). We can say that most natural processes are irreversible.

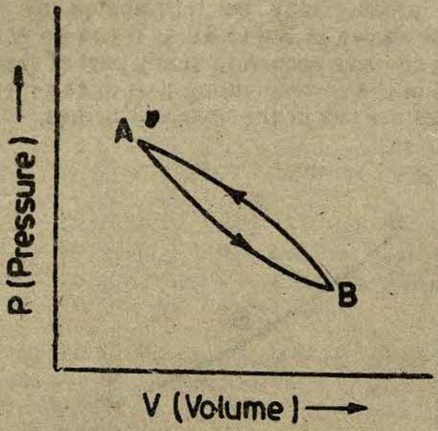


Fig. 5.9 Irreversible Process

5.14 CARNOT'S CYCLE—*An ideal reversible heat engine* :

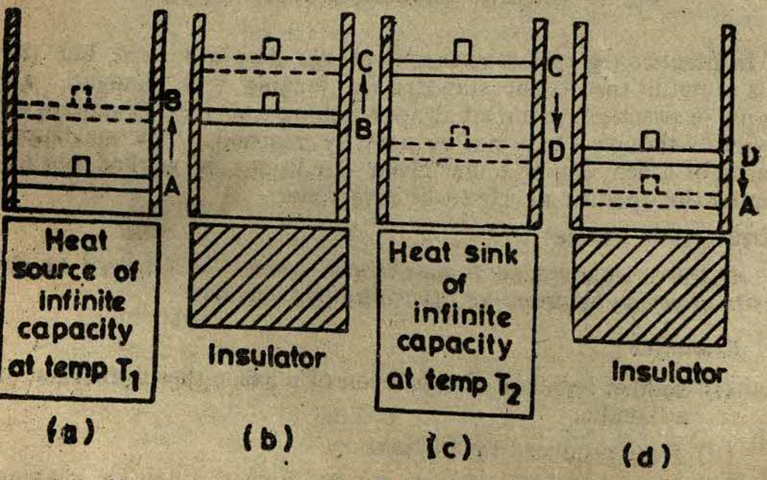

B
A
Heat source of infinite capacity at temp T_1
(a)
C
B
Insulator
(b)
C
D
Heat Sink of infinite capacity at temp T_2
(c)
D
A
Insulator
(d)

Fig. 5.10

Carnot's cycle constitute a *cycle of operations* for an ideal heat engine which though never realisable in practice provides the best guide for the construction of actual heat engines and its improvement. It bears an immortal name, *SADI CARNOT*, a French engineer who died young at 26 in 1832. He was the first to approach the problem of the efficiency of the heat engine (which was only 5 to 55%) from a fundamental stand point. Before him, the attempts we find on record made by different persons at the improvement of the efficiency of steam engines were only haphazardly tried, sometimes meeting with partial success and often not. He tried to bring out the fundamental features in the action of the steam engine leaving out the details about the construction and operation of the engine. The main features of an *idealised carnot engine* are the following :

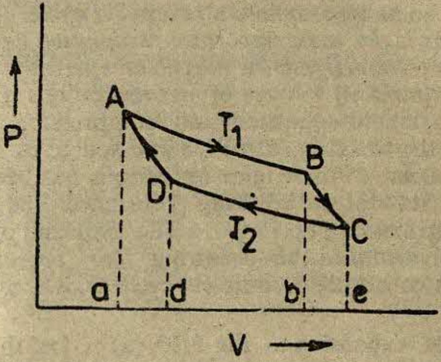


Fig. 5.11 Indicator diagram of carnot's cycle

(*i*) The engine operates in a *cyclic process* between two constant temperatures, one provided by a higher temperature body, called the *source*, and the other provided by a lower temperature body, called the *sink* or *refrigerator*.

(*ii*) It receives heat from the source at constant temperature.

(*iii*) It performs some mechanical work in each cycle, the corresponding energy being a part of the heat drawn from the source.

(*iv*) The balance of heat is rejected to the sink.

For the organisation of the engine, suppose a perfectly non-conducting piston is fitted in a cylinder which is also perfectly non-conducting everywhere excepting at the bottom which is made of a perfect conductor. Take also a heat source at temperature T_1 and heat sink at a lower temperature T_2, both being of infinite capacity for heat, so that their temperatures may remain constant inspite of any withdrawal or rejection of heat from or to either of them.

Let us suppose, for convenience, that one gm-molecule of a *perfect gas* is taken in the cylinder as working substance and the working substance is taken through the following four successive reversible operations so as to complete a reversible cycle. *The reversibility of the operations is the most important assumption* here, for *the aim of the engineer is to investigate the maximum efficiency* attainable by an engine which requires all sources of irreversibilities (*i.e.*, wastage due to friction, unbalanced expansion and compression, temperature difference between the working substance and source or sink etc.) to be absent. The scheme of the engine operations has been shown in Fig (5.10), and the state of the working substance has been shown in an indicator diagram in Fig (5.11). The working of the engine can be distinctly divided into the following *four operations*. These series of operations constitute a complete cycle known as the Carnot's *cycle* :

(1) **Isothermal expansion** : Fig 5.10 (a) : Let the bottom of the cylinder be in perfect contact with the source of heat at temperature T_1. Suppose the state of the gas at start now is represented on the P-V indicator diagram (Fig 5.11) by the point A, and the pressure, volume and temperature are P_1, V_1, T_1, respectively. Let the gas be allowed to expand such that the piston moves *very slowly*, the gas expands steadily and the process is reversible. As the gas expands its temperature tends to fall. But since it is kept in thermal contact with the heat source, it will absorb a quantity of heat Q_1. So all throughout the process of expansion the temperature of the gas will remain T_1. Incidentally any process in which the temperature remains the same throughout is called an *isothermal process*. During this isothermal expansion let the representative point move from A to B along the isothermal curve AB where the volume of the gas is V_2 and pressure is P_2. By the first law of thermodynamics, Q_1 is equal to the external *work done* (W_1) *by the gas* in driving the piston forward

$$W_1=\int_{V_1}^{V_2} PdV=\text{area ABba}=Q_1 \qquad \ldots(i)$$

which is the heat received from the source.

(2) **Adiabatic expansion** [Fig 5.10 (b)] :—Let now the bottom of the cylinder be in contact with a perfect insulator so that *there is no heat transfer from or to the gas*. But the piston will continue to move through some distance because of *inertia*. This motion is reversible. The gas thus expands *adiabatically* till its temperature falls to T_2. This expansion of the gas is represented by the adiabatic curve BC on the indicator diagram where the representative point moves during this adiabatic process from B to C. Let P_3, V_3 be the pressure and volume of the gas at C, then the work done (W_2) by the gas on the piston at the expense of the internal energy is given by

$$W_2=\int_{V_2}^{V_3} PdV=\text{area BC eb} \qquad \ldots(ii)$$

Since the pressure of the gas is now very much reduced, in order to make it recover its capacity for doing work, it must be brought back to its original condition. To achieve this the gas is compressed in two stages, first isothermally and then adiabatically.

(3) **Isothermal Compression** [Fig. 5.10 (c)] :—Let now the bottom of the cylinder be in perfect contact with the heat sink at temperature T_2. The gas is compressed isothermally by moving the piston very slowly until its pressure and volume become P_4, V_4 represented by the point D on the indicator diagram. The heat developed owing to compression will pass to the sink, so that the change takes place at constant temperature T_2. This isothermal compression is represented by the curve CD on the indicator diagram. The amount of heat Q_2 absorbed by the sink during the process is equal to the work done (W_3) *on the gas* to compress it from P_3, V_3 state at C to P_4, V_4 state at D.

$$Q_2 = W_3 = \int_{V_3}^{V_4} PdV = \text{area CD de} \ldots\ldots \qquad \ldots(iii)$$

(4) **Adiabatic Compression** : [Fig 5.10 (d)] :—Let now the cylinder be placed in contact with the insulator again. The gas is further compressed in a reversible process. Since the process is adiabatic, the temperature of the gas rises to T_1 and the gas attains its initial pressure and volume P_1, V_1 *i e.*, **its starting** state point A is reached. This adiabatic change is represented by the curve DA on the indicator diagram.

The work done (W_4) *on the gas* during the operation is given by

$$W_4 = \int_{V_4}^{V_1} PdV = \text{area DA ad} \qquad \ldots(iv)$$

There is no heat transfer between the gas and the outside.

Thus the cycle is completed. During the cycle, the gas has done *net external work*

$$\begin{aligned} W &= W_1 + W_2 - W_3 - W_4 \\ &= \text{area AB ba} + \text{area BC eb} - \text{area CD de} - \text{area DA ad.} \\ &= \text{area ABCD} \end{aligned}$$

The gas can be taken through the cycle again and again and more and more work can be done.

5.15 EFFICIENCY OF CARNOT'S ENGINE

Since in the Carnot's cycle discussed above, the system is brought back to the initial stage, *there is no change in its internal energy* i.e., $dU = 0$. Hence from the first law of thermodynamics the net external work done (W) by the gas will be given by

$$dQ = dU + dW$$

$$\text{or } (Q_1 - Q_2) = 0 + W$$

or $\boxed{W = Q_1 - Q_2}$ i.e., the difference between the heat supplied to the system and the heat rejected by the system. The arrangement described above is a heat engine as it converts heat into useful work. All the heat absorbed from the source is not converted into work; much of it is rejected into the sink.

Now efficiency of the Carnot's engine

$$\eta = \frac{\text{Heat converted into useful work}}{\text{Heat drawn from the source}}$$

$$= \frac{\text{Output}}{\text{Input}} = \frac{W}{Q_1}$$

$$= \frac{Q_1 - Q_2}{Q_1}$$

or $$\boxed{\eta = 1 - \frac{Q_2}{Q_1}}$$

It can be shown that if the working substance in a Carnot's engine is a perfect gas, then $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$ where T_1 and T_2 are the temperatures on the perfect gas scale.

Hence $$\boxed{\eta = 1 - \frac{T_2}{T_1}}$$

Note that the thermal efficiency (η) of the Carnot's engine depends only on two temperatures T_1 and T_2 between which the cycle operates and is always less than 1 For a given value of T_1, the efficiency is greater, the lower the value of T_2 *i.e.*, the lower the temperature of the sink, the greater is the portion of the heat absorbed from the source converted to work.

5.16 REVERSED CARNOT'S CYCLE (*Carnot's Refrigerator*)

Carnot's ideal engine is *perfectly reversible* on account of the total absence of all friction between the piston and the cylinder, the extreme slowness of the operations performed on the gas, the strictly constant temperatures under which the isothermal changes are made by the use of a perfectly conducting base for the cylinder and of a source and sink with an indefinitely large capacity and the prevention of loss of heat by conduction from the gas to the piston and cylinder by employing perfectly heat insulating material for the piston and the walls of the cylinder.

With such an arrangement, *the Carnot's cycle becomes reversible at each stage i.e.*, instead of abstracting the heat Q_1 from a source at T_1 and transferring a part of it Q_2 to the sink at T_2 and converting the net amount of heat (Q_1-Q_2) into work, one can proceed in the opposite way where the machine abstracts the heat Q_2 from the sink at T_2, then work is performed on the machine and Q_1 is transferred to the source at T_1. In terms of the indicator (P, V) diagram (Fig 5.11), this reverse process is achieved by performing the cycle in the *opposite* direction ADCBA *i e.*, first allowing the gas to expand adiabatically from A to D with the cylinder placed on the insulator then allowing it to expand isothermally from D to C in contact with the sink at T_2, the heat Q_2 being extracted in the process. Then the gas is compressed adiabatically with the cylinder on the insulator till the point B at Temperature T_1 is reached and finally the gas is further compressed isothermally till the point A is reached in contact with the source at T_1, the heat Q_1 developed in the process being transferred to the source at the higher temperature T_1 (Fig. 5.11). The machine, therefore, now acts as a *refrigerator i.e.*, by doing work on it, it is made to extract the heat Q_2 from a colder body at T_2. So it follows that Carnot's engine is a reversible engine

The working of the reversed cycle measured by a term called the *coefficient of Performance* which is defined as the ratio,

$$\frac{\text{Heat extracted from the sink}}{\text{Work Spent}}=\frac{Q_2}{W}$$

By the principle of conservation of energy,

$Q_1 = Q_2+W$ *i.e.*, heat transfer to the hot body is equal to the heat extracted from the cold body plus the heat equivalent of work done in driving the machine.

$$W=Q_1-Q_2.$$

Hence $\dfrac{Q_2}{W}=\dfrac{Q_2}{Q_1-Q_2}=\dfrac{T_2}{T_1-T_2}$

Heat extracted from $\text{sink}=Q_2=W\times\dfrac{T_2}{(T_1-T_2)}$

The reversed Carnot's engine is really an ideal refrigerating machine.

5.17 CARNOT'S THEOREM

Carnot, having thus established the possibility of a perfectly reversible engine capable of performing reversible cycles, proceeded to show that no engine can be more efficient than the perfectly, reversible engine, the nature of the working substance being immaterial. This is known as *Carnot's Theorem* and may be stated as follows :—

> *"No heat engine working between given temperatures can have efficiency greater than that of a reversible engine working between the same temperatures."*

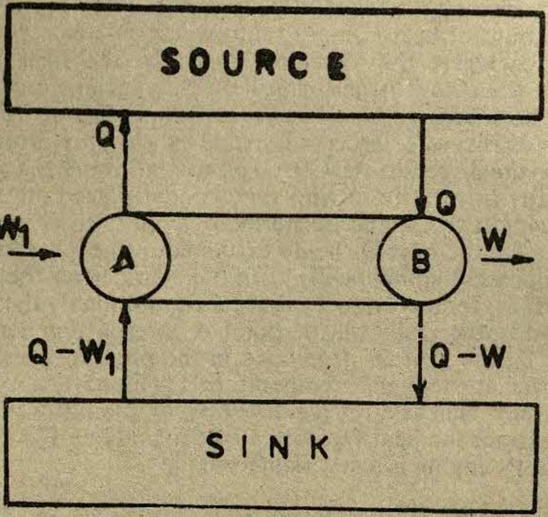


Fig. 5.12

Suppose we have two engines A and B (Fig 5.12), A being reversible and B irreversible. If possible let B be more efficient than A. Suppose B absorbs the heat Q from the source, converts a part W into work and rejects the rest Q—W into the sink. Let B be coupled to A and used to drive A in the reverse direction so that A acts as a refrigerator. Hence A abstracts a certain amount of heat from the sink, has the work W_1 done on it and returns the same heat Q to the source. The amount of heat A abstracts from the sink must then be equal to $Q-W_1$. Now since B is assumed to be more efficient than A, $W>W_1$ and hence $(Q-W_1)>(Q-W)$. This means that A abstracts more heat from the sink than B restores! to it. The net result is that the compound engine AB abstracts $(W-W_1)$ heat per cycle from the sink and converts the whole of it into work, since the source is unaffected. This is against Second Law of Thermodynamics. There is an unlimited amount of heat in the air, ocean, etc. and if this process were feasible, one can get work without any expenditure of energy by the use of some sort of a perpetual motion machine. But no one has ever succeeded to do such a thing so far. Such being the human experience, one is obliged to conclude that our assumption is wrong. No engine can be more efficient than the reversible engine, whose efficiency therefore, is maximum. All engines such as steam engine, diesel engine, etc., are irreversible and their efficiency is less than that of Carnot's engine.

5.18 ABSOLUTE SCALE OF TEMPERATURE :

The scale for temperatures measurement as given by ordinary thermometers are all arbitrary and no two of them agree with each other except at the calibration temperatures, the nature of the scale in each case being dependent on the property of the working substance (*e.g.*, expansion of a solid, a liquid or a gas, change of electrical resistance, development of e.m.f.; change in vapour density etc.). Thermometers based on these variations have a limited range of application. *Lord Kelvin deduced from thermodynamical considerations a scale of temperature which is defined in terms of energy and is independent of the property of the working substance. It is, therefore, regarded as an absolute scale.* The new scale, often referred to as work scale, has been found *identical* with the *perfect gas scale.*

Since the efficiency of all reversible engines working between any two temperatures θ_1 and θ_2 is a function of these two temperatures alone, we may write.

$$\eta = f(\theta_1, \theta_2)$$

$$\text{But } \eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

where Q_1=amount of heat absorbed at the higher temperature θ_1 and Q_2=amount of heat rejected at the lower temperature θ_2. θ_1 and θ_2 being measured on any arbitrary scale.

$$1 - \frac{Q_2}{Q_1} = f(\theta_1, \theta_2)$$

$$\text{or} \quad \frac{Q_1}{Q_2} = \frac{1}{1 - f(\theta_1, \theta_2)} = F(\theta_1, \theta_2) \qquad \text{...(1)}$$

Where F denotes some other function of θ_1 and θ_2. Similarly, if the reversible engine works between the temperatures θ_2 and θ_3, $(\theta_2 > \theta_3)$, we have

$$\frac{Q_2}{Q_3} = F(\theta_2, \theta_3) \qquad \text{...(2)}$$

and if it works between θ_1 and θ_3, $(\theta_1 > \theta_3)$

$$\frac{Q_1}{Q_3} = F(\theta_1, \theta_3) \qquad \text{.. (3)}$$

Multiplying (1) and (2)

$$\frac{Q_1}{Q_2} \times \frac{Q_2}{Q_3} = \frac{Q_1}{Q_3} = F(\theta_1, \theta_2) \times F(\theta_2, \theta_3)$$

comparing this with (3)

$$F(\theta_1, \theta_3) = F(\theta_1, \theta_2) \times F(\theta_2\ \theta_3) \qquad \text{...(4)}$$

If this relation (4) is to be satisfied, $F(\theta_1, \theta_2)$ must be of the form $\frac{\psi(\theta_1)}{\psi(\theta_2)}$ where is another function of temperature.

Therefore, for any reversible engine

$$\frac{Q_1}{Q_2}=\frac{(\theta_1)}{(\theta_2)} \quad \ldots(5)$$

Since $\theta_1>\theta_2$ and $Q_1>Q_2$, $(\theta_1)>(\theta_2)$ which means that (θ) is a *linear function of θ and may be used to measure temperature.*

Representing, therefore, (θ) by τ, which would be some multiple of θ,

$$\boxed{\frac{Q_1}{Q_2}=\frac{\tau_1}{\tau_2}} \quad \ldots(6)$$

This relation (6) can be used to define a new scale of temperature, which does not depend on the properties of any particular substance. The ratio of any two temperatures on this scale is equal to the ratio of the heat taken in and heat rejected by an engine working reversibly between the two temperatures. Hence it is called the ***absolute*** or the ***thermodynamic scale.***

Relation (6) can be written as

$$\frac{Q_1-Q_2}{Q_1}=\frac{\tau_1-\tau_2}{\tau_2} \quad \ldots(7)$$

Since (Q_1-Q_2) represents the work W done by the reversible engine between the two temperatures τ_1 and τ_2 the new scale is called also the work scale.

The efficiency of the reversible engine on the absolute scale is given by

$$\eta=1-\frac{Q_2}{Q_1}=1-\frac{\tau_2}{\tau_1} \quad \ldots(8)$$

In order that η may be equal to unity, τ_2 should be zero. But the efficiency can be unity only when all the heat taken by the engine is converted into work *i.e.*, when $Q_1=W$ or $Q_2=0$ or $\tau_2=0$. This represents then the *zero of the absolute scale.* A temperature less than $\tau_2=0$ is not possible since then τ_2 will be negative which means that the efficiency η will be greater than one, which is impossible.

The absolute scale agrees completely with the perfect gas scale as follows :—

We have already seen that for a reversible engine using a perfect gas as the working substance, the effiiciency is given by

$$\eta=1-\frac{Q_2}{Q_1}=1-\frac{T_2}{T_1} \quad \ldots(9)$$

where T_1 and T_2 are the temperatures of the source and sink, measured on the perfect gas scale.

Comparing (9) with (8), we have

$$\boxed{\frac{T_1}{T_2}=\frac{\tau_1}{\tau_2}} \quad \ldots(10)$$

The relation (10) indicates that ratios of any two temperatures on the perfect gas scale and the thermodynamic scale are equal. Since, if $\tau_2 = 0$, T_2 is also $= 0$, the zero of the thermodynamic scale coincides with the zero of the perfect gas scale. If T_1 is the temperature of boiling water and T_2 that of melting ice, measured on the perfect gas scale

$$T_1 - T_2 = 100$$

on the Absolute Scale we have for the same two fixed points, as shown above,

$$\tau_1 - \tau_2 = 100$$

using the perfect gas scale, the efficiency

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{100}{T_1}$$

and in terms of the absolute scale

$$\eta = \frac{\tau_1 - \tau_2}{\tau_1} = \frac{100}{\tau_1}$$

Since the efficiency is the same

$$\frac{100}{T_1} = \frac{100}{\tau_1}$$

This means that the temperatures of the boiling points of water and the melting point of ice are identical on the two scales. In a similar manner, it can be shown that any temperature has the same value on the *two scales* which are, therefore, identical. (The ice point on the Kelvin Scale is 273·16 *i.e.*, triple point of water).

Example 5.3 :

A perfect Carnot engine utilizes an ideal gas. The source temperature is 500 K and the sink temperature is 375 K. If the engine takes 600 k cal per cycle from the source, compute (i) the efficiency of the engine, (ii) work done per cycle, and (iii) the heat rejected to the sink per cycle.

Solution :

(*i*) The thermal efficiency of the Carnot's engine

$$= \eta = \frac{T_1 - T_2}{T_1}$$

$$= \frac{500 - 375}{500}$$

$$= 0.25$$

$$= 25\%$$

(*ii*) Also, $\eta = \frac{W}{Q_1}$

$$\therefore \; W = \eta Q_1$$

Here $Q_1 = 600 \text{ k cal}$

$$W = \frac{25}{100} \times 600$$

$$= 150 \text{ k cal.} = 150 \times 10^3 \text{ cal}$$

$$= 150 \times 10^3 \times 4{\cdot}18 \text{ joules}$$

$$= 6.27 \times 10^5 \text{ joules}$$

(*ii*) Since $Q_1 - Q_2 = W$

$$Q_2 = Q_1 - W = 600 - 150 = 450 \text{ k cal}$$

$$= 1{\cdot}881 \times 10^6 \text{ joules}$$

Example 5.4

In a refrigerator heat from inside at 277K is transferred to a room at 300K. How many joules of heat will be delivered to the room for each joule of electric energy consumed ideally ? Also compute co-efficient of performance of this refrigerator.

Solution :

We know that for an ideal refrigerator

$$\frac{W}{Q_1} = \frac{T_1 - T_2}{T_1}$$

Here

$$W = 1 \text{ joule}$$

$$T_1 = 300K$$

$$T_2 = 277K$$

$$\frac{1}{Q_1} = \frac{300 - 277}{300}$$

or

$$Q_1 = 13{\cdot}04 \text{ joules}$$

$$\text{Coeffi. of performance} = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

$$= \frac{277}{300 - 277}$$

$$= 12{\cdot}04$$

5.19 RADIATION

We know that when we feel cold in winter we sit in the sun. The warmth that we get is not due to the temperature of air surrounding us. This fact can be proved by placing a screen between the un and our body, when we no longer feel warm. Hence, we come to onclusion that the sensation of warmth is due to the heat energy nat directly comes to us from the sun. The distance of sun from ne earth is 1.49×10^8 km and our atmosphere does not extend beond a distance of more than about 800 km. The space beyond this stance can be supposed to be almost a perfect vacuum and there is no material medium. Thus the heat from the sun does not come to us by conduction or convection, we get it by a third process known as *radiation.*

Hence radiation is that process of transmission of heat in which heat travels from one point to another in straight lines, with the velocity of light without heating the intervening medium.

The properties of heat radiation tell us that heat is a form of wave motion similar to light. It travels through empty space with the velocity of light 3×10^8 ms^{-1} and does not require any material medium for its transmission.

We know that light travels in the form of *electromagnetic waves.* Hence thermal radiations also travel in the form of electromagnetic waves.

The molecules of a hot body are in a state of rapid vibration, which produce waves. The waves travel through space and set the molecules of a colder body into vibrations, when they are absorbed by it, and thus raise its temperature.

All substances do not allow the heat radiations to pass equally through them. Some allow heat radiation to pass more readily than others. The substances which allow heat radiations to pass through them are called *diathermanous,* whereas those which absorb heat radiations and are themselves heated are called ***athermanous.***

No material medium is perfectly diathermanous, air is only approximately so. Water, wood and other liquids and solids are mostly athermanous. 'Ordinary glass, which is transparent to light absorbs about 90% of radiant heat, when it is coming from a source at a low temperature, but when radiant heat is coming from a source at a high temperature it can pass through it. Due to this property of glass it is used in "Green houses" to keep the plants warm. It allows the heat radiations from the sun to pass through it but does not allow the heat radiations from the plants to escape from inside.

It is not necessary that a substance which is opaque to light must be athermanous. A solution of iodine in carbon disulphide is quite dark and does not allow light to pass through, but heat radiaions can pass through it.

Detection of thermal radiations : Radiation is merely a form of energy which can easily be converted into heat by causing it to fall on any body which can absorb. An instrument used for detecting radiations must be able to measure a small quantity of heat *i.e.,* small rise of temperature.

(*i*) *Blackened bulb thermometer* : If the bulb of a thermometer is painted black it shows a considerable rise of temperature when heat radiations fall on it.

(*ii*) *Leslie's differential air thermoscope* :— The blackened bulb thermometer is not very sensitive. It was soon replaced by a differential air thermoscope. It consists of two glass bulbs connected by a U—tube. The bulbs and part of the tube contain air and the bend of the tube contains a coloured liquid like sulphuric acid because it is neither volatile nor heavy and viscous. One of the bulbs is coated with lamp black so that it may absorb a large amount of heat radiations.

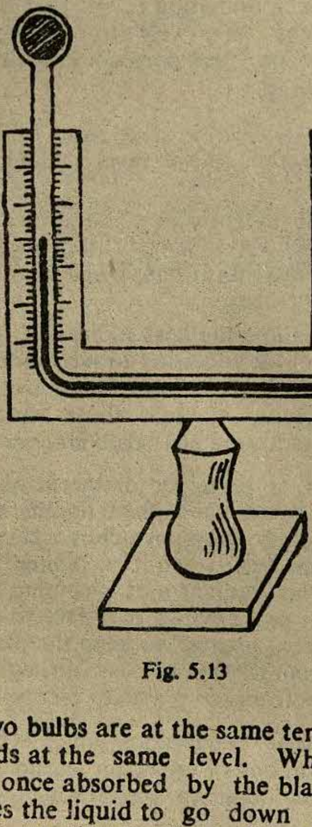


Fig. 5.13

When the two bulbs are at the same temperatures the liquid in the two arms stands at the same level. When heat radiations fall on it, these are at once absorbed by the black bulb. The air inside expands and causes the liquid to go down The liquid on the other side rises up. Greater the amount of heat radiations falling on the instrument, greater is the difference in the two levels.

(iii) Crooke's radiometer : It consists of two light aluminium rods RR fixed at right angles to form a cross and carry at their free ends thin vertical vanes VV of mica or tin-foils. Similar faces of the vanes are coated with lamp-black. The arrangement is suspended inside an evacuated glass vessel so that it can rotate about a vertical axis. When radiations fall on the blackened faces, the vanes begin to rotate in such a direction that the blackened face continually recedes away from the source of radiation. The cause of this motion is as follows :

The blackened face absorbs the incident radiations and thereby its temperature is raised, the clear face remaining at a lower temperature. The molecules bombarding the blackened face become more heated than the molecules bombarding the clear face, thereby exerting greater pressure on the vane. This results in a net effective force repelling the vanes away from the incident radiation. This effect is known as the radiometer effect. The velocity of rotation is a measure of the intensity of the incident radiation.

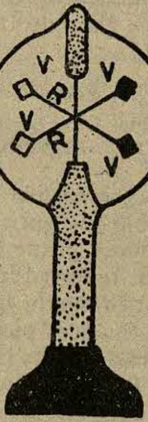


Fig. 5.14

(*iv*) *Thermopile*—The instruments described above have been superseded by electrical instruments of which thermopile is an example. A thermopile consists of a number of thermocouples made out of antimony and bismuth rods connected in series as shown in Fig 5.15 (*a*). One set of junctions marked 'hot' is exposed to the radiations while the other set marked 'cold' is properly shielded. The ends of the pile are connected to a sensitive galvanometer. When the radiations fall on the hot face, it absorbs heat energy and a thermo-electric current flows from bismuth (Bi) to antimony (Sb) across the hot junction and finally through the galvanometer as shown, thereby causing a deflection. The deflection is proportional to the current which is in turn proportional to the intensity of heat radiations.

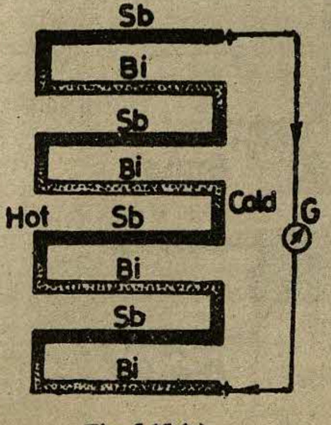


Fig. 5.15 (a)

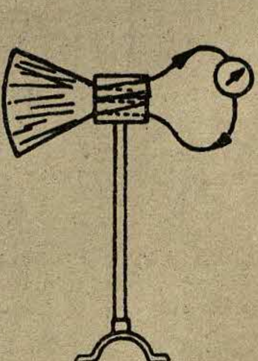
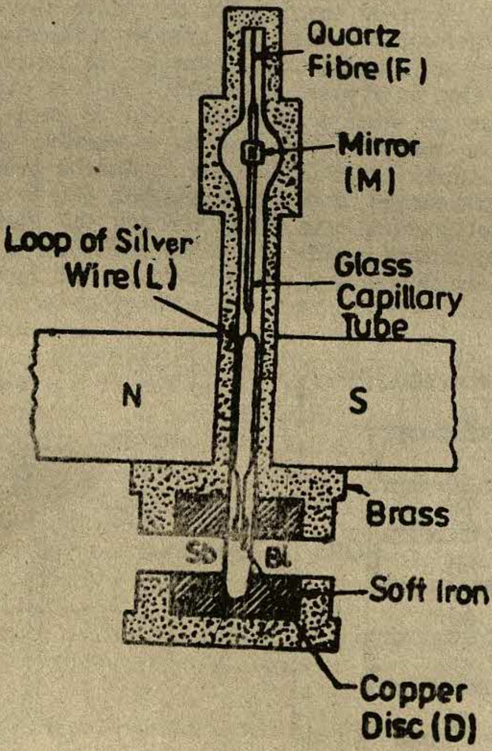


Fig. 5.15 (b)

The improved form of the thermo-pile consists of bars of antimony and bismuth arranged to form a cube, with all the hot junctions at one face and the cold junctions at the opposite face. The junctions are soldered and the different layers are insulated with paraffined paper or mica. The face of the hot junction is coated with lamp-black and provided with conical protector to protect it from stray radiations, while the cold face is covered with a metallic cap as shown in Fig 5 15 (*b*).

Measurement of Heat Fadiation : The methods of detecting heat radiations are explained above. For the quantitative measurement of heat radiation a radio-micrometer or a bolometer is used.

Radiomicrometer : A thermopile has a large thermal capacity and it absorbs a considerable amount of heat before any appreciable rise of temperature is produced in the metal near one of the exposed surfaces. This drawback is removed in the radiomicrometer designed by Prof. Boys. It consists of a single loop L of silver wire suspended by a quartz fibre F within the pole pices N and S of a strong permanent magnet. The lower ends of the loop are joined to a bismuth-antimony thermocouple. Below the junction of the thermocouple is attached a thin blackened copper disc. D A mirror M is attached as shown to measure the deflection of the loop.



5. 6

When radiations fall horizontally on the copper disc the Bi—Sb junction is heated and a thermo-electric current flows through the silver loop producing a deflection. The deflection is proportional to the current which depends upon the intensity of the energy received. To prevent disturbance due to diamagnetism of bismuth, the rods of the thermocouple are surrounded by a mass of soft iron.

Bolometer : The uniform increase in resistance with temperature of a platinum strip was first used by Langley for measuring thermal radiations. He named it the Bolometer. It essentially consists of two gratings such as shown in Fig 5·17, punched from very thin platinum foils (1 to 2×10^{-3} mm. thick) and covered with a layer of platinum black. Such a grid has a resistance of about 50 ohms, but a low thermal capacity.

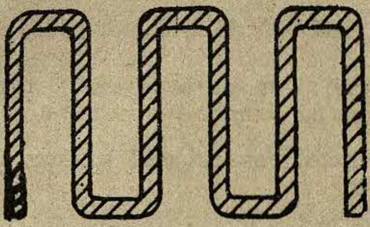


Fig. 5.17

In Langley's bolometer two such grids, exactly identical, are arranged to form the two ratio arms of a Wheatstone bridge. One of the grids is exposed to the radiation and the other is well screened. The resistance of the exposed grid increases, thereby disturbing the bridge balance and produces a deflection in the galvanometer. A change of temperature of 0.0001°C produces a measurable deflection.

Lummer and Kurlbaum improved Langley's bolometer by using four identical grids instead of two. Grids 1 and 3 put in the opposite arms of the bridge are so arranged one behind the other that the strips of 3 cover the spaces in 1, and thus receive the radiations passing through the spaces in 1. By this arrangement twice as much radiation falls on the bolometer as with a single grid, so that the effect of change in resistance is doubled. The other two grids are arranged in the remaining two arms 2 and 4 of the bridge and are shielded from the radiations.

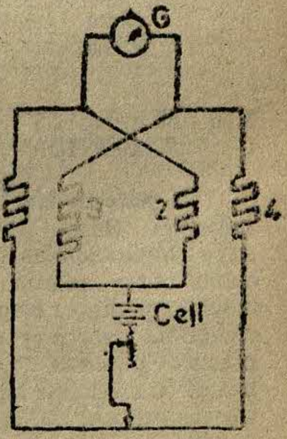


Fig. 5.18

The bridge is first balanced by the sliding contact with all the four grids shielded from radiations. The grids 1 and 3 are then exposed to the radiations. The deflection of the galvanometer due to the upsetting of the balance is proportional to the amount of radiations falling on the bolometer in unit time.

Properties of heat radiations : The following is the summary of their properties in a homogeneous medium.

(*i*) Heat radiations travel in straight line.

(*ii*) Heat radiations require no material medium for their transmission and can travel through vacuum.

(*iii*) In a homogeneous medium heat radiations are transmitted equally in all directions.

(*iv*) Heat radiations travel with the velocity of light *i.e.*, $3 \times 10^8 ms^{-1}$

(*v*) The intensity of heat radiations varies inversely as the square of the distance from the source.

(*vi*) Heat radiations can be reflected from polished surfaces and obey the laws of reflection.

(*vii*) Heat radiations can be refracted and obey the laws of refraction.

(*viii*) Heat radiations do not heat the medium through which they pass.

(*ix*) They affect a special photographic plate and are least scattered. They are used for long distance photography through fog and mist.

(*x*) They produce the phenomena of interference ; diffraction and polarisation.

(*xi*) They are absorbed by glass, but rock-salt, sylvine and a solution of iodine in carbon-bisulphide are transparent to them. Glass is partially transparent to solar heat radiation.

5.20 PERFECTLY BLACK BODY AND BLACK BODY RADIATION :

A perfectly black body is one which absorbs completely all the radiation which fall on it. It neither reflects nor does it transmit any radiation and as a consequence appears black irrespective of the colour of the radiation incident on it. A surface coated with lamp-black or platinum black absorbs from 96% to 98% of the incident radiations and may be considered as a perfectly black body for practical purposes. On the other hand if a perfectly black body is heated to a suitable high temperature it will give out radiations of all wavelength *i.e.*, *full* or *total* radiations.

No actual black surface fulfils these requirements entirely. A closed metallic chamber of brass or platinum say a hollow sphere having a very narrow opening at one point O is a perfect absorber and is used in practice as a *black body.* If the radiation enter through the narrow opening O it is lost inside by successive reflection on the inner wall and whole of it is absorbed. The type of the black body shown in Fig (5.19) was designed by Fery. It is closed hollow sphere painted black from inside having a small opening O and a projection P opposite to the opening. This projection protects direct reflection of any of the radiation into the opening O from the surface opposite the hole.

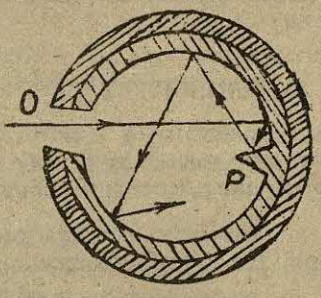


Fig. 5.19 Black Body

If the enclosure is heated to a uniform high temperature the radiations coming out of the hole O are *full radiations* or ***black body radiations.***

The radiation within the constant temperature enclosure is *independent of the nature of the substance of which the walls of the enclosure is made* and is only dependent on the *equilibrium temperature* of the enclosure. Any body placed inside, it will in the steady state attain the temperature of this enclosure and will emit the black body radiations characteristic of the temperature of the enclosure.

5.21 ABSORBING AND REFLECTING POWERS

When heat radiations fall on a body, they are partly reflected and partly absorbed.

Those surfaces which absorb more heat than what they reflect are called *good absorbers* while those surfaces which reflect more heat than what they absorb are called *good reflectors.*

If Q units of radiant heat between λ and $\lambda + d\lambda$ are incident on a body per second and Q_1 is the portion which is absorbed then the ratio Q_1/Q is called the absorbing power (a_λ) Hence *absorbing power is the ratio of the amount of heat radiations absorbed in a given time by the surface to the total amount of heat radiations incident on it in the same time.*

If Q_2 is the portion of the incident radiations Q which is reflected by a surface, then the ratio Q_2/Q is called the *reflecting power of the surface. Hence reflecting power of a surface is measured by the ratio of the amount of heat radiations reflected by the surface to the amount of heat radiations incident on it in the given time.* From the definition of reflecting and absorbing powers it is clear that those substances which reflect more of the incident radiations, absorb less. Thus we see that *good absorbers are bad reflectors.* Dull black surfaces are good absorbers but bad reflectors whereas polished white surfaces are bad absorbers but good reflectors.

The absorbing power of a perfectly black body is unity as it absorbs all the radiations incident on it. Its reflecting power is. zero

5.22 EMISSIVITY

Emissivity of a surface is defined as the amount of heat radiated per second by a unit area of the surface when the difference in temperature between the body and its surrounding is 1°C.

The amount of heat radiated per second by a surface depends upon the following factors :

(i) Area of the surface.

(ii) Difference of temperature between the surface and the surroundings.

(iii) Nature of the surface. Dull black surface are good radiators whereas polished white surfaces are bad radiators.

The *emissive power* or *radiating power of a surface is defined as the ratio of the amount of radiations emitted per second by a unit area of the surface to the amount emitted per second by a unit area of a perfectly black body, the emissions taking place under identical conditions.*

If e_λ is the spectral emissive power of the surface then the radiant energy emitted per square metre per second between spectral wavelengths λ and $\lambda+d\lambda$ will be $e_\lambda \, d\lambda$.

The total emissive power is the total energy emitted per square metre per second for that given temperature and is equal to $\int e_\lambda \, d\lambda$ for all wavelengths.

5.23 KIRCHOFF'S LAW .

We have seen above that the initial researches on the emission and absorption of thermal radiation showed that they vary in a similar manner, good emitters being also good absorbers and bad emitters bad absorbers. In 1859 Kirchoff deduced from such observations an important law which may be stated as follows :

> "*At any given temperature and for radiations of the same wavelength, the ratio of the emissive power to the absorptive power is the same for all substances and is equal to the emissive power of a perfectly black body.*"

Kirchoff's own proof of the law is somewhat complicated. A much simpler proof can be had in the following manner, based on the fundamental characteristics of thermal radiation.

Let us consider a body placed inside a uniformly heated enclosure maintained at temperature T. The radiation within the enclosure will be independent of the nature of the walls of the enclosure and of the presence of the body.

Let dQ be the quantity of radiant energy, of wavelength lying between λ and $\lambda+d\lambda$, falling on a body per unit area of its surface per second. Let a_λ be the *absorptive power* of the body. Then the amount of radiant energy absorbed by the body per unit area per second is $a_\lambda\, dQ$. The remainder $(1-a_\lambda)\, dQ$ is reflected or transmitted.

If e_λ be the *emissive power* of the body, the amount of energy radiated per unit area per second under the same conditions of temperature and wavelength is $e_\lambda\, d\lambda$. Hence the *total energy given out by the body per unit area per second* $=(1-a_\lambda)\, dQ+e_\lambda\, d\lambda$.

Since the body is in temperature equilibrium with the enclosute, the radiant energy given out must be equal to that received.

$$dQ=(1-a_\lambda)\, dQ+e_\lambda\, d\lambda$$

$$\text{or} \quad a_\lambda\, dQ= e_\lambda\, d\lambda \qquad \text{...(i)}$$

For a perfectly black body, since $a_\lambda=1$ and e_λ has a maximum value, which we may denote by E_λ.

$$dQ=E_\lambda\, d\lambda \qquad \text{...(2)}$$

From (1) and (2) we have

$$a_\lambda\, E_\lambda\, d\lambda=e_\lambda\, d\lambda$$

$$\boxed{\frac{e_\lambda}{a_\lambda}=E_\lambda} \qquad \text{...(3)}$$

The relation (3) shows that, at any given temperature T and for radiations for the some wavelength λ, the ratio of the emissive power to the absorptive power of a substance is constant and equal to the emissive power of a perfectly black body. This is Kirchoff's Law

Since E_λ is a constant, if e_λ is large a_λ also is large and vice versa. *This shows that good emitters are also good absorbers.*

Experimental Verification of Kirchoff's Law

Ritchie, demonstrating with the following arrangement that good emitters are also good absorbers, was able to confirm experimentally Kirchoff's Law.

Ritchie's apparatus consists of a differential air thermoscope whose bulbs have been replaced by the two identical cylindrical metal vessels P and Q, fig (5.20). A third cylinder RS, of the same area of cross-section as that of the P and Q, is supported in the middle of the vessels P and Q. This can be filled with hot water and can also be rotated about its vertical support. The left hand faces of all the three vessels are painted black, while the right hand faces are polished. Fill the central vessel with boiling water, and it is found that the liquid levels remain the same in the two limbs. It shows that the heat radiations received by the vessel P and Q are the same.

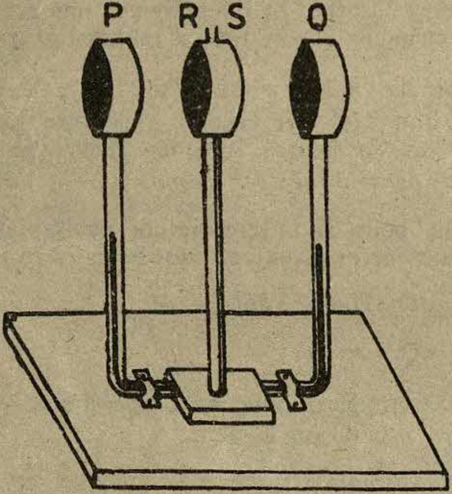


Fig. 5.20

Now rotate the central vessel RS through 180°, so that now the shining face of RS faces the shining face of P and the black face of RS, faces the black face of Q. In this case the liquid level in the tube connected with Q will be lower than in the tube connected with P. This shows that the heat radiations received by the vessel P is lesser than the heat radiations received by Q, because the good radiating surface of the vessel RS is facing the good absorbing surface of Q and the poor radiating surface of the RS is facing the poor radiating surface of P. This shows that good radiators are good absorbers and bad radiators are bad absorbers.

Applications of Kirchoff's Law :

A direct outcome of the Kirchoff's Law is that if a body is capable of emitting a certain radiation, it will absorb them when they fall on it.

(*i*) If a piece of decorated china is heated in a furnace to about 1000 K and then taken out suddenly in a dark room, the decorations appear much brighter than the white china. The decorations being dark are better absorbers at ordinary temperatures and also emit much greater light at higher temperatures.

(*ii*) If we take a polished metal ball and make a black spot on it with platinum black, then on heating the ball to about 1000 K, and suddenly taking it out in a dark room, it will be found that the black spot shines much more brilliantly than the polished surface.

(iii) A green glass appears green because it absorbs red light strongly and reflects green light; red and green being complementary colours.* If we heat the green glass in a furnace and then take it out it is found to glow with red light. Similarly if a red glass is heated to a high temperature it will glow with green light.

(iv) There are many such cases illustrating Kirchoff's Law, but the one classical example regarding the explanation of the *Fraunhofer lines* in the Solar Spectrum given by Kirchoff on the basis of his law deserves special mention.

Fraunhofer was the first, in 1801, to discover a large number of dark lines in the solar spectrum. Although he carefully measured their wavelengths realising their importance, he was unable to explain how they originated. He also investigated the light from stars and found similar dark lines in their spectra. Other workers, notably Fizeau,found that, if the solar spectrum was examined side by side with spectrum from a sodium flame, the two yellow linesof the sodium spectrum occupied the same place as the Fraunhofer D-lines in the solar spectrum. A similar coincidence was also observed in the case of hydrogen spectrum. Although these facts paved the way for a possible solution, the dark lines of Fraunhofer remained unexplained, until Kirchoff completely solved the problem on the basis of his law as follows :—

Since sodium can emit the D lines when it is excited, it can also absorb the same light when white light falls on it and produce the corresponding dark lines. As a matter of fact, when white light from an arc lamp is passed through relatively cool sodium vapour (Fig 5.21) and analysed in a spectrometer, the spectrum is found to be crossed by two dark lines in precisely the same position as the yellow lines in the emission spectrum of sodium. This suggests a correct explanation of the Fraunhofer lines in the solar spectrum. The light from the central mass of the sun is such that it emits a continuous spectrum without dark lines. But this light has to pass through a

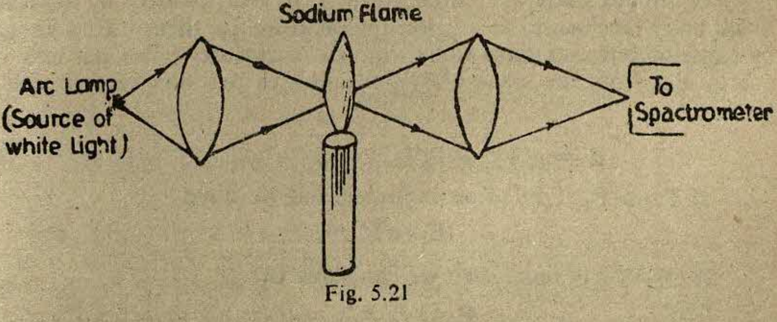


Fig. 5.21

cooler atmosphere containing a number of elements such as H_2, N_2, O_2, Na, Cu, etc., in the gaseous form, which absorb effectively the lines that they themselves can emit and thus give rise to the Fraunhofer lines. The presence of dark lines in the spectra of stars is accounted for in a similar manner. It is by the analysis of such dark lines, one is able to tell of what elements the cool atmosphere surrounding the sun and stars are composed.

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*Complementary colours are those colours which when mixed in certain proportions give white light.

Kirchoff's discovery is of much greater importance than the mere success in explaining the dark absorption lines in solar and stellar spectra would indicate. For it clearly shows that every type of atom, when properly excited, emits light of definite wavelength which is characteristic of the atom.

5.24 STEFAN'S LAW

Stefan, in 1819, deduced from experimental data a law which states that the *total energy emitted by a black body per second per unit area is proportional to the fourth power of its absolute temperature.*

If E is the total energy radiated of all wavelengths per unit area per second by a black body at thermodynamic temperature T, then

$$E \propto T^4$$

$$\text{or} \quad \boxed{E = \sigma T^4} \qquad \ldots(i)$$

where σ is constant, called *Stefan's constant* If E is in Wm^{-2} (*i.e.*, $Js^{-1}\ m^{-2}$) and T is in K (Kelvin) then σ has units $Wm^{-2}\ K^{-4}$ Its value is

$$\sigma = 5{\cdot}735 \times 10^{-8}\ Wm^{-2}K^{-4}.$$

Equation (i) is true for an isolated black body.

Boltzmann, on the supposition that the black body radiation behaves as a perfect gas, applied laws of thermodynamics to radiation and theoretically deduced the above law. So the law is generally known as *Stefan Boltzmann Law.*

If a black body at temperature T is surrounded by another black body (enclosure) at a lower temperature T_0, then it loses energy by emission but also gains some from the enclosure. The net loss of energy E_{net} from the black body per unit area per unit second is thus given by

$$E = \sigma(T^4 - T_0^4) \qquad \ldots(ii)$$

If $T >> T_0$, T_0^4 can be neglected and we have

$$E = \sigma T^4$$

If $(T - T_0)$ is *small* then we can show that

$$E \simeq 4\sigma T_0^3\,(T - T_0)$$

Thus, let $x = (T - T_0)$ and so

$$E = \sigma(T^4 - T_0^4) = \sigma[(T_0 + x)^4 - T_0^4]$$

$$= \sigma[T_0^4 + 4T_0^3x + 6T_0^2x^2 + 4T_0x^3 + x^4 - T_0^4]$$

$$= 4\sigma T_0^3 x \text{ since } x^2,\ x^3 \ \&\ x^4 \text{ are negligible}$$

$$\approx 4\sigma T_0^3(T - T_0)$$

or $$E \propto (T - T_0) \text{ as } \sigma \ \&\ T_0 \text{ are constant.}$$

Thus we find that *the rate of loss of energy by radiation is proportional to the temperature difference of the hot body and the surroundings if this is small.* This is *Newton's Law of Cooling* :

For a *non-black body* Stefans' law can be applied in the form

$$\boxed{E = \epsilon \sigma T^4} \quad \text{...(iii)}$$

where ϵ is called the total emissivity of the body and has a value between 0 and 1.

5.25. ENERGY DISTRIBUTION OF BLACK BODY RADIATION

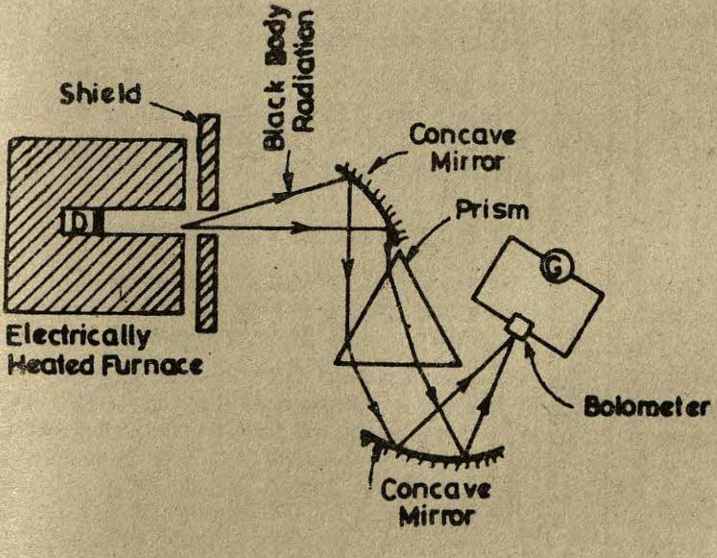


Fig. 5.22 Experiment on Black Body Radiation

In 1899 Lummer and Pringsheim carried out an experiment to measure how the energy from a black body radiator was distributed among the various wavelengths. They placed a blackened disc D in the middle of a blackened porcelain tube, which was surrounded by other concentric porcelain tubes with pl tinum ribbon wrapped round them (Fig 5.22). The furnace was heated electrically, and black body radiation was emitted from D and refracted (spectrum obtained) by a fluorite or flourspar prism, which is, transparent to heat radiation (infra-red). The temperature of the furnace was measured by a thermocouple. The distribution of energy among the various wavelengths was measured by exposing in turn parts of the spectrum to a narrow strip of platinum called a *bolometer*, whose resistance change is a measure of the heat or energy falling on it. At a particular part of the spectrum, the energy measured is that in a narrow range of wavelengths λ to $\lambda + d\lambda$, so that on the average, the energy can be called "E_λ". The wavelength λ can be measured by a diffraction grating.

The distribution of energy of the spectrum for different temperatures in the range 723 K to 1646 K are shown by the curves in Fig (5.23). The wavelength in microns is plotted along the abscissa and the emissive power E_λ along the ordinate.

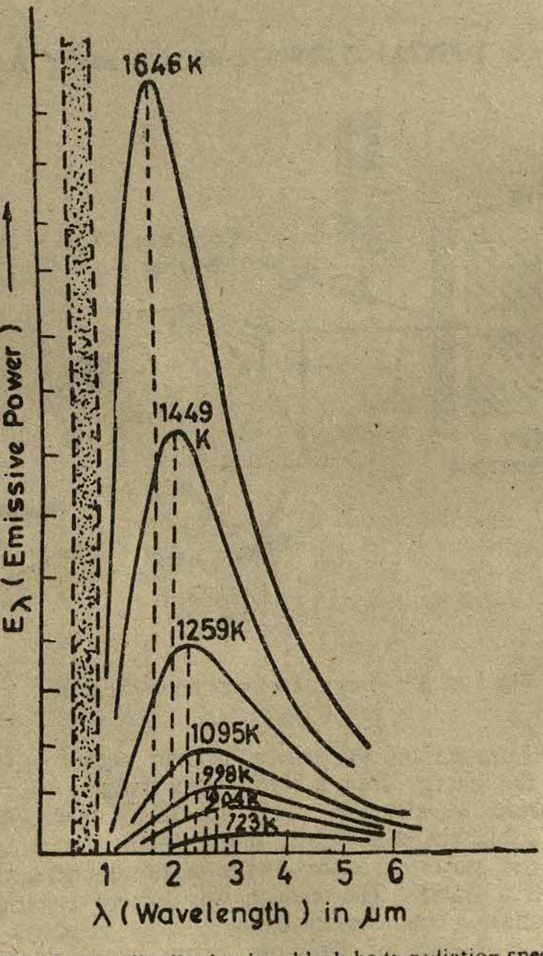


Fig. 5.23 Energy distribution in a black body radiation spectrum

From the study of the curves the following points emerge :

(*i*) As the temperature rises the energy (E_λ) emitted in each band of wavelengths, increases *i.e.*, the body becomes 'brighter'.

(*ii*) Even at 1646 K only a small fraction of the radiation is light. (The maximum does not lie in the visible region until about 4000 K).

(*iii*) At each temperature T the energy radiated is a *maximum* for a certain wavelength λ_{max} which *decreases* with rising temperature. In fact $\lambda_{max} \propto \frac{1}{T}$ *i.e.*, λ_{max} T = constant—a statement known as Wein's *displacement law*. This explains why a body appears successively red-hot, yellow-hot and white hot etc. Sirius (the Dog Star) looks blue, not white. Why ?

(*iv*) The maximum of each curve shifts towards shorter wavelength as the temperature is raised.

(*v*) Total radiation E at any absolute temperature T ($= \int E_\lambda \, d\lambda$) is represented by the area enclosed between the curve and the abscissa (*i.e.*, λ-axis). This area is found to be proportional to T^4 agreeing with Stefan's Law.

5.26 WIEN'S DISPLACEMENT LAW

We have already mentioned above that the intensity of radiation emitted by a black body is not uniformly distributed over the whole range of wavelengths involved.

A general solution of the problem is readily obtained from the fact that when the temperature of a body is steadily raised, the colour of the radiation emitted by it changes. Thus, for instance, when a blackened platinum wire is raised in temperature by electrical heating, it begins to appear dull-red at about 525°C, changes to cherry red at about 900°C, becomes orange red at 1100°C ; yellow at 1250°C and finally white at about 1600°C. This shows that the colour emitted becomes richer in waves of shorter wavelength. In other words, *the maximum intensity of emission shifts towards the shorter wavelength side as the temperature is raised.*

Wien, in 1893, tackled the problem analytically by applying the principles of thermodynamics to the black body radiation and obtained the following relation.

$$\boxed{\lambda_m T = b}$$

where b = constant

λ_m=wavelength at which the emissive power E_λ is a maximum (E_m) at the absolute temperature T.

This statement is known as *Wien's Displacement Law i.e.*, as the temperature increases the maximum intensity of radiation emitted shifts towards shorter wavelengths.

The value of the constant b has been found to be $2{\cdot}892\times10^{-3}$ mK.

Wien's Law furnishes us with a simple method of determining the temperature of the heavenly bodies. Thus for the moon, $\lambda_m=14\mu$ and hence

$$T=\frac{2{\cdot}892\times10^{-3}}{14\times10^{-6}}\simeq200\text{ K}$$

Similarly, we can calculate the temperature of the sun.

5.27 PYROMETERS

Radiations emitted by a hot body depend upon the temperature of the hot body and therefore it is possible to determine the temperature of a hot body from the study of its thermal radiations. Devices which work on this principle are known as **pyrometers**

There are two types of Pyrometers :

(*i*) *Total Radiation Pyrometers* based on black body radiation of all wavelengths. Stefan's Law is made use of in calculating the temperature.

(*ii*) *Optical Pyrometers* : In these, energy $E_\lambda\, d\lambda$ emitted in a particular portion of the spectrum is measured and the temperature is calculated by using Wien's Law for the distribution of energy in the radiation spectrum.

Total Radiation Pyrometer :

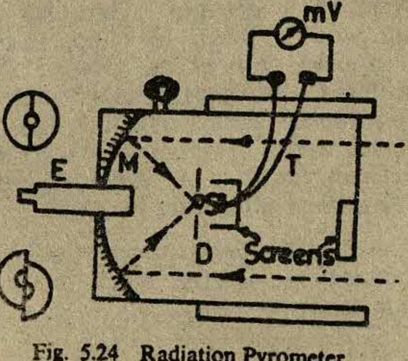


Fig. 5.24 Radiation Pyrometer

Fery was the first to devise this type of Pyrometer, which is therefore ordinarily known as Fery's Pyrometer. A modern version of this Pyrometer is shown in Fig (5.24). It consists of a concave Mirror M made of copper and nickeled in the front surface, having a small opening in the centre, into which an eyepiece E is fitted. In front of M a small diaphragm D is mounted and immediately behind it a metallic strip S is arranged. The front surface of S facing D is blackened, while to its back surface one junction of a thermocouple T is fixed. The other junction of T as well as S are protected from the direct radiations by suitable screens. The leads from the thermocouple are connected to a millivoltmeter mV which measures the e.m.f. developed when the radiation from the source whose temperature is to be measured, after being focussed by M, falls on the receiver S through D. The mirror M can be moved forwards and backwards for the purpose of focussing the radiations on S.

In order to obtain accurate focussing the following device is employed. The diaphragm D is constituted by two semi-circular mirrors inclined to each other at an angle of about 5° with a central hole of about 1.5 mm to allow the incident radiation to pass. When this diaphragm is viewed through the eyepiece E, if the focussing is perfect, the images of the two halves of the diaphragm coincide to form a complete circle but if the focussing is not accurate, the two halves will not coincide but appear displaced as shown in fig (5.24). The concave mirror M is moved till this relative displacement of the two halves disappears.

In making a measurement, if V is the reading indicated by the millivoltmeter, then

$V \propto (T^4 - T_0^4)$ where T and T_0 are the temperatures of the hot body and the receiving junction respectively. Since T is very large as compared to T_0 ; T_0^4 can be neglected in comparison to T^4.

In practice it is found that E.M.F. V is not exactly proportional to the fourth power of the absolute temperature of the source. Some of the reasons for this discrepancy are :

(*a*) The source is not a black body

(*b*) T_0^4 is not zero

(*c*) Spurious effects produced by stray and diffuse reflections etc.

(*d*) Conduction effect along the leads of the thermocouple.

So, before use, the pyrometer has to be suitably calibrated.

By using *rotating sector device* (Fig 5.25), the instrument enables us to measure even *very high temperatures.* An opaque disc, from which a sector making an angle θ at the centre is cut off and is rotated about the axis of the Pyrometer tube in the path of the incident radiation, so that only a fraction of the radiation enters the pyrometer per revolution. If T_1 is the temperature, now measured by the instrument then the actual temperature T of the source, according to Stefan's Law, is given by

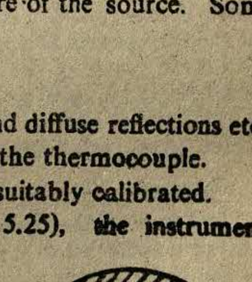


Fig 5.25

$$\frac{T_1^4}{T^4}=\frac{\theta}{360}$$

$$T=T_1\left(\frac{360}{\theta}\right)^{\frac{1}{4}}$$

Suppose that $\theta=22{\cdot}5^\circ$ and $T_1=1800$ K

$$T=1800\left(\frac{360}{22{\cdot}5}\right)^{\frac{1}{4}}$$

$$=3600 \text{ K}$$

Optical Pyrometer :

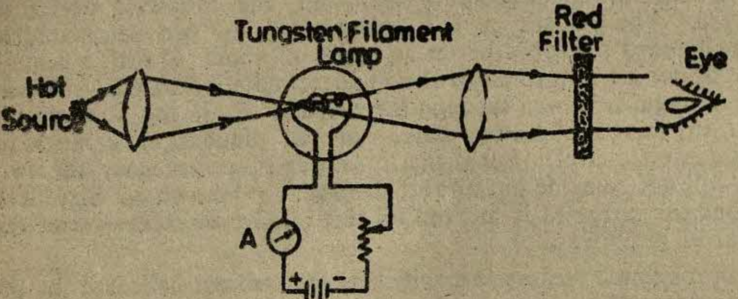


Fig. 5.26 Optical Pyrometer

This is the popular form of pyrometer. The principle of the *disappearing filament* optical pyrometer, which responds to light only is shown in Fig (5.26). Light from the hot source and the tungsten filament lamp passes through a red filter of a known wavelength range before reaching the eye ; both appear red. The current through the filament is adjusted until it appears as bright as the background of light from the source. The temperature is then read off from the ammeter, previously calibrated in K.

Calibration may be done against a gas thermometer upto 1800 K. The range of the pyrometer can be extended upto about 3000 K by having a sectored disc rotating in front of the source which cuts down the radiation in a known proportion.

Pyrometers are used to measure the temperatures of the interiors of furnaces—without getting too close—and of the surfaces of molten metals.

Pyrometers differ from other instruments such as thermometers, thermocouples etc., in that they are not to be in contact with the hot body. They can measure temperature howsoever high and the lower practical limit for radiation pyrometers is about 900 K. Further no extrapolation is required in estimating very high temperatures as the formulae on which the pyrometers are based are found to be valid at all temperatures.

The pyrometers, however, suffer from two drawbacks :

(*a*) Since they are based on laws (*i.e.*, Stefan's Law and Wien's Law) that govern strictly black body radiation alone so the temperature actually measured is the *black body temperature* of the source which is evidently less than its *actual* temperature.

(*b*) There is a lower practical limit (900K) below which pyrometers cannot be used for accurate measurement, since the emission from bodies becomes too small to be estimated correctly at lower temperatures.

Example 5.5

Calculate the radiant emittance of a black body at a temperature of 4000 K.

$$(\sigma = 5{\cdot}672 \times 10^{-8} \mathrm{W}m^{-2}\mathrm{K}^{-4})$$

Solution :

$$\text{Since } E = \sigma T^4$$

$$E = 5{\cdot}672 \times 10^{-8} \times (4000)^4$$

$$= 14520000 \ \mathrm{W}m^{-2}$$

Example 5.6

If a physician is interested in fluctuations in Skin temperature of 0.1 deg about a normal value of 300 K, what must be the sensitivity of his equipment for producing thermograms ?

Solution :

$$\text{Since } E = \sigma T^4$$

so the rate of change of E with T is given by

$$\frac{dE}{dT} = 4\sigma T^3 = \sigma T^4 . \frac{4}{T} = \frac{4E}{T}$$

Therefore, for small changes δE and δT, we have approximately

$$\frac{\delta E}{\delta T} \simeq \frac{4E}{T}$$

or

$$\frac{\delta E}{E} = \frac{4\delta T}{T} = \frac{4 \times 0{\cdot}1}{300}$$

$$= 0{\cdot}13\%$$

So his detectors must be able to reliably detect fluctuations of 0·13% in the radiation emitted by the skin.

SUMMARY

1. If two bodies are in thermal equilibrium with a third body, they are in thermal equilibrium with each other (Zeroth Law).
2. Temperature is the numerical indication of hotness and coldness.

3. Heat and work are closely in relation. They represent energy in transit. Heat is the flow of energy from one body to the other, by virtue of their temperature difference alone.

4. Internal energy is an important characteristic of a thermodynamic system.

5. The first law of thermodynamics states that the amount of heat added to a system equals the sum of the increase in the system's internal energy and the work done by the system against its surroundings. This law extends the conservations of energy to process that involve heat.

6. An amount of mechanical energy equal to 4·186 joules is always equivalent to one calorie of heat.

7. While mechanical energy can always be converted entirely into heat, the reverse is not true. Attempts to convert heat completely into energy always produce some waste heat. This observation is the basis of the Second Law of Thermodynamics.

8. The Second Law of Thermodynamics limits the efficiency of engines to below 100 percent. The most efficient possible engine is one based on the carnot cycle.

9. Reversible process is that process which can be exactly reversed so that the system passes through the same intermediate states as in the forward process. A reversible process implies no wastage of energy.

10. Cyclic process is a sequence of processes as a result of which a system returns to its original state. For a cyclic process the change in internal energy is zero.

11. An absolute scale of temperature is a scale which is independent of characteristics of any particular substance. It coincides with the perfect gas scale.

12. Energy is transported from a hot body to a cold body by a process known as radiation. In this mode of transference of heat, the intervening medium does not get heated.

13. A black body is one that absorbs all the radiations that fall on it.

14. **The ratio of the emissive power to the absorptive power is constant for all bodies and is equal to the emissive power of a black body. (Kirchoff's Law).**

15. **The total energy radiated per second per unit area by a black body is proportional to the fourth power of the absolute temperature. (Stefan-Boltzmann Law).**

16. **At each temperature T the energy radiated by a black body is a maximum for certain wavelength λ_{max} which decreases with rising temperature. (Wien's Law).**

17. **Thermal radiation also can be used to measure high temperatures as the radiation from a black body depends solely on its temperature. Instruments which work on this principle are known as Pyrometers.**

QUESTIONS

1. **Distinguish between (*i*) heat and temperature and (*ii*) Heat and Work. Is it possible to measure heat in joules ? If so what is the relation between calories and joules ?**

2. Is heat and internal energy the same thing? Define internal energy of a system and state first law of thermodynamics.

3. Distinguish between isothermal and adiabatic process. Does a gas work when it expands adiabatically? If so, what is the source of energy? [*A.I S S.C. 1986*]

4. Define the first law and the second law of thermodynamics. Explain carefully the significance of both of them

5. If two bodies are in thermal equilibrium with one another. do they have (*a*) the same T, (*b*) the same U?

6. Can two bodies at different pressures be in thermal equilibrium with one another? If you think so, quote an example.

7 Is a saucepan of water steadily boiling on a stove a case of thermal equilibrium?

8. Would a universe in complete thermal equilibrium be an interesting place to live in? List the ways in which it would differ from the actual universe.

9. Give several examples familiar in every day life of (*a*) work being converted into heat, (*b*) heat being converted into work.

10. Describe qualitatively how the first law of thermodynamics can be applied to (*a*) warming a poker in a fire. (*b*) boiling away a saucepan of water on a stove, (*c*) the boy scout method of lighting a fire by rubbing together two sticks.

11 A balloon containing helium gas is punctured and the helium diffuses throughout the whole room. How does this illustrate the Second Law of Thermodynamics?

12. What do you understand by a reversible process and an irreversible process? Give three examples in each case.

13. (*a*) What is a reversible cycle?

    (*b*) Describe the Carnot's cycle. Is the efficiency of the cycle dependent on the nature of the working medium?

14. Describe a Carnot's engine. Draw the P V indicator diagram for a cycle of its operations between two given temperatures and deduce from it the thermal efficiency of the engine. [*A.I.S.S.C. 1986*]

15. Explain.—

    (*i*) No engine can be more efficient, man the reversible Carnot engine.

    (*ii*) The efficiency of all reversible engines working between the same two temperatures depends only on the temperatures of the hot and cold bodies.

16. What do you understand by absolute scale of temperature ? Show that it agrees with the ideal gas scale.

17. State the laws relating to (*i*) the radiation and (*ii*) the temperature of a radiating body.

18. What is a black body ? What are the characteristics of a black body radiation ? How has a black body been realised in practice ?

19. Show that the radiation in a uniform temperature enclosure depends only on the temperature of the enclosure and not on the material of which it is made or on the presence of any other bodies in the enclosure. Why is such radiation called "*full radiation*" and why is it the same as "black body radiation"?

20. (*a*) Describe a sensitive instrument for the detection and measurement of radiant heat.

    (*b*) What are the principle points of resemblance and difference between radiant heat and visible light ?

21. Define emissive power and absorptive power. Deduce that at any temperature the ratio of the emissive power to the absorptive power of a substance is constant and is equal to the emissive power of a perfectly black body.

22. Explain Kirchoff's Law concerning the relation between the absorptive and emissive powers of a body ? Discuss some of its important applications.

23. In what respects does the radiation from a black body at 2000K differ from that from a black body at 1000K ? How would you devise a black body to radiate at 1000K ?

24. (*a*) How may the distribution of energy in the spectrum of the radiation emitted by a black body be investigated experimentally ?

    (*b*) Draw labelled curves which show how the energy in the spectrum of the radiation from a black body varies with wavelength at various temperatures. List the various points that emerge from the curves.
    (*Delhi Senior Sec. 1980*)
    [*A.I.S.S.C. 1986*]

25. (*a*) State (*i*) Stefan's Law ; (*ii*) Wien's Displacement Law.
    (*b*) How can we derive Newton's Law of cooling from Stefan's Law. Mention the practical applications of these laws.

    (*c*) How Wien's displacement law is used to determine the temperature of the Sun ? [*A.I.S.S.C. 1986*]

26. (*a*) What is a Pyrometer ? Mention the principle on which it works. How does it differ from (*i*) a mercury thermometer and (*ii*) a thermocouple ?

    (*b*) Mention the uses and the drawbacks of a Pyrometer.

27. Discusss a method that has been used to measure high temperatures in the neighbourhood of 2000 K.

28. Write short notes on :—

(*i*) Internal Energy of a system
(*ii*) Carnot's refrigerator
(*iii*) Fraunhofer lines
(*iv*) Nature of thermal radiations

29. Apply the first law of thermodynamics to obtain :—

(*i*) an expression for the change in the internal energy during the melting process.

(*ii*) an expression for the change in the internal energy during the boiling process.

(*iii*) $C_p - C_v = R$ where the symbols have their usual meanings. (*Delhi Senior Sec. 1980*)

30. Discuss whether the following phenomena are reversible (*i*) Electrolysis, (*ii*) Rusting of iron, (*iii*) Water fall, (*iv*) Mixing salt with water, (*v*) Frictionless movement, (*vi*) Conduction of heat.

31. Why does a piece of red glass, when heated to 1000K and taken out, glow with a green light ?

32. (*i*) What is ice point on Kelvin scale ? If the temperature of a body fall by 1°C, what will be the change in temperature on Kelvin Scale ?

(*ii*) Can a refrigerator be employed as a heat pump to heat a house in winter ?

(*iii*) Radiant energy with wavelength longer than that of visible light produces heat. What about shorter wavelengths ?

(*iv*) A green house keeps plants warmer. Explain

PROBLEMS

(Take J=4.18 joule/calorie unless specified otherwise.)

1. A water fall is 500 m high. If the whole of the energy of water remains in it, how much hotter is the water at the bottom than at the top ? (J=4.2 joule/Cal.)

(1.167 K)

2. A lead bullet strikes a target with a velocity of 630 ms^{-1} and the bullet falls dead. Calculate the rise in temperature of the bullet, assuming that 25% of the heat produced is used in heating the bullet. (Sp. heat of lead 0.03)

(393.75K)

3. Four moles of an ideal gas at 375K is compressed isothermally from an initial volume of 7.5 litres to a final volume of 1.5 litres. Calculate the work done on the system.

(-2.006×10^4 J *i.e.*, work is done on the system)

4. 2 kg of water at 100°C is converted into steam at the same temperature. The volume of 1 cm^3 of water changes to

1671 cm^3 of steam on boiling. Calculate the change in the internal energy of the system, if the latent heat of vaporization of water is 540 cal/g

(10.799 × 10^5 cal)

5. For each adiabatic process find the change in the internal energy (*i*) A gas does 167.2 J of external work while expanding adiabatically, (*ii*) during an adiabatic compression 167.2 J of work is done on the gas.

(—40 cal ; 40 cal.)

6. Compute the efficiency of a Carnot engine operating between 450K and 300K. If this engine takes 500 000 J from the source, how much work does it perform ?

(33.3%; 1.6 × 10^5 J)

7. In a Petrol engine the rate of production of heat due to the combustion of the fuel is 5 × 10^6 calories / hour. The efficiency of the engine is 30%. Calculate the horse power of the engine. Given J=4.2 joule/calorie and 1 H.P.=746 watts.

(2.346)

8. A Carnot refrigerator takes 60K cal from a freezing chamber at 250K and rejects heat at 300K. How much work must be done ? How many calories are rejected ?

(50160 J; 72 k cal)

9. Wavelength corresponding to E_{max} for the sun is 4753 A°. Compute the temperature of the sun. Given that Wien's constant=2.884 × 10^{-3} m. deg.

(6067 K)

10. To what temperature must a black body be raised in order to double its total radiation if its original temperature is 1000 K ?

(1190.5 K)

11. A spherical body diameter 2 cm is maintained at 873 K. Assuming that it radiates as if it were a black body, at what rate is energy radiated from its surface ? Given $\sigma = 5.735 \times 10^{-8}$ Wm^{-2} K^{-4}.

(41.86 Js^{-1})

12. In a refrigerator heat from inside at 277 K is transferred to a room at 300 K. How many joules of heat will be delivered to the room for each joule of electric energy consumed ideally ?

(13.04 J)

13. The temperature of a tungsten filament in an incandescent lamp is 3000K. Find the surface area of the filament of 100 W lamp. The emissivity of the filament is 0.30. ($\sigma = 5.735 \times 10^{-8}$ Wm^{-2} K^{-4})

(7.175 × 10^{-6} m^2)

Unit 6

LIQUIDS

6.1 INTRODUCTION

We know that the molecules in a liquid are in constant motion (Kinetic Theory of Matter), but they do not possess enough energy to overcome the strong attractive forces due to the rest of the molecules. They are therefore unable to escape out of the liquid. They exist as a fluid aggregate. This gives a definite volume to a liquid but does not give any definite shape. Liquids, therefore, conform to the shape of the container. It is difficult to compress a liquid.

Such curious effects are associated with the surface of a liquid which we shall study in this unit.

Like gases liquids can flow readily but their rate of flow is much lower. How fast a liquid can flow depends on the internal friction known as viscosity. We shall study the flow of liquids in this unit as well.

6.2 GENERAL IDEAS OF INTERMOLECULAR FORCES :

A comparison of the densities of a substance in the solid, liquid and gaseous states gives us an idea of the distance between molecules in the three states of matter. The distances between molecules in the solid and liquid states are found to differ only slightly, no more than several per cent, while in gases the distances (at not very high pressures) were about ten times as great. Hence, the molecules in liquids and solids are said to be closely packed. Owing to close packing liquids and solids are much *less compressible* than gases. The compressibility of liquids and solids is only about one thousand to one hundred thousandth of that of gases. This is evident from *Table 6.1* which lists the density and compressibility of certain liquids and solids at the same value of the surplus pressure (P=20 atm).

TABLE 6.1

Substance	Density (kg m^{-3})	Compressibility	Substance	Density kgm^{-3}	Compressibility
Water (0°C)	999.9	48.9×10^{-11}	Copper	8930	0.73×10^{-11}
Ice (0°C)	916.8	40×10^{-11}	Aluminium	2690	1.37×10^{-11}
Ethyl alcohol (15°C)	790	76×10^{-11}	Diamond	3600	0.23×10^{-11}
Mercury (20°C)	13545.9	3.8×10^{-11}	Graphite	2300	3.0×10^{-11}

An analysis of data on the compressibility of liquids and solids and also of many phenomena (wetting of solids by liquids etc.) indicates that there are considerable forces between molecules. They are called *molecular forces* These forces possess many interesting features.

As the molecules move away from each other molecular interaction is manifested in the form of forces of *attraction* ; when brought together to distances of the order of the linear dimensions of the molecules themselves, interaction takes the form of forces of *repulsion*. Consequently, there is a definite distance between the molecules where the forces of attraction and repulsion balance each other and their resultant equals zero. *A system of molecules located at distance d from one another is in a state of stable equilibrium and their interaction energy is minimum.*

Molecular forces are displayed only when the distance between the molecules is very small, *i.e.*, of the order of the effective size of the molecules themselves. When the distances are increased by several times, the forces become practically equal to zero. This explains why the action of molecular forces can be neglected in most cases in analyzing phenomena in gases where distances between molecules are about tenfold those in liquids and solids.

The magnitude of the molecular forces does not depend upon the total number of molecules. Thus the density or elastic properties of liquids and solids do not depend upon the size of the test piece, a drop of water and water in the ocean have exactly the same density and compressibility at the same temperature and external pressure etc. *In this, molecular forces differ in principle from gravitational or coulomb forces in which the resultant is determined from the action of all the bodies making up the system. The reason why molecular forces act only between adjacent molecules* will be cleared up as follows :

Careful research conducted by many scientists in the first half of the 20th century has led to the conclusion that *molecular forces are of electrical origin*

On the face of it such a conclusion may seem strange because we know that molecules are neutral and that electrical interaction is typical only of charged bodies. However it has been shown that *electrical interaction is developed between dipoles* : neutral particles with a symmetrical arrangement of charges, which are equal in magnitude but of opposite sign (Fig 6.1). This enable us to understand the nature of the electrical interaction between molecules. Molecules having a dipole moment induce a dipole moment in neighbouring molecules The force of attraction between them is inversely proportional to the seventh power of the distance between the molecules

$$\left(i.e.\ F \propto \frac{1}{r^7} \right)$$

This provides an explanation for intermolecular interaction in real gases known as *Vander Waal's interaction or forces*. Interaction between induced dipoles is the simplest case of molecular attraction. Actually, forces of attraction between molecules may also be due to various other causes ; which are beyond the scope of this book. Any rigorous theory of molecular forces must be based only on quantum mechanics. It was found, however, that in all cases the force of molecular attraction was inversely proportional to the seventh power of the distance between the molecules.

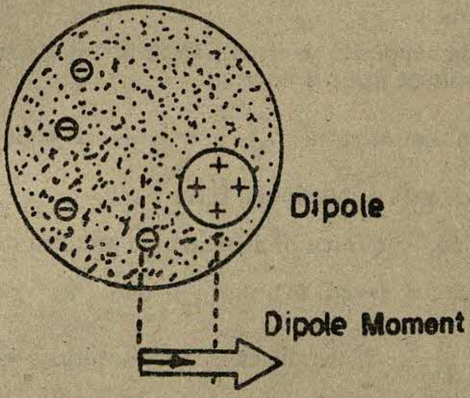


Fig. 6.1

$$F_{att} = -\frac{a}{r^7} \qquad ...(i)$$

Here the minus sign indicates that we are dealing with a force of attraction and 'a' is a factor depenuing upon the structure of the molecules and the kind of attraction force between them.

Experiments in the compression of bodies show that forces of repulsion appear between molecules that are brought close together. These forces are also of electrical origin. It was found that here the forces of repulsion between positively charged nuclei begin to play an essential role. Besides, quantum mechanical effects, are manifested, which are also beyond the scope of our book.

As the molecules are brought close together, the forces of repulsion vary rapidly, much faster than the forces of attraction. This can be expressed approximately as

$$F\ rep. = \frac{b}{r^9} \qquad ...(ii)$$

Here the factor b depends upon the same things that factor a does. A more accurate formula is obtianed in quantum mechanics but we shall not concern ourselves with it here.

Naturally, the molecule is subjected to the action of the resultant force

$$F = F_{att} + F_{rep}$$

or

$$\boxed{F = -\frac{a}{r^7} + \frac{b}{r^9}} \qquad ...(iii)$$

The sign of this force depends upon the distance between the molecules. A certain distance known as *normal distance* $r=d$ exists at which the repulsion forces counterbalance the attraction forces. It follows from equation (*iii*) that $F=0$ if $d=\sqrt{\frac{b}{a}}$.

When $r<d$ (*i.e.*, the distance between the molecules is less than the normal distance) the resultant force is *positive i.e.*, the repulsion force exceeds attraction.

When $r>d$, the opposite is true : attraction is stronger than repulsion and the resultant force is *negative*.

It can be shown that at $r_0=d\sqrt{\frac{9}{7}}=1.134\ d$ molecules are attracted to each other with a maximum force $F_{max}=-\frac{a}{11\ d^7}$ (Fig 6.2). At $r=1.5d$, the force of attraction is reduced to about $\frac{1}{3}$ of the maximum value ; at $r=2d$, to about $\frac{1}{16}$ and at $r=3d$ to almost as small as $\frac{1}{250}$. If the distance is reduced to $r=0.9d$, the force of repulsion is $F_{rep}\approx 5\ |F_{max}|$ and at 0.7d, it is equal to 140 $|F_{max}|$. The nature of intermolecular forces has been shown in Fig (6.2).

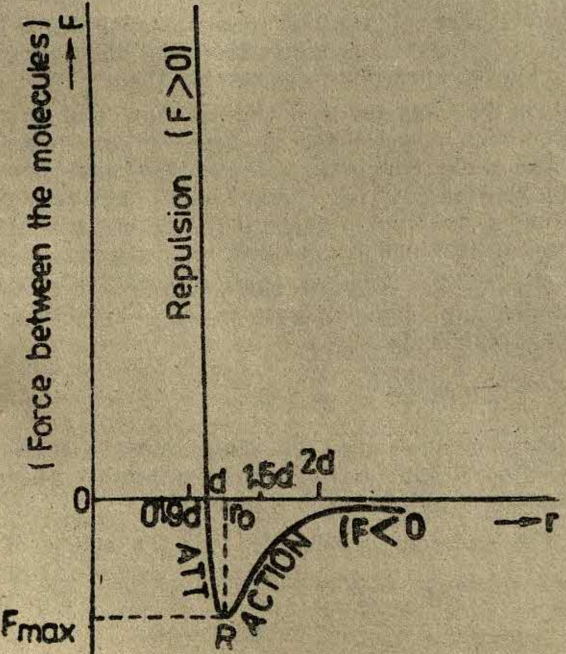


Fig. 6.2 Nature of Intermolecular forces

Such a strong dependence of the force on the distance signifies that molecular forces are *short ranged* : molecules interact on with their closest neighbours. The molecular forces are (i) attractive at large distances and (ii) have necessarily to be repulsive at very close distances. If the molecules are represented by closely packed spheres (Fig 6.3), which, in general, conforms to the packing of molecules in many liquids and solids, the central black molecules in Fig (6.3): interacts only with the first layer of molecules surrounding it. With an error not exceeding 6 per cent we can neglect the interaction of the molecule with the second layer, and, all the more, with the third and subsequent layers.

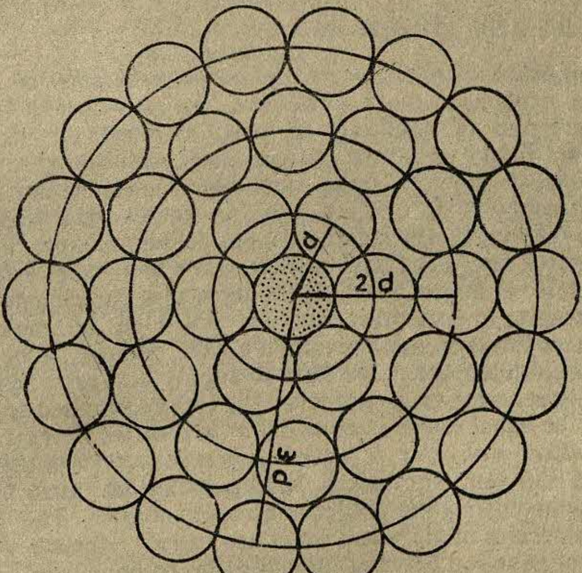


Fig 6.3.

This is what explains all the features of molecular forces—in particular, the independence of the magnitude of the molecular forces from the total number of molecules in the system and the lack of molecular interaction in gases (except when they are under very high pressure).

6.3 NATURE OF LIQUIDS

The combination of the intermolecular forces and the thermal motion gives rise to three states of matter *i.e.* solid, liquid and gas. In solids the attractive forces are quite strong and thermal motions cannot break them away. Molecules may stay at one place and vibrate. In gases the intermolecular attractive forces are so weak that random thermal motion can very easily overcome them and molecules are almost free to move about anywhere. *Liquids fall in between these two cases.* For the sake of simplicity a liquid is sometimes visualized as a dense gas. The molecules in a liquid are neither forced to stay permanently in an equilibrium position nor they are free to leave the company of other molecules A molecule inside a liquid can slide freely over others without any effort.

Liquids, however, differ very widely in the time which they take to yield to the deforming forces. Water and alcohol, for example, take up the shape of the vessel in which they are placed with great rapidity ; honey and pitch take much longer.

6.4 COHESION AND ADHESION

The force with which like molecules attract each other is called **Cohesion**. As explained above, cohesive forces which hold the molecules of a liquid or solid are in part at least of electrical origin and do not follow an inverse square law. Within the solids the forces of cohesion are very large since the molecules are close together. The cohesion forces are less for liquids and least for gases. This is the reason why solids have got definite shape, liquids definite free surface and gases have neither. We can thus define a liquid as a state of matter in which the molecules can change their position with respect to each other but are restricted by molecular forces or cohesive forces so as to maintain a fixed volume. If we want to convert a liquid into vapour, we have to supply an extra amount of energy and to separate two molecules the energy supplied to them should *at least* be equal to the *intermolecular interaction Energy. The attraction of unlike molecules is called adhesion.* At the common surface of two different substances unlike molecules attract each other. Glue adheres to wood, solder adheres to brass. The forces between the molecules of glass and water are called adhesive forces. It is the force of adhesion between the molecules of honey and finger which we have to overcome when we exert to lick the honey off the finger

6.5 SURFACE TENSION :

We know that liquids when in small quantities always assume spherical shapes. For example, rain drops, dew drops, droplets of mercury are all spherical [Fig 6.4 (a)]. However, when liquids are in somewhat large quantities *e.g.*, mercury [Fig 6.4 (b)], the shape is oval and not spherical. The reason is that when liquids are in small quantities, external forces such as gravity acting on them are negligibly small whereas they are appreciable when liquids are in large quantities. Thus, we conclude that when liquids are in

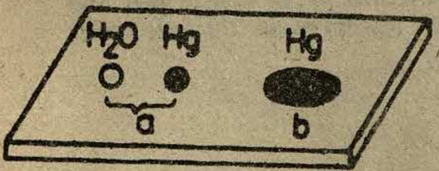


Fig. 6.4

small quantities and free from external forces, they assume spherical shapes And as for a given volume the surface area of a sphere is minimum, *it means that a liquid free from external forces tends to have least possible surface area, i.e. spherical shape.*

The students might have seen small insects running about on the surface of water in ponds. Again, if you place gently a sewing needle on the surface of water, you will find that it floats even when it is eight times heavier than water (Fig 6.5). The experiment becomes easier to perform if the needle is slightly smeared with oil or grease. On watching closely the surface of water is found to be depressed slightly under the needle in the same way as a thin rubber sheet will do when needle is placed over it. The place where the surface is depressed, the forces of cohesion between water molecules give rise to restoring forces, equal and opposite to the weight of the needle, tending to restore the horizontal surface. It is the restoring force developed in the surface under the weight of the needle because of force of cohesion between the molecules of water which counter-balances the weight of the needle. *Thus, the surface of a liquid behaves like an elastic skin or stretched membrane.*

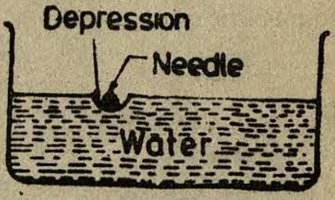


Fig. 6.5

From this we conclude that free surface of a liquid behaves like an elastic membrane under tension, tending to contract so as to have minimum surface area. This behaviour of a liquid surface suggests that there exists a force in the surface of the liquid which tends to shrink the liquid surface to have minimum area.

The phenomenon by virtue of which the free surface of a liquid behaves like an elastic membrane under tension tending to contract so as to have minimum surface area is called **Surface Tension.**

In order to define surface tension of a liquid, consider an imaginary line AB drawn in the free surface of the liquid (Fig 6.6). The molecules lying just on its one side try to pull away from the molecules lying just on the other side in order to decrease the surface area. *Hence surface tension is measured as the force per unit length acting on either side of a line in the liquid surface in equilibrium, the direction of the force being tangential to the surface and perpendicular to the line.*

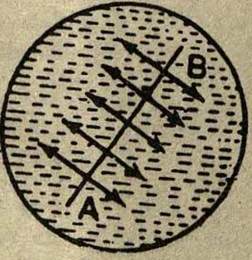


Fig. 6.6

If F is the total force acting on either side of the line AB of length l, then.

$$\text{Surface Tension } \sigma = \frac{\text{Force}}{\text{length}} = \frac{F}{l}$$

The unit of surface tension is newton/metre (Nm^{-1}) in SI system and dynes / cm in C.G.S. system. Its dimensions are MT^{-2}.

The value of surface tension depends on, among other things, the temperature of the liquid. At 20°C, for water $\sigma = 72.6 \times 10^{-3}$ Nm^{-1} and for mercury $\sigma = 465 \times 10^{-3}$ Nm^{-1}

Molecular theory of surface tension : – The molecules of a liquid attract each other with a force called the force of cohesion. It decreases as the distance between the molecules increases. The maximum distance up to which force of cohesion between two molecules can act is called their *molecular range*. It is of the order of 10^{-9} m being different for different substances. A sphere drawn with the molecule at its centre and having a radius equal to the molecular range is called the *sphere of influence*.

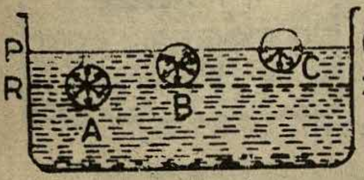


Fig. 6.7

First consider a molecule A lying well within the liquid (Fig 6.7) It is attracted equally in all directions by molecules lying within its sphere of influence. *Hence the resultant cohesive force acting* on it is zero.

Now consider a molecule B, whose sphere of influence lies partly outside the liquid. The number of molecules in the lower half attracting it downwards is more than the number of molecules in the upper half attracting it upwards. Hence there is a resultant downward force of cohesion on it.

The sphere of influence of the molecule C lies exactly half outside the liquid and half inside it. This molecule has the maximum downward force of cohesion acting perpendicular to the surface.

If we draw a plane RS parallel to the free surface PQ at a distance equal to its molecular range then the layer of liquid between these two planes PQ and RS is called the *Surface film*. All molecules lying in this surface film experiences a resultant downward cohesive force which increases as we go from the planes RS towards the free surface. *If any molecule is brought from within the liquid to this surface film some work is done against the downward cohesive force which increases the potential energy of the molecule.* Hence the potential energy of the molecules lying in the surface film is greater than the potential energy of the molecules lying below. But a system in *equilibrium* always tries to have the lowest possible energy. Thus in order to decrease potential energy of the molecules in the surface film the area of the film *must be the least. Since, for a given volume* a sphere has the least surface area, the free surface of the liquid tends to assume a spherical shape. This makes the surface of the liquid to behave like a stretched membrane giving rise to surface tension.

Examples of surface tension : (*i*) It has already been stated that due to surface tension the free surface of a liquid tries to have the minimum possible area. For a given volume a sphere has the least surface area. Hence the liquid assumes a spherical shape. That is why the rain drops and the mercury globules are spherical in nature. Ordinarily the effect is not so marked due to the force of gravity. If however, the force of gravity is elminated, the liquid will assume a perfcetly spherical shape This can be shown by Platau's experiment. If a large drop of olive oil is introduced in a mixture of alcohol and water having the same density as olive-oil, the drop assumes a perfectly spherical shape (Fig 6.8). It is because the upward thrust of the mixture on the drop is equal to the downward gravitational force acting on it and hence the effect of gravity is eliminated and the drop assumes a perfectly spherical shape.

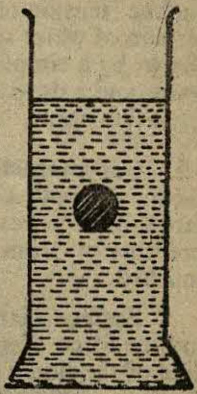


Fig. 6.8

(*ii*) The tendency of a liquid film to contract due to the surface tension can be shown by the following experiment. Dip a circular wire frame in the soap solution to form a thin soap-film. Place a wet cotton locp on the film. The loop adopts an irregular shape because the soap film lies both inside and outside the loop and at every point of contact of the loop, there is a force acting outwards at the surface of the film and an equal and opposite force acts inward due to the inside film. There is therefore, no resultant force acting on it due to surface tension. If the film inside the loop is pierced with a sharp needle it vanishes. The film outside the loop tries to contract and pulls the loop equally at all points so that the loop assumes a circular shape because for a given perimeter a circle has the greatest area.

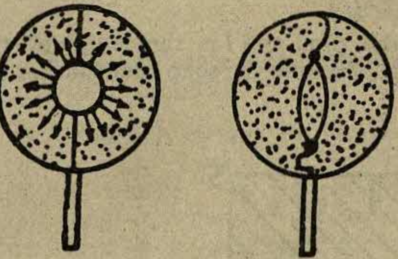


Fig. 6.9

(*iii*) If a soap bubble is blown at the end of a thin glass tube, and allowed to stand for some time, it gradually shrinks in volume. This shows that the surface tension acting on the surface of the bubble tends to reduce the surface area to a minimum.

(*iv*) When a camel-hair brush is dipped in water, the individual hair get separated and project in different direction. As soon as it is removed from the water the hair are found to be drawn together as if the hair are now connected by a stretched membrane.

(*v*) Different liquids have got different values of surface tension. When a drop of oil or spirit is placed on the surface of water the surface tension decreases considerably. The oil drop is pulled in all directions until it spreads over the entire surface. This is because the surface tension of oil is much less than that of water. The greater tension of water stretches the oil in all directions. This can be further shown by a simple experiment. If a needle floats over the surface of water and a drop of oil is poured close to it the needle is drawn away from that spot.

(*vi*) Umbrellas, raincoats, tents and canvas have tiny holes in them. Inspite of this, when rain falls the water does not pass through the pores. This is because the surface tension of water prevents it from passing through the fabric unless the inside is touched.

Surface-energy : The surface of a liquid is in a state of tension and the force of surface tension tends to decrease the surface. If the area of the liquid surface is increased, work is done against the surface tension. *This work done is stored as the potential energy in the surface.* Consider a rectangular wire frame PQRS in which the wire PQ is movable. Dip the wire in the soap solution. A film is formed which pulls the wire PQ inward due to surface tension by a force given by

$$\sigma \times 2l$$

where σ is the surface tension of the film and l is the length of the wire PQ. The length is taken twice because the soap film touches the wire above as well as below.

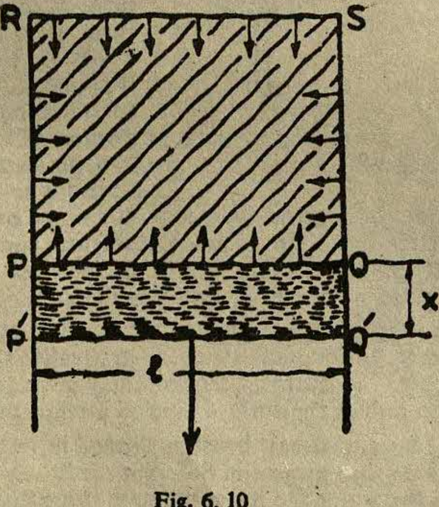


Fig. 6. 10

The film is stretched by a small distance x to the position P'Q' keeping the temperature constant. Then the work done is given by

$$\sigma \times 2l \times x.$$

But $21 \times x$ is the total increase in area $\triangle s$ of the soap film on both the sides. Therefore

work done $= \sigma \times$ increase in area $= \sigma . \triangle S$

or work done per unit area $= \sigma$

This work is stored as the surface energy in the soap film. Hence surface tension is numerically equal to the surface energy per unit area of the surface. This way it is measured in Jm^{-2}.

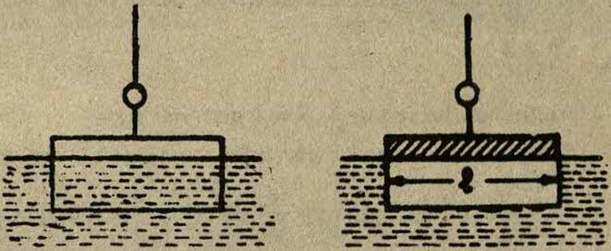


Fig. 6.11

In actual practice the force acting on the frame of wire can be measured as follows :

A carefully cleaned wire frame, in the form of a rectangle about 5 cm long and 2 cm high (Fig 6.11) is suspended from one arm of a sensitive balance. A beaker of water is placed such that the wire frame hangs in water with the top about 3 mm above the surface of water when the beam is horizontal and the balance is counterpoised. The beaker is then raised a little so that the frame is completely under the surface of water and then lowered carefully. The portion of the frame outside water now carries a film of water and more weights are required to restore equilibrium. Let this additional weight be m gms.

The film of water has two surfaces, front and back. The ***downward force on the frame*** (parallel to the film and normal to the wire above water) exerted by the film neglecting the weight of the film raised, is $2\sigma l$ dynes, where l is the length of the frame (parallel to the surface of water in the beaker). This is balanced by the added weight mg dynes.

Thus $2\sigma l = mg$

$$\therefore \quad \sigma = \frac{mg}{2l}$$

This experiment illustrates the action of surface tension and its measurement in a simple manner.

The concept of surface tension as energy per unit area affords us an explanation why small drops of liquid are spherical. The potential energy of a system always tends to assume a minimum value for stability. For a given volume the spherical shape provides the minimum surface area and hence the minimum potential energy in the case of a small drop whose weight is negligible. Large drops become flat (as in the case of a small pool of mercury) as centre of gravity must also fall in order that the P.E. due to gravity may also be small and that the total P. E. may be the minimum.

Example 6.1 :

A sphere of water of radius one mm is sprayed into 1000 drops all of the same size. Find the energy spent in doing so. Surface tension of water is 72 dynes/cm.

Solution :

Radius of the sphere $R = 1 \text{ mm} = 0.1 \text{ cm}$

Surface area of the sphere $4\pi R^2 = 4\pi \times (0.1)^2$ sq. cm

Volume of the sphere $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (0.1)^3$ c.c

Since the sphere is broken into 1000 drops all of the same size of radius r

$$\text{Volume of each} = \frac{4}{3}\pi r^3$$

or $$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (0.1)^3$$

$$(10\,r)^3 = (0.1)^3$$

$$r = 0.01 \text{ cm}$$

Surface area of small drop $= 4\pi(.01)^2$ sq. cm

Total area of 1000 small drops $= 1000 \times 4\pi\,(.01)^2$ sq. cm

$$= 4\pi\,(.1)$$

Increase in surface area $= 4\pi \times .1 - 4\pi \times (.1)^2$

$$= 4\pi \times 0.09 \text{ sq. cm}$$

The Energy spent $= \sigma \times$ increase in area

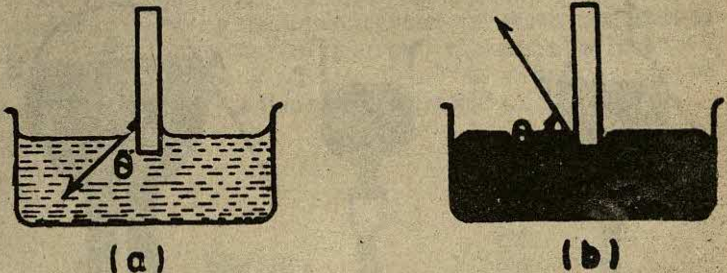
$$= 72 \times 4\pi \times .09$$

$$= 81.4 \text{ ergs}$$

6·7 ANGLE OF CONTACT

When a liquid meet, a solid, its surface near the plane of contact of the solid is generally curved. If a plate of glass is dipped in water with its sides vertical, water is drawn up and assumes a curved surface as shown in Fig 6. 2 (a). On the other hand if the same glass plate is dipped in mercury, the surface of mercury is again curved but is depressed below the horizontal surface as shown in Fig. 6.12 (b). *The angle θ between the tangent to the liquid surface at the*

point of contact and the solid surface inside the liquid is called the angle of contact for that pair of solid and liquid.



(a) (b)

Fig. 6.12

The angle of contact between mercury and glass when air is above mercury is obtuse being about 140°. It is an acute angle in the case of water which wets the solid surface. For a pure water and a clean glass the angle of contact is zero, whereas for ordinary water it is about 8°. The angle of contact between paraffin and water is 107°, between methyl iodide and glass 29° and between silver and water 90°.

In general, we find that liquids which wet the surface of glass rise along the surface and angle of contact is acute. On the other hand those liquids which do not wet the surface of the glass *e.g.*, mercury the angle is obtuse and liquids are depressed.

For dying sometimes we use wetting agents. They are added to make the angle of contact between the fibres and the dye small so that dye may penetrate well. On the other hand for water proofing waxy substances are added to create large angle of contact between the water and fibres.

Determination of angle of contact : For mercury and glass the angle of contact is determined by the following simple method due to Gay Lussac. The apparatus consists of a round-bottomed glass-flask filled more than three-fifth of it with mercury. Its mouth is tightly closed with a rubber-cork through which a glass rod is passing. The glass rod is moved in or out until the surface of the mercury in the flask is plane. This can be tested by holding a printed sheet of paper against the flask and observing its image by the light reflected at grazing incidence from the mercury surface. If the image is undistorted the surface of mercury is plane. The mercury surface in the flask forms a circular sheet of radius AC=r. It will be half of the diameter AB which can be measured by a pair of callipers. Let R be the radius of the flask also measured by the callipers. Referring to Fig 6.13 (a), we have

$$\cos\phi = \frac{r}{R}$$

or $$\phi=\cos^{-1}\frac{r}{R}$$

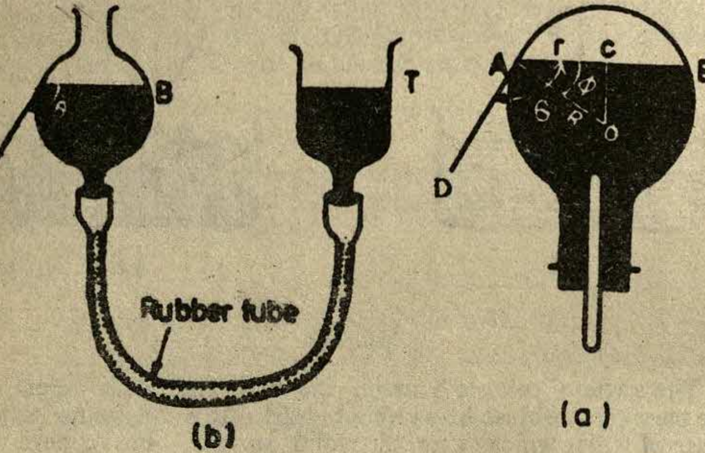


Fig. 6.13

If we draw a tangent AD to the surface of the glass at the point A, then the angle of contact θ for mercury and glass is given by the angle CAD.

$$\text{Angle of contact } \theta=\angle CAD=\angle CAO+\angle OAD$$

$$=\phi+90°.$$

A more convenient method of adjusting the level of mercury in the flask is as shown in the Fig 6.13 (b). In this case the level of the mercury can be raised or lowered by moving the reservoir T connected to the flask by a piece of rubber-tubing.

6.8 CAPILLARITY

The word capillary literally means as fine as a hair and thus a capillary tube is a tube of very fine bore. Take a capillary tube open at both ends and dip in water,the water will rise in the tube. Not only water but other liquids which wet the surface of the glass also rise in it. Then dip this tube in mercury, the mercury is depressed below the outside level.

This phenomenon of rise or fall of a liquid in a tube of very narrow bore is called capillarity. The rise and fall of the liquid can be explained on the basis of the difference in the values of cohesion and adhesion and surface tension. In case of water the force of adhesion between glass and water is greater than the force of cohesion between molecules of water. Thus water rises up till the downward weight of the water in the capillary tube above the outer surface of water in the vessel balances the upward force due to surface tension acting along the line of contact of water and glass in the capillary tube. In the case of mercury, the force of cohesion between mercury molecules is greater than the adhesion between mercury and glass, mercury is therefore, depressed in the tube.

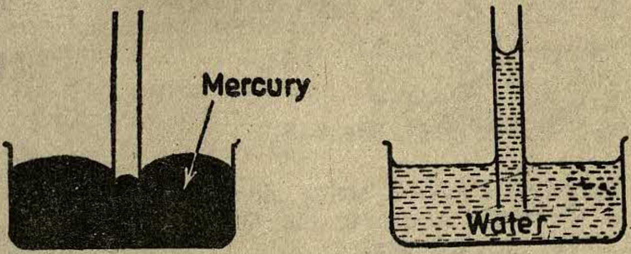


Fig. 6.14

Smaller the radius of the capillary tube the greater is the rise or fall of the liquid in it.

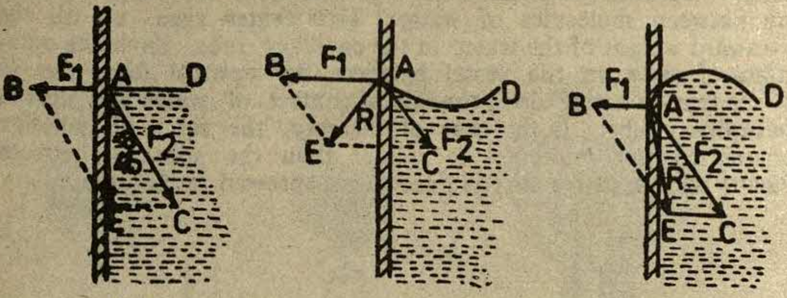
Examples of capillarity : (*i*) The rise of oil in the wick of the lamp, the rise of sap in plants, or the rise of melted wax in a candle or the absorption of ink by blotting paper are familiar examples of the capillary action.

(*ii*) Bricks and mortar are porous and get damped by the water of the soil rising through them by capillarity. This is avoided by laying a layer of cement concrete over the brick foundation. This layer is called damp-proof.

(*iii*) Capillary action plays an important part in farming. In compact porous soil, water rises to the surface by capillarity and evaporates. To prevent this loss, the soil is loosend and broken up into small pieces so that the spaces between the pieces become large and rise of water to the surface, by capillarity is avoided. This operation is very essential in dry-farming.

6.9 SHAPE OF MENISCUS IN A CAPILLARY TUBE

We know that the liquid meniscus in a capillary tube is concave for a liquid like water and convex for a liquid like mercury. To explain how it happens, consider a molecule A on its *surface touching the walls* of the glass tube. It is acted upon by the following *two* forces.



(i) (ii) (iii)

Fig. 6.15

(*i*) The outward force F_1 acting along AB at right angles to the walls of the tube at A, due to the *force of adhesion* exerted by the molecules of the glass tube.

(*ii*) The inward force F_2 acting along AC making an angle of 45° due to the force of cohesion exerted by other molecules of the liquid. The reason for it is that the molecule at A is pulled along AD by the molecules in the surface, along AE by the molecules vertically below it with the same force and along directions between AD and AE by other molecules. The resultant of all these forces due to cohesion is therefore inclined to the vertical at an angle of 45°.

The direction of the resultant R of the forces F_1 and F_2 depend upon the magnitudes of F_1 and F_2.

(*a*) If $F_1 = F_2 \sin 45° \text{ or } \frac{F_2}{\sqrt{2}}$

The resultant R acts vertically downwards and hence the liquid surface AD remains horizontal as shown in Fig 6.15 (*i*)

(*b*) If $F_1 7 \frac{F_2}{\sqrt{2}}$

The resultant R lies outwards and hence the liquid surface AD becomes *concave* as shown in Fig 6.15 (*ii*)

(*c*) If $F_1 < \frac{F_2}{\sqrt{2}}$

The resultant R lies inward and hence the liquid surface AD will become *convex* as shown in Fig 6.15 (*iii*)

This is because a liquid cannot permanently withstand a shearing force. Its surface, at every point, will set itself at right angles to the resultant force there when the liquid attains the position of equilibrium.

In the case of water and also other liquids which wet the walls the meniscus is concave as given by Fig 6.15 (*ii*) and in the case of mercury and other liquids which do not wet the walls, the meniscus is convex as given by the Fig 6.15 (*iii*).

6.10 NEUMANN'S TRIANGLE

If two liquids, not miscible with each other, be brought into contact as at 0, (Fig 6.16), both being in contact with air, three surface tensions are to be taken into consideration.

(*a*) that of the surface between air and liquid 1, *viz.*, σ_1 ;

(*b*) that of the surface between air and liquid II *viz.*, σ_2 ;

and (*c*) that between liquid I and liquid II, *viz.*,σ_3.

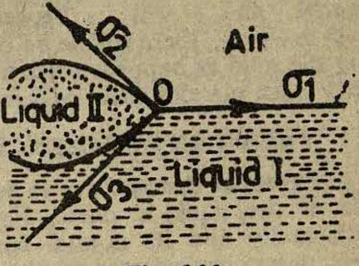


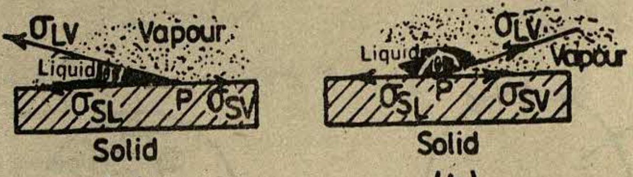
Fig. 6.16

For equilibrium σ_1, σ_2, and σ_3 should be represented by the three sides of a triangle, taken in order. This triangle of forces is known as *Neumann's triangle*. In actual practice, we come across no two pure liquids for which the Neumann's triangle may be constructed,—one of the surface tensions being always greater than the other two ; so that the equilibrium condition shown in the Fig. (6.16) is never attained.

Neumann's triangle and the Angle of Contact :

(Case of a solid, liquid and vapour in contact)

Let σ_{SV}, σ_{LV} and σ_{SL} be the surface tensions for solid-vapour, liquid-vapour and solid-liquid surfaces respectively (Fig 6.17). Let θ be the angle of contact of the liquid with the solid—acute in case Fig 6.17 (*a*) and *obtuse* in case Fig 6.17 (*b*).



(a) (b)

Fig. 6.17

For equilibrium, therefore, σ_{SL} *plus* the component $\sigma_{LV}\cos\theta$ of σ_{LV}, in the direction of σ_{SL}, must be balanced by σ_{SV} *i.e.*.

$$\boxed{\sigma_{SL}+\sigma_{LV}\cos\theta=\sigma_{SV}}$$

or
$$\sigma_{LV} \cos\theta = \sigma_{SV} - \sigma_{SL} \text{ whence}$$

$$\boxed{\cos\theta = \frac{\sigma_{SV} - \sigma_{SL}}{\sigma_{LV}}}$$

clearly, therefore, if σ_{SV} is greater than σ_{SL}, cos θ will be *positive i.e.* θ will be less than 90°. and if σ_{SV} is less than σ_{SL}, cos θ will be *negative*, and θ will lie between 90° and 180°. If however, $\sigma_{SV} > \sigma_{LV} + \sigma_{SL}$, there *will be no equilibrium* and the liquid will spread over the solid as happens when a water drop is placed over a perfectly clean plate of glass, or a grease free mercury surface.

6.11 PRESSURE DIFFERENCE ACROSS A SURFACE FILM

Excess of Pressure due to Surface tension : When the free surface of the liquid is plane, Fig (6.18) (*i*), the surface tension acts horizontally and its normal component is zero. Thus no extra pressure is communicated to the inside or outside. But when the surface is convex, Fig 6.18 (*ii*), the forces due to surface tension acting on both sides of a line on the surface have components acting downwards which go to increase the pressure inside the liquid. Similarly when the surface is concave, Fig 6.18 (*iii*), the pressure inside the liquid is decreased. *Thus there is always an excess of pressure on the concave side.* The excess of pressure depends upon surface tension and radius of curvature.

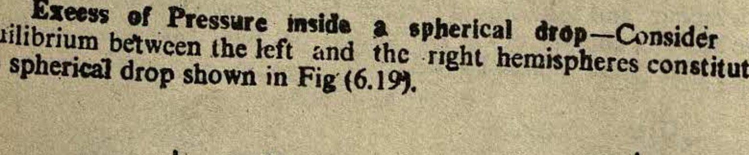

(i) (ii) (iii)

Fig. 6.18

Excess of Pressure inside a spherical drop—Consider the equilibrium between the left and the right hemispheres constituting the spherical drop shown in Fig (6.19).

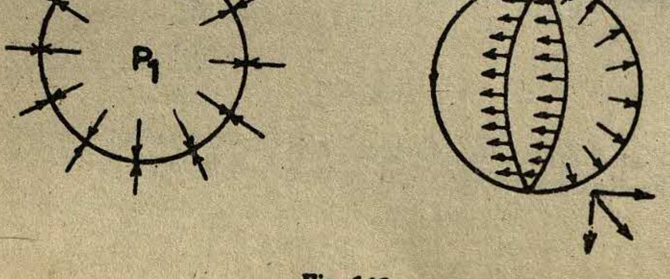


Fig. 6.19

Let P_1, P_2 be the pressures inside and outside the drop and let σ be the surface tension.

The right hemisphere pulls the left hemisphere towards itself with a force $2\pi r\sigma$, where r is the radius of the drop. This force is exerted on account of the surface tension on the surface of the right hemisphere, which acts along the circumeference, as shown in the figure. This force is balanced by the thrust $(P_1-P_2)\pi r^2$ exerted on the left hemisphere towards the left on account of pressure difference. In calculating this thrust we have taken into account the area of the plane circular face of the hemisphere and not the curved surface on which the excess pressure actually acts. This is because the vertical components of pressure at different points will cancel each other, as is clear from the figure, while the horizontal components add up. The effective area on which the horizontal components ac is the projection of the curved area in a direction normal to the horizontai pressures, which is the area, πr^2, of the circular interface of the two hemispheres.

Thus $\sigma\times 2\pi r=(P_1-P_2)\pi r^2$

$$\boxed{P_1-P_2=P=\frac{2\sigma}{r}}$$

Thus the difference of pressure (P) is *inversely proportional to the radius.*

Excess Pressure inside a spherical bubble : In this case there are two free surfaces of the right hemisphere which pull at the left hemisphere, as shown in Fig (6.20).

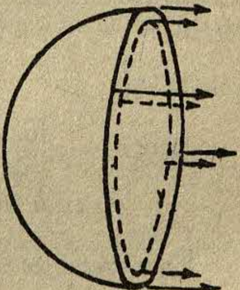


Fig. 6.20

Force due to surface tension

$$=2\sigma\times 2\pi r=4\pi\sigma r$$

For equilibrium,

$$4\pi\sigma r=(P_1-P_2)\pi r^2$$

$$\therefore\ P_1-P_2=\frac{4\sigma}{r}\text{ Nm}^{-2}$$

or $P=4\sigma/r$ where P is excess of pressure.

Alternative Proof :

Suppose the radius of the bubble is increased by a small value dr. To do so work is done on the bubble. It is equal to the product of excess of pressure and the volume swept. If dw be this work,

$$dw=(P_2-P_1)\times 4\pi r^2.\ dr.\text{ ergs}$$

The increase in surtace area, ds due to increase dr in the radius, taking into account both surfaces of the bubble is given by

$ds = d(8\pi r^2)$ ($\because$ surface area $s = 4\pi r^2$) of one surface

$\frac{ds}{dr} = 16\pi r$ $\rightarrow ds = 16\pi r . dr$

$\therefore$ Increase in potential energy

$= 16\pi r \, dr \sigma.$

By the principle of conservation of energy,

$$16\pi r \, dr \sigma = (P_1 - P_2) \, 4\pi r^2 \, dr.$$

$$\therefore \quad P_1 - P_2 = \frac{4\sigma}{r}$$

or $\boxed{P = \frac{4\sigma}{r}}$ as before

It follows from this result that if the surface tension (σ) remains constant, *the pressure difference* (*P*) *is larger for smaller values of r.* Due to this large internal pressure, tiny fog drops have rigidity properties like those of solids. For the same reason it is easier to walk on wet sand than on a dry one. Ice skaters slide over the surface of smooth ice because under the enormous pressure exerted on the ice, ice melts and the tiny drops act as ball bearings.

Example 6·2 :

Find the difference in air pressure between inside and outside of a soap bubble of 4 mm in diameter. ($\sigma = 1.6 \text{ Nm}^{-1}$).

Solution :

Here $r = \frac{4\text{mm}}{2} = 2\text{mm} = 0.002\text{m}$

$\sigma = 1.6 \text{ Nm}^{-1}$

Since excess of pressure $P = \frac{4\sigma}{r}$ for a soap bubble

$$P = \frac{4 \times 1.6}{0.002}$$

or $$P = 3200 \text{ Nm}^{-2}$$

Example 6.3 :

A glass tube of 2 mm bore is dipped vertically into a container of mercury upto a certain depth. What should be the depth of glass tube in mercury, in order to blow off a hemispherical bubble at its lower end if the gauge pressure is 80 cm of mercury. ($\sigma = 0.49 \text{ Nm}^{-1}$ and $g = 9.8 \text{ ms}^{-2}$).

Solution :

Let x be the depth of the glass tube in mercury, P be the gauge pressure, r be the radius of the bubble, ρ be the density of mercury and σ be the surface tension. Then

$$P=\frac{2\sigma}{r}+x\rho g$$

$$\therefore \quad 0.80\times 13600\times 9.8=\frac{2\times 0.49}{001}+x\times 13600\times 9.8$$

or $$x=0.79 \text{ m}$$

6.12 HEIGHT OF LIQUID IN A CAPILLARY TUBE

(*i*) **Rise on the basis of Pressure difference :**

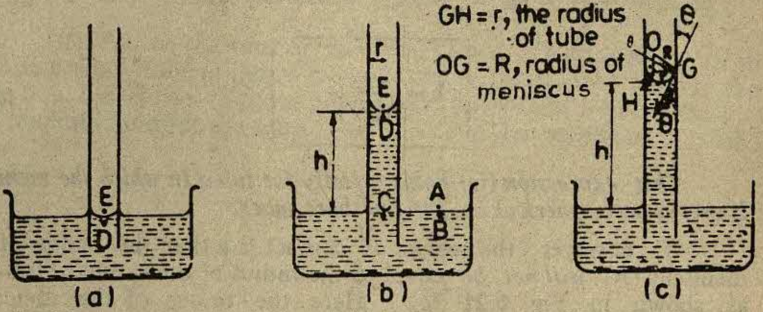


Fig. 6.21

Consider a liquid (say water) of surface tension σ in a capillary tube of radius r. If the meniscus is everywhere spherical (as it will be in a very narrow tube), then the radius of the meniscus will be equal to the radius of the tube if the angle of contact is *zero.*

When the tube is first placed in the liquid of density ρ, the curvature of the surface makes downward pressure at D just below the meniscus *less* than that at E, which is atmospheric Fig 6.21 (a). Liquid will flow into the tube because of this pressure difference and capillary rise occurs.

Let P_A, P_B, P_C, P_D and P_E be the pressures at A, B, C, D, and E respectively Fig 6.21 (b). We have, once equilibrium is established,

$$P_A=P_B \qquad \text{.. }(i)$$

(*i.e.* on pressure difference across a flat surface).

$$=P_C$$

or $$\boxed{P_A=P_D+h\rho g} \qquad \text{...}(ii)$$

where h is the equilibrium capillary rise (neglecting the effect of liquid contained in the meniscus)

Also, excess of pressure $= \boxed{P_E - P_D = \frac{2\sigma}{r}}$ ...(iii)

But $P_E = P_A$ as both are atmospheric pressure which is constant if we ignore the presseure due to the column AE of air.

From (ii) and (iii)

$$h\rho g = \frac{2\sigma}{r}$$

$$\boxed{h = \frac{2\sigma}{\rho r g}} \quad ...(iv)$$

This expression (iv) holds strictly for tubes in which the meniscus is everywhere spherical i.e. for fine bore tubes.

If, however, the angle of contact is θ then the radius of the meniscus (R) *will not* be equal to the radius of the capillary tube (r) as shown in Fig 6.21 (*c*), Here the radius of the meniscus $R = r \sec\theta$.

The excess of pressure ($P_E - P_D$) is given by

$$P_E - P_D = \frac{2\sigma}{R} = \frac{2\sigma}{r\sec\theta} = \frac{2\sigma\cos\theta}{r} \quad ...(v)$$

Therefore from eq. (*ii*) and (*v*)

$$h\rho g = \frac{2\sigma\cos\theta}{r}$$

or $$\boxed{h = \frac{2\sigma\cos\theta}{\rho r g}} \quad ...(vi)$$

$$\text{Surface Tension } (\sigma) = \frac{h\rho r g}{2\cos\theta} \quad ...(vii)$$

The same expressions (vi) & (vii) hold for *capillary depression* (*h*) *i.e.* when the capillary tube is put in mercury.

From (vi)

$$r.h = \frac{2\sigma\cos\theta}{\rho g} = \text{constant}$$

This means that the height upto which a particuler liquid rises in different capillary tubes depends upon the radius of the bore $\left(r \propto \frac{1}{h} \right)$. The liquid rises to a greater height in a narrow tube.

If the effect of liquid contained in the meniscus is taken into account then eq. (vii) takes the form

$$\sigma = \frac{r\,\rho g \left(h + \frac{r}{3} \right)}{2 \cos \theta}$$

because capillary rise will become $h + \frac{\left(\pi r^3 - \frac{2}{3} \pi r^3 \right)}{\pi r^2} = (h + \frac{1}{3} r)$.

(*ii*) *Rise on the basis of Balance of forces* : When one end of the glass capillary tube of radius r dipped into a liquid (say water), then the liquid level rises in the capillary tube to a height h above the level of the liquid outside the tube, Fig (6.22). The meniscus of such a

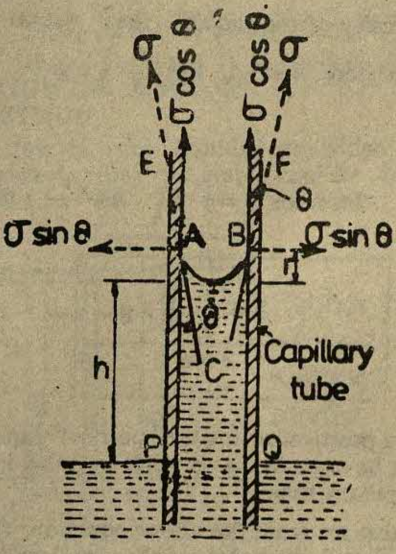


Fig. 6.23

liquid is concave in shape and is almost hemispherical. Then ∠PAC is the angle of contact between the liquid and the material of the capillary tube. The force of surface tension, σ, acts along the tangent to the liquid surface inside the capillary tube (along AC) at every point of contact between the liquid surface and the capillary tube. In this case all the points of contact will lie on the circumference of the circle of diameter AB. This force of surface tension acting on the liquid molecules along the circumference of the circle of diameter

AB gives rise to a reaction force σ, as marked in the diagram, along the circumference of the circle of diameter AB. Resolving this force of reaction into two rectangular components, $\sigma \cos\theta$ and $\sigma \sin\theta$ acting along vertical and horizontal directions respectively. As the reaction acts along the circumference of a circle of diameter AB, so (*i*) total upward force is $(2\pi\ r \times \sigma\ \cos\ \theta)$ and this balances the downward weight of the liquid column of height h inside the capillary tube ; and (*ii*) the horizontal components $\sigma \sin\theta$ cancel each other.

$2\pi\ r \times \sigma \cos\theta$ = weight of the liquid column of height h. Volume of the liquid column in the capillary tube above the liquid level= (Volume of the cylinder of height h and radius r)+ (Volume of the cylinder of height r and radius r)−(Volume of the hemisphere of radius r).

$$=\pi r^2 h+\pi r^2 r-\frac{2}{3}\pi r^3$$

$$=\pi r^2\left(h+\frac{r}{3}\right)$$

If ρ be the density of the liquid, then weight of the liquid column supported $=\pi r^2\left(h+\frac{r}{3}\right)\rho g$

In equilibrium,

$$2\pi r\sigma\cos\theta=\pi r^2\left(h+\frac{r}{3}\right)\rho g$$

$$\boxed{\sigma=r\frac{\left(h+\frac{r}{3}\right)\rho g}{2\cos\theta}}$$

As σ is always positive. so for the pair of liquid and solid if $\theta > 90°$ then h must be *negative i.e.* in such case the liquid would be depressed in the capillary tube.

Further, we also see from the above equation that for a pair of liquid and solid, if the diameter of the capillary tube is decreased the height of liquid (h) in it would increase and vice versa.

For pure water and glass $\theta = 0$

$$\boxed{\sigma=r\frac{\left(h+\frac{r}{3}\right)\rho g}{2}}$$

6.13 CAPILLARY RISE IN A TUBE OF INSUFFICIENT LENGTH :

As derived before

$$\sigma=\frac{h\rho gr}{2\cos\theta}=\frac{h\rho g\,(R\cos\theta)}{2\cos\theta}$$
$$=\frac{h\rho gR}{2}$$

where R is the radius of the meniscus and h is the height of the liquid column in the tube.

$$hR=\frac{2\sigma}{\rho g}=\text{constant}$$

Now, with the tube sufficiently longer than h, it is the value of h alone that changes to satisfy the above relation for ρ. But, if the tube be smaller than the calculated value of h, the only variable in the above relation is R. because now $h=l$. the length of the tube (a constant) and so is θ a constant for the given liquid and the tube. The liquid thus just spreads over the walls of the tube at the top and its meniscus requires a new radius of curvature R_1 so that $R_1 l=\frac{2\sigma}{\rho g}$ or $R_1 l=R.h=$ a constant $(R_1>R)$

Since $h=\frac{2\sigma}{\rho gr\sec\theta}$ where $r=$ radius of capillary, if the height is less than h the angle of contact or (θ) will increase.

6.14 DETERMINATION OF SURFACE TENSION BY CAPILLARY RISE METHOD :

The surface tension of such liquids for which the angle of contact with glass is acute *i.e.* they wet glass, can be determined by this capillary rise method.

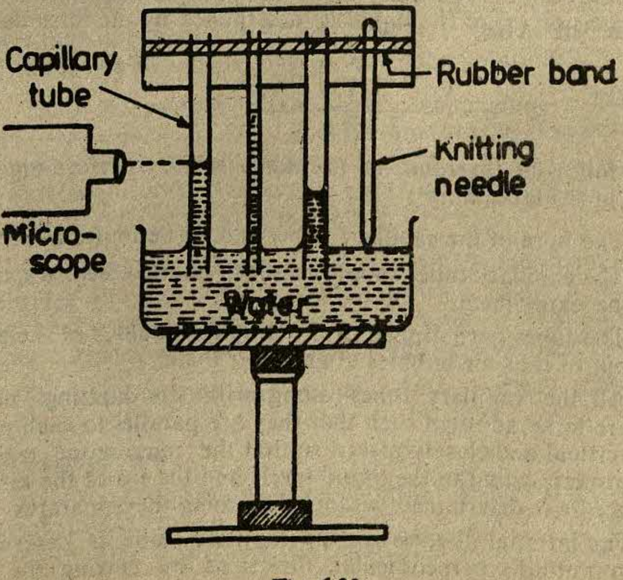


Fig. 6.23

Uniform capillary tubes of different bores are taken and washed first with caustic soda and then with tap water. These tubes are then dried. Three such capillary tubes along with a knitting needle are fixed parallel to one another on a mirror strip by means of a rubber band. The lower end of the knitting needle is slightly higher than the lower ends of the capillary tubes. The liquid is taken in a glass vessel and the mirror strip attached with the capillary tubes is so held in a clamp stand, that

(*i*) the lower end of the capillary tubes dip into the liquid,

(*ii*) the Capillary tubes and knitting needle are vertical, and

(*iii*) the lower tip of the knitting needle just touches the liquid surface.

Then with the help of a travelling microscope find the readings of the horizontal part of the concave meniscus of the liquid in the different tubes. Then the glass vessel containing the liquid is removed gently without disturbing the rest of the apparatus, and the reading of the microscope corresponding to the lower tip of knitting needle is noted. Subtracting this from the readings for the meniscus of various capillary tubes we get the height of the liquid column, (h), in various capillary tubes.

The capillary tubes are cut perpendicular to the length at the position of the liquid meniscus. The internal diameter along two mutually perpendicular position is measured with the help of the travelling microscope. The mean diameter and hence radius (r) of the capillary is then determined. The value of the density (ρ) of the liquid at the room temperature and the angle of contact (θ) between glass and the liquid is determined from the table of constants. Then the surface tension (σ) of the liquid is determined by the formula

$$\sigma = \frac{\left(h+\frac{r}{3} \right) r\rho g}{2 \cos \theta}$$

The following *precautions* are taken while performing this experiment in the laboratory :

(1) The bore of the capillary tubes should be uniform.

(2) The capillary tubes should be clean and dry before starting the experiment.

(3) The tubes once dipped in the liquid should not be taken out of it as air bubbles might enter them.

(4) All the capillary tubes along with the knitting needle are to be adjusted such that they are parallel to each other, vertical and closely placed so that the microscope readings corresponding to the liquid levels and the tip of the knitting needle is determined without disturbing the apparatus.

(5) The internal diameter of the capillary tube is determined in mutually perpendicular directions by cutting the tube perpendicular to its length at the place upto which the liquid has risen.

(6) Density of the liquid be noted corresponding to the room temperature.

6.15. EFFECT OF CONTAMINATING A LIQUID SURFACE

(*i*) Place two matches parallel to each other and 3 cm or 4 cm apart on the surface of water in a clean glass dish. Allow a drop of soap solution or alcohol to fall on the water between them. The matches spring apart.

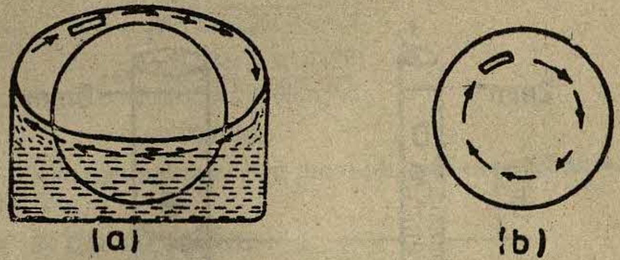


Fig. 6.24

(*ii*) Rub one end of the match stick with soap. Now place it on the surface of the water contained in a fairly wide trough and watch what happens to it. You will find that the match stick starts moving immediately in *one direction only* and that is in such a way that the end that is not rubbed with soap is always in front (Fig 6.24) Any attempt to make it move in the opposite direction will fail. The reason is that the soap at the end of the stick immediately dissolves in water, thereby lowering the surface tension at that end of the match stick. The surface tension at the other end being now greater pulls the match stick and makes it move in that direction until the whole surface is covered with soap solution.

These observations show that *contaminating a liquid surface reduces the surface tension.* For the same reason if some small pieces of camphor are scattered on to the surface of water in a clean dish, they dart about on the surface in a very lively manner.

6.16 EFFECT OF TEMPERATURE

Sprinkle some flowers of sulphur and lycopodium powder on the surface of water in a flat metal dish. With a small bunsen flame heat one side of the dish and observe what happens. The floating particles are swept away from the heated portion.

This experiment indicates that *surface tension diminishes with rise in temperature.*

Example 6·4 :

A U-tube with limbs of diameters 5.0 mm and 2.0 mm contains water of surface tension 7.0×10^{-2} Nm^{-1}, angle of contact zero and density 10^3 kgm^{-3}. Find the difference in levels (g = 9.8 ms^{-2}).

Solution :

If the menisci are spherical they will be hemispheres since the angle of contact is zero ; their radii will then equal the radii of the limbs. The pressure on the concave side of each surface exceeds that on the convex side by $\frac{2\sigma}{r}$ when σ is the surface tension and r is the radius of the limb concerned. Let ρ be the density of the liquid.

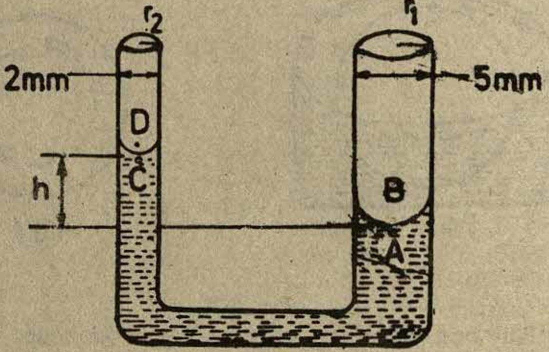


Fig. 6.25

Now $r_1 = 2.5\text{mm} = 2.5 \times 10^{-3}\text{m}$

and $r_2 = 1\text{mm} = 1.0 \times 10^{-3}\text{m}$

Hence from fig. (6·25) it is clear that

$$P_B - P_A = \frac{2 \times 7.0 \times 10^{-2}}{2.5 \times 10^{-3}} = 56 \text{ pascal}$$

$P_A = P - 56$ where $P_B = P =$ atmospheric pressure

Also $P_D - P_C = \frac{2 \times 7.0 \times 10^{-2}}{1.0 \times 10^{-3}} = 140$ pascal

$\therefore P_C = P - 140$ $(\because P_D = P)$

$$P_A - P_C = (P - 56) - (P - 140)$$
$$= 84 \text{ pascal}$$

But $P_A = P_C + h\rho g$ where h is the difference in levels

$h\rho g = P_A - P_C = 84$

or $h = \frac{84}{\rho g} = \frac{84}{10^3 \times 9.8}$

or $h = 8.4\text{mm}$

Example 6.3 :

A mercury barometer tube is 3mm in diameter. Calculate the error in reading the barometer due to surface tension. Angle o

contact of mercury with glass = 130° ; surface tension of mercury is $0.49 Nm^{-1}$; density of mercury = $13600 \ kgm^{-3}$ and $g = 9.8 \ ms^{-2}$.

Solution :

Here radius of the tube $= r = 1.5 \ mm = .0015 \ m$

$$h = \frac{2\sigma \cos \theta}{r \rho g}$$

$$= \frac{2 \times 0.49 \times \cos 130^\circ}{0.0015 \times 13600 \times 9.8}$$

$$= \frac{2 \times 0.49 \times (-0.6428)}{0.0015 \times 13600 \times 9.8}$$

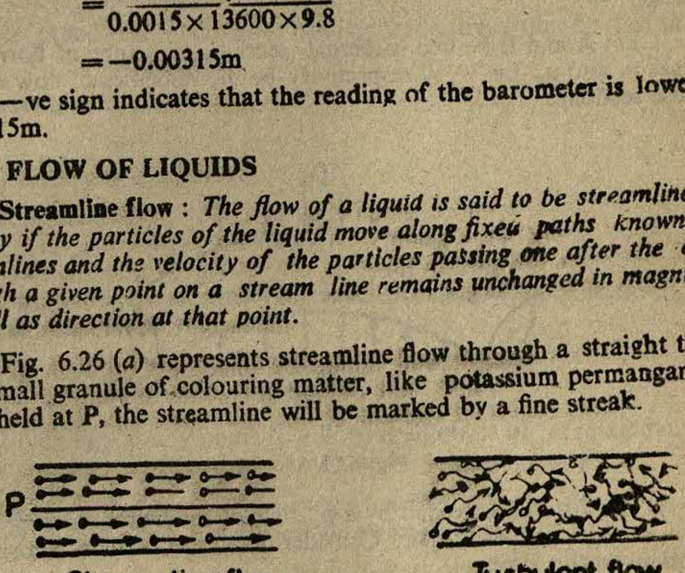
$$= -0.00315 m$$

—ve sign indicates that the reading of the barometer is lower by 0.00315m.

6.17 FLOW OF LIQUIDS

Streamline flow : *The flow of a liquid is said to be streamlined or orderly if the particles of the liquid move along fixed paths known as streamlines and the velocity of the particles passing one after the other through a given point on a stream line remains unchanged in magnitude as well as direction at that point.*

Fig. 6.26 (*a*) represents streamline flow through a straight tube. If a small granule of colouring matter, like potassium permanganate, were held at P, the streamline will be marked by a fine streak.



Stream line flow
(a)

Turbulent flow
(b)

Fig. 6.26

Fig. 6.26 (*b*) represents *turbulent* flow of a liquid through a straight tube. A colouring granule placed at P in this case will colour uniformily a considerable portion of the water flowing along the tube. In this case eddies are formed in the liquid and the velocity of a water particle passing through a point is not the same as of the particle which preceded it

A streamline in general follows the shape of the tube through which the liquid flows. Thus it may be straight or curved and whatever the shape it remains fixed in position.

A *tube of flow* consists of a number of streamlines forming a tube. The liquid in one tube flows quite independently of the liquid flowing in a neighbouring tube.

The necessary condition for streamline flow is that the velocity of flow should be less than a certain *critical velocity* depending upon the nature of the liquid and radius of the tube. The more viscous a liquid and smaller the radius, the higher is the critical velocity.

In the case of a straight streamline no force acts upon it in a direction perpendicular to the direction of flow. In a curved tube of flow, the change in direction is due to resultant force acting on it towards the centre of curvature (centripetal force). It means that the normal pressure, P_1 acting on the convex boundary is greater than that, (P_2) acting on the concave boundary fig (6.27).

The velocity of streamline flow in a tube of varying cross section :—

Let A and B be two different sections of a tube of flow having areas a_1 and a_2. Let v_1, v_2 respectively be the velocities of flow through these sections.

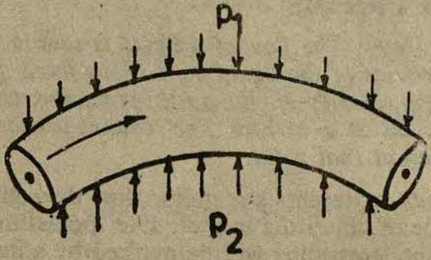


Fig. 6.27

The distance travelled by a particle of the liquid crossing section A in a small time δt is $v_1 \delta t$. Considering all the particles crossing the sections in this time.

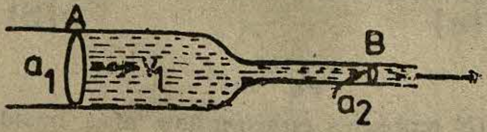


Fig. 6.28

Volume of liquid crossing

A$=a_1 v_1 \delta t$.

If ρ_1 is the density of the liquid at A, mass of liquid crossing A

$=\rho_1 a_1 v_1 \delta t$

Similarly, in time δt

mass of liquid crossing B$=\rho_2 a_2 v_2 \delta t$

where ρ_2=density of the liquid at B

By the law of conservation of matter, since there is no accumulation of mass anywhere in the tube.

$$\rho_1\, a_1\, v_1\, \delta t = \rho_2\, a_2\, v_2\, \delta t$$

$$\rho_1\, a_1\, v_1 = \rho_2\, a_2\, v_2$$

For a liquid, since it is almost incompressible.

$$\rho_1 = \rho_2$$

$$a_1\, v_1 = a_2 v_2$$

$$\text{or} \quad \boxed{av = \text{constant}}$$

This is called the *equation of continuity* for streamline flow of a liquid.

we have

$$v_2 = v_1\, \frac{a_1}{a_2}$$

If $a_2 < a_1$, evidently

$$v_2 > v_1$$

Thus the velocity of flow increases as the channel becomes narrower and vice versa.

6.18 TOTAL ENERGY OF A LIQUID

The total energy at any point in a flowing liquid is of three kinds: (a) ***Pressure Energy*** : (*b*) *Potential Energy* and (*c*) *Kinetic Energy*

(a) **Pressure Energy** : *The energy possessed by a liquid by virtue of the pressure acting on it is called pressure energy.*

By virtue of the pressure experienced by a liquid it is able to do work, say in driving a piston P against an external resistance. Let 'a' be the area of cross section of the piston and let δx be the distance moved by it.

Force acting on the piston $= pa$, where p is the pressure on the piston transmitted by the liquid.

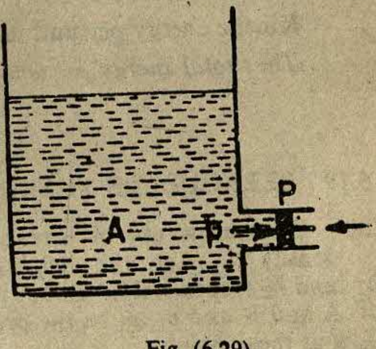


Fig (6.29)

work done by $pa = pa.\ \delta x$

But $a\,\delta x =$ volume swept by the piston

$$= \delta v$$

$$\text{work done} = p.\ \delta v$$

By the law of conservation of energy, this work is equivalent to the energy contained in volume δv of the liquid on account of its pressure, *i.e.* its pressure energy.

$$\text{pressure energy in volume } \delta v = p.\,\delta v$$

$$\text{pressure energy per unit volume} = \frac{p.\delta v}{\delta v}$$

$$= p$$

It is usual to express pressure energy for unit mass of a liquid.

Let ρ be the density of the liquid.

$$\text{Then volume of unit mass} = \frac{\text{mass}}{\text{density}} = \frac{1}{\rho}$$

$$\text{pressure energy per unit mass} = \frac{p}{\rho}$$

(*b*) **Potential energy :** *The energy possessed by a liquid by virtue of its elevation above some arbitrary datum level is called potential energy.*

When a liquid of mass m is raised through a height h work done on it, against gravity is mgh. This is stored in the liquid as potential energy of the liquid.

Potential energy of mass m of the liquid $= mgh$.

Potential energy per unit mass $= gh$.

(*c*) **Kinetic Energy :** *The energy possessed by a liquid by virtue of its motion is called kinetic energy.*

Kinetic energy for mass $m = \frac{1}{2} mv^2$.

where v is the velocity of flow.

Kinetic energy per unit mass $= \frac{1}{2}v^2$

Thus total energy per unit mass at any point of a flowing liquid

$$= \frac{p}{\rho} + gh + \frac{v^2}{2}$$

6.19 BERNOULLI'S THEOREM AND ITS APPLICATIONS

Consider a tube of streamline flow AB, Fig (6.30), of a liquid density ρ. Let the heights of A and B above ground level be h_1, and h_2 respectively. Let a_1 and a_2 be the areas of cross section of A and B and p_1, p_2 be the pressures and v_1, v_2 be the velocities of flow at these points.

Let δm_1 be the mass of liquid crossing the section A in time δt.

$\therefore \ \delta m_1 = \rho a_1 v_1 \, \delta t$.

Let δm_2 be the mass of liquid crossing the section B in the same time δt.

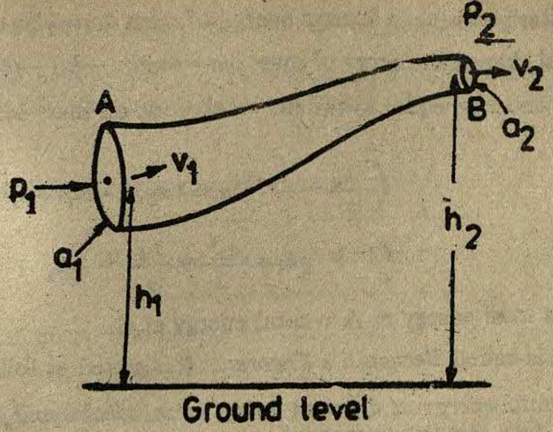


Fig. 6.30

$\therefore\ \delta m_2 = \rho a_2 v_2\ \delta t.$

As no accumulation of mass takes place anvwhere,

$\delta m_1 = \delta m_2 = \delta m$

$$a_1 v_1\ \delta t = \frac{\delta m}{\rho} = a_2 v_2\ \delta t$$

The force acting on section A $= a_1 p_1$.

Work done by force $a_1\ p_1$ in driving mass δm across section A

$$= \text{force} \times \text{distance}$$
$$= a_1 p \times v_1 .\ \delta t$$
$$= p_1 \cdot \frac{\delta m}{\rho}.$$

Work done by liquid in driving a mass δm through section B against the pressure p_2 or force $a_2\ p_2$

$$= a_2 p_2 v_2 . \delta t$$
$$= p_2 \frac{\delta m}{\rho}$$

Net work done by the pressure forces in *driving mass* δm

$$= p_1 \frac{\delta m}{\rho} - p_2 \frac{\delta m}{\rho}$$
$$= \left(\frac{p_1}{\rho} - \frac{p_2}{\rho} \right) \delta m.$$

Now potential energy of mass δm at A $= \delta\ ngh$.

and potential energy of mass δm at B $= \delta mgh_2$

$\therefore$ change in potential energy $= \delta mg(h_1 - h_2)$.

Similarly change in kinetic energy of mass $\delta m = \frac{1}{2}\delta m\,(v_1^2 - v_2^2)$

Total change in energy of mass $\delta m = \delta mg(h_1 - h_1) + \frac{1}{2}\delta m(v_1^2 - v_2^2)$.

This must be equal to the net work done on mass δm as found already.

i.e.
$$\left(\frac{p_1}{\rho} - \frac{p_2}{\rho}\right)\delta m = \delta mg(h_1 - h_2) + \frac{1}{2}\delta m(v_1^2 - V_2^2)$$

$$\frac{p_1}{\rho} + gh_1 + \frac{v_1^2}{2} = \frac{p_2}{\rho} + gh_2 + \frac{v_2^2}{2}.$$

Thus total energy at A = total energy at B.

This is called *Bernoulli's Theorem.* It is stated as follows :

The total energy at different points in an ideal liquid, undergoing streamline flow, remains unchanged.

Note : In an ideal liquid there is supposed to be no viscous drag between the neighbouring tubes of flow nor is there any friction offered by the walls of the tube on the liquid When these conditions are satisfied, all the particles of the liquid in a section of the tube of flow possess identical velocities (In actual practice the velocity of flow is zero at the walls of the containing tube and maximum at the centre on account of the viscous forces and friction)

According to Bernuolli's Theorem,

$$\frac{p}{\rho} + gh + \frac{v^2}{2} = \text{constant.}$$

Dividing throughout by g we have

$$\frac{p}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$$

All the terms in this relation have dimensions of length shown below :

$$\frac{p}{\rho g} = \frac{ML^{-1}T^{-2}}{ML^{-3}LT^{-2}} = L$$

$$\frac{v^2}{2g} = \frac{L^2T^{-2}}{LT^{-2}} = L$$

$$h = L$$

$\frac{p}{\rho g}$ is called *pressure head* ;

h is called *gravitational head*, and

$\frac{v^2}{2g}$ **is called *velocity head.***

Thus *Bernoulli's Theorem* can also be stated as follows :

At all points along a streamline of an ideal liquid the sum of pressure head, the gravitational head and the velocity head remains constant.

Bernoulli's Theorem can also be written as

$$\boxed{p+\rho gh+\tfrac{1}{2}\rho v^2=\text{constant}}$$

Which gives total energy per unit volume

p is called *static pressure*, and

$\frac{1}{2}\rho v^2$ is called *dynamic pressure.*

Torricelli's Theorem: The *velocity of efflux* of a liquid through an orifice is equal to the velocity attained by a body in falling freely from the free surface to the orifice. This result was first obtained by Torricelli in 1644 and is known as *Torricelli's Theorem after his name.*

Proof : The proof of Torricelli's theorem is easily afforded by Bernoulli's theorem as shown below :

Let the free surface, A and the orifice, B, be at heights h_1 and h_2 from ground level, Fig (6.31). Let p be the atmospheric pressure and let ρ be the density of the liquid.

Let $h_2 - h_1 = h$ = the height of A above B.

When the liquid flows out through B let its velocity be v. As the area of the free surface of the liquid in the vessel is very large as compared to the area of the orifice, the velocity of the liquid at A is negligible and can be regarded as zero.

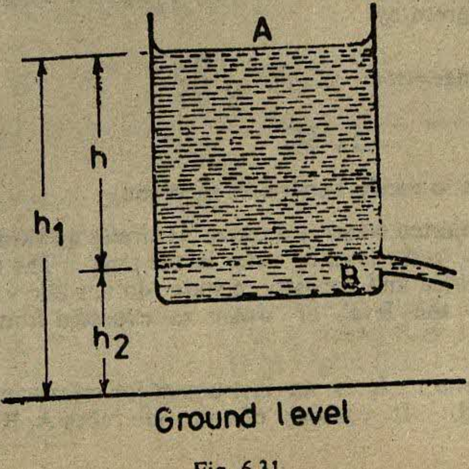


Fig. 6.31

Applying Bernoulli's theorem to points A and B, we have total energy per unit mass *at* A=total energy per unit mass at B.

$$\frac{p}{\rho}+gh_1+0=\frac{p}{\rho}+gh_2+\tfrac{1}{2}v^2,$$

-taking the pressure at the orifice to be also atmospheric.

$$\therefore gh_1=gh_2+\tfrac{1}{2}v^2.$$

Thus, whereas the potential energy at the orifice is less than that at A' the kinetic energy is correspondingly increased.

$$\tfrac{1}{2}v^2=g(h_1-h_2).$$
$$=gh.$$
$$v=\sqrt{2gh}.$$

In the case of a body let fall freely from level A to level B, the height traversed $=h_1-h_2=h$. The velocity, v', at B is given by

$$v'=2gh.$$
$$v'=\sqrt{2gh}=v.$$

Thus the velocity of efflux is the same as if the particles of the liquid had fallen freely through height h and then ejected through the orifice.

Verification : (1) Let us suppose the tank contains water. The path of the ejected water will be a parabola. As h_2 is the height of the orifice from ground level, the time taken by the water to fall to the ground will be given by

$$h_2=\tfrac{1}{2}gt^2$$

or
$$t=\sqrt{\frac{2h_2}{g}}.$$

The horizontal distance travelled by ejected water in this time, *i.e.* its range is given by

$$\text{Range}=vt=\sqrt{2gh}.\sqrt{\frac{2h_2}{g}}$$
$$=2\sqrt{hh_2}$$

which enables us to verify Torricelli's theorem.

(2) If the ejected water is directed vertically upwards by a jet, it will rise nearly to the level of water in the tank. The resistance of the air and orifice and viscosity of water do not allow the water to rise exactly to the level of water as expected from Torricelli's theorem.

Venturi Meter : It is an instrument for measuring the rate of flow of a liquid. It consists of two wide tubes A, B joined by a narrow tube C known as the throat. A and C are fitted with manometer tubes as shown in Fig (6.32). The apparatus is interposed horizontally in the pipe in which the flow is to be measured so that the potential energy of the liquid may be the same at all points along the axis of the venturimeter.

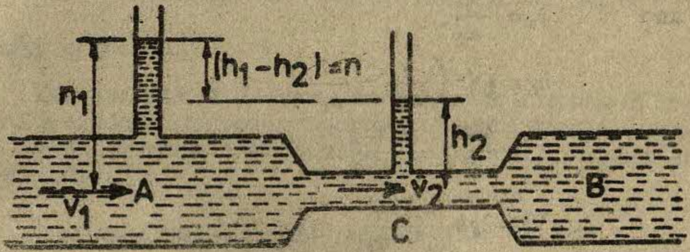


Fig. 6.32

Let a_1, a_2 be the areas of cross-section of tubes A and C and let v_1, v_2 be the velocities of flow of the liquid through these tubes. If V be the volume of the liquid flowing through any section in one second, then

$$V = a_1 v_1 = a_2 v_2$$

$$v_2 = v_1 \frac{a_1}{a_2}$$

As $a_2 < a_1$,

∴ $v_2 > v_1$,

Thus the velocity of flow through C is much greater than that through A, as the throat is quite narrow.

If p_1 and p_2 be the pressures in A and C and ρ be the density of the liquid, then be Bernoulli's theorem,

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} + gh' = \frac{p_2}{\rho} + \frac{v_2^2}{2} + gh'$$

where h' is the height of the axis of the venturimeter above ground level.

$$p_1 - p_2 = \tfrac{1}{2}(v_2^2 - v_1^2)\,\rho$$

As $v_2 < v_1$

∴ $p_1 < p_2$

The manometers connected to the tubes A and C indicate the pressures, p_1 and p_2 respectively. As p_1 is greater than p_2, the level of liquid in the manometer of A will be higher than that of C. Let the difference in levels be h. Then

$$p_1 - p_2 = h\rho g$$

$$g\rho h = \tfrac{1}{2}\,\rho(v_2^2 - v_1^2)$$

or $$hg = \tfrac{1}{2}\,(v_2^2 - v_1^2).$$

But $$v_1 = \frac{V}{a_1}$$

and $$v_2 = \frac{V}{a_2}.$$

$$hg = \tfrac{1}{2}V^2\left(\frac{1}{a_2^2} - \frac{1}{a_1^2}\right)$$

$$= \frac{V^2}{2}\;\frac{a_1^2 - a_2^2}{a_1^2\,a_2^2}$$

$$\therefore\quad V^2 = 2gh\,\frac{a_1^2\,a_2^2}{a_1^2 - a_2^2}.$$

or $$V = \frac{a_1\,a_2}{\sqrt{a_1^2 - a_2^2}}\cdot\sqrt{2g}\;\;\sqrt{h}$$

$$= k\sqrt{h}.$$

where $$k = a_1 a_2.\;\sqrt{\frac{2g}{a_1^2 - a_2^2}} = \text{constant}.$$

Thus $V \propto \sqrt{h}$

Knowing a_1, a_2 and g, the constant k for the meter may be calculated once for all and hence V can be calculated for any value of h.

Pitot's Tube: It is an arrangement for measuring the velocity of flow in a liquid. Its working is also based on Bernoulli's theorem. It

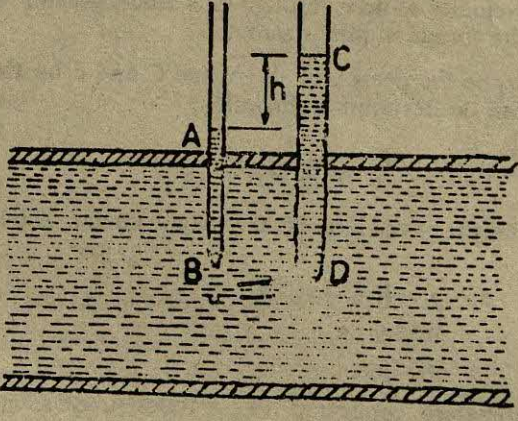


Fig. 6.33

consists of two vertical tubes AB and CD, Fig (6.33) with small apertures at their lower ends dipping in the liquid flowing in the pipe.

The aperture of the AB is parallel to the direction of flow of the liquid and that of CD is normal to it. If v be the velocity of flow

Kinetic energy per unit mass at $B=\frac{v^2}{2}$

Pressure energy per unit mass at $B=\frac{p}{\rho}$, say where ρ is the density of the liquid.

The aperture at D being normal to the direction of flow, the liquid is suddenly stopped in its plane and hence kinetic energy per unit mass at aperture D will be zero. Let p_2 be the pressure at this point. Then

pressure energy per unit mass at $D=\frac{p_2}{\rho}$

As potential energy per unit mass remains unchanged for horizontal flow, we have by **Bernoulli's theorem**, for a streamline through B and D,

$$\frac{P_1}{\rho}+\frac{v^2}{2}=\frac{P_2}{\rho}+0$$

$$v^2=2\,\frac{p_2-p_1}{\rho}$$

p_2 being greater than p_1, the level of liquid in the tube CD will be higher than that in AB. Let h be the difference in levels.

$$p_2-p_1=h\rho g$$

$$v^2=2\,.\,\frac{h\rho g}{\rho}=2gh$$

$$\therefore\ v=\sqrt{2gh}$$

Filter Pump or Aspirator: It is a simple device for reducing the pressure in a vessel and works on Bernoulli's principle according to which pressure decreases if the velocity of flow increases.

The apparatus consists of a wide tube A. Fig. (6.34), carrying a tapering tube B at the top. The tube B ends into a jet. A tube C with its cross section increasing downwards is fitted to the lower end of B and a tube T is fitted to the side. The tube T is connected to the vessel to be exhausted and water is poured into the tube B from a top. As the area of cross section at the inner end of B is very small, water issues out of it with a very high velocity and hence at a very low pressure. On account of the low pressure at the nozzle of B, air rushes towards it from the vessel through the side tube T and is carried away by the stream of water down the discharge tube C. The pressure in the vessel can thus be made to fall to about 2 cm of mercury in a short time.

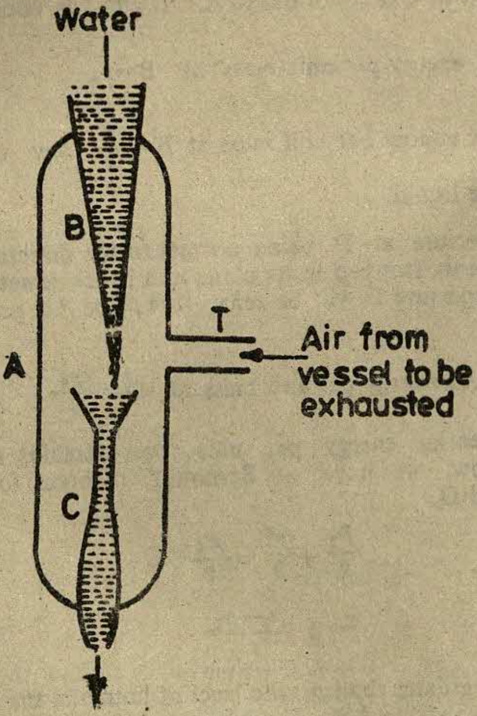


Fig. 6.34

On account of the conical shape of tube C with diameter increasing downwards the velocity of flow of discharged water decreases and therefore the pressure increases and the tendency of the outer air to rush into the tube through C diminishes.

The Atomizer or sprayer : Atomizer, illustrated in Fig (6.35) is yet another application of Bernoulli's principle as applied to air. Air

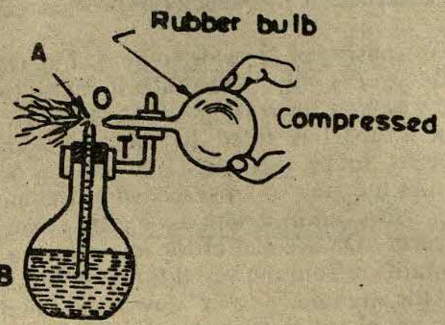


Fig. 6.35

is blown through a jet tube T by compressing a rubber bulb fitted the end of the tube. While rushing out of the narrow aperture O of tube T, air acquires a high velocity and hence the pressure in the vicinity of O is greatly reduced. Since the nozzle O lies directly above the nozzle of a vertical tube A dipping in the liquid in the vessel B the liquid rises up on account of the difference of pressure and issues out of the aperture at the top where it is blown into a fine spray by the air stream issuing out of O.

The ***Bunsen burner***, Fig. (6.36) sucks in air through the air hole on the same principle. As the gas issues through the jet hole inside the air hole a low pressure zone is created in its vicinity and air is sucked inwards.

Further Illustrations of Bernoulli's Theorem :

(1) **Lifting a Disc :** A tube T carries a disc A at its lower end Fig 6.37 (*a*). B is a carboard disc placed below it. On blowing air into the tube a low pressure area is created, by a Bernoulli's principle, between the two discs when air flows out rapidly through the narrow space between the two discs and the external atmospheric pressure pushes the disc B up.

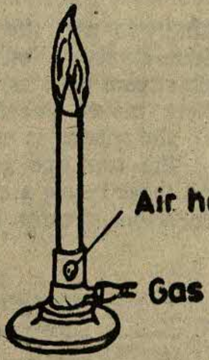
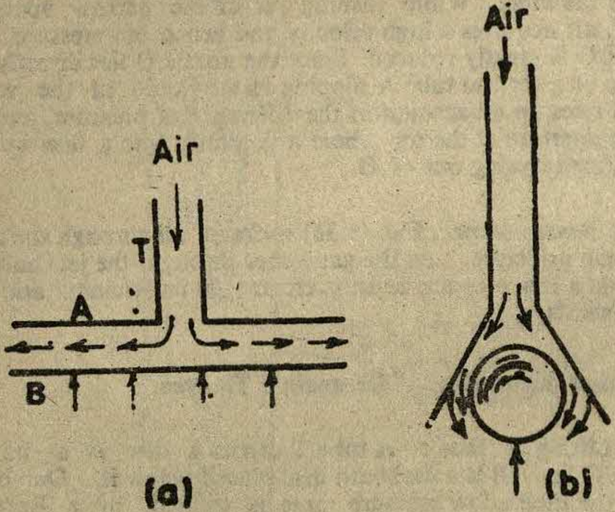


Fig. 6.36

Similarly when two ping pong balls are suspended side by side and air is blown up through the space between the two, the air flows rapidly through the narrow space between the balls and the pressure falls. The atmospheric pressure from the sides brings the balls together.

The arrangement in Fig. (6.37) (*b*) illustrates a similar effect in the case of a ping pong ball in a funnel. When air is blown into the stem, the ball is sucked up by the difference of pressure.

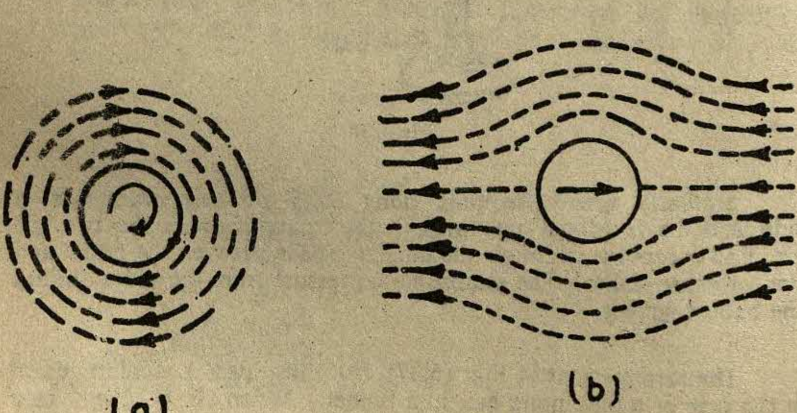


(a) (b)

Fig. 6.37

(2) Magnus Effect :

When a sphere spins rapidly in air, the air surrounding it is also set into rotation in the same direction on account of friction of the sphere, forming concentric stream lines as shown in Fig. 6.38 (*a*). On the other hand if the sphere moves linearly the streamlines are as shown in Fig. 6.38 (*b*). If the sphere is moving linearly as well as rotating the streamlines at the top due to both types of motion are opposed to each other and those below are in the same direction. The velocity of air flow is therefore, greater below than above the



(a) (b)

Fig. 6.38

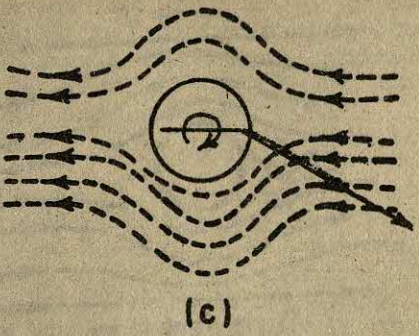
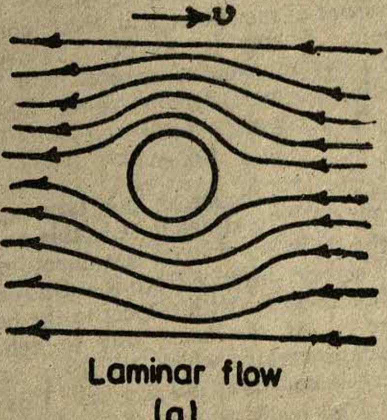


Fig. 6.38

sphere. Since an increase in vleocity is accompanied by a lowering of pressure, the pressure on the lower side will be less than the pressure above and hence a resultant force acts upon the sphere at right angles to linear motion in the downward direction. This acts as a centripetal force and the sphere moves along a curved path. This is called *Magnus Effect*. A cricket ball bowled with an initial spin in the hand, a tennis ball struck out of centre and a pistol bullet show this effect. It is to avoid Magnus effect that for distance shooting the bullets are given cylindrical shape and they are given an initial spin about their axis inside the barrel of the gun by guiding them along a screw thread inside the barrel. In this case, as the streamlines due to rotation are at right angles to the streamlines due to linear flight the velocity of air flow and hence pressure at the sides is uniform and therefore the flight is straight.

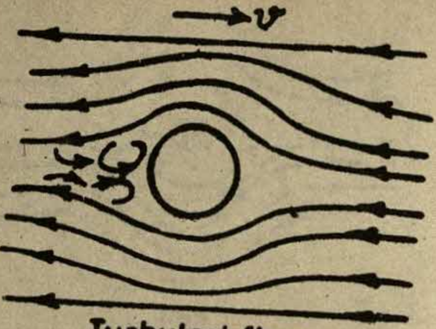
(3) **Streamlining** : When a body moves through a fluid (liquid or air) there is a relative flow of the fluid with respect to the body. For simplicity consider the case of a spherical body moving horizontally through still air, Fig (6.39), with a small velocity *v*.



Laminar flow

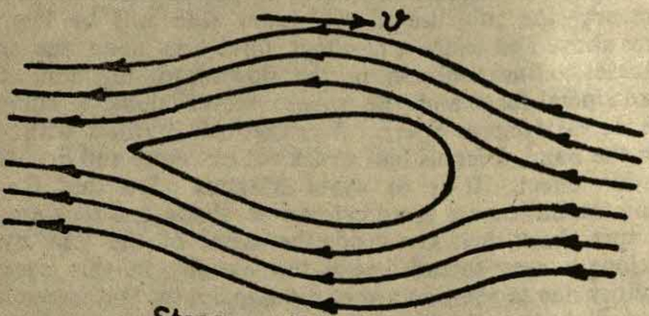
(a)

Fig. 6.39



Turbulent flow

(b)



Stream line flow

(c)

Fig. 6.39

The stream lines of air flow as shown in the figure, divide symmetrically in front of the sphere and then combine immediately behind the sphere. Energy is expanded in overcoming friction of the air and in carrying some of mass of air in front of the sphere. Such a flow is called *'laminar' flow*. The retarding force in this case, is proportional to the velocity. Or

$$f \propto v.$$

At higher values of v, the streamlines take up the shape illustrated in Fig. (6.39) (*b*). The streamlines in this case divide symmetrically in front of the body but they do not meet immediately behind the body but at a certain distance from it. On the front surface the fluid is stopped momentarily (with respect to the body) and therefore, the pressure is great. At the back, due to high velocity of flow, a zone of low pressure is created which tends to suck air into it—a phenomenon which is known as *tail suction*. This tail suction acts in two ways : (1) It gives rise to violent eddies in the fluid when the fluid from the sides rushes into it and (2) it tends to drag the body backwards. The eddies tend to reduce the pressure still further behind the body thus helping to increase the drag. Thus more energy is expended in overcoming the extra forces so produced. This type of fluid flow is called 'turbulent' flow and in this case the retarding force is proportional to the square of velocity. Thus

$$f \propto v^2.$$

By shaping a body to the streamlines of the fluid through which it has to move, Fig. 6.39 (*c*), the retarding force may be considerably reduced. In that case the tendency to form *eddies* or *vortices* is greatly reduced and the body, as it were, slips through the fluid with a minimum disturbance. This is called ***streamlining***. The ***bodies*** of ***airplanes, torpedoes, ships, bombs*** and ***automobiles are stream-lined to avoid wastage of energy in movement through the fluid***, especially if they are to travel at high speeds. The man riding a race-bicycle bends his body forward in order to have a streamlined shape.

Lift on an aircraft wing :

Fig. (6.40) shows the cross section of the wing of an aircraft. When the aircraft moves, the air flows both from under and above

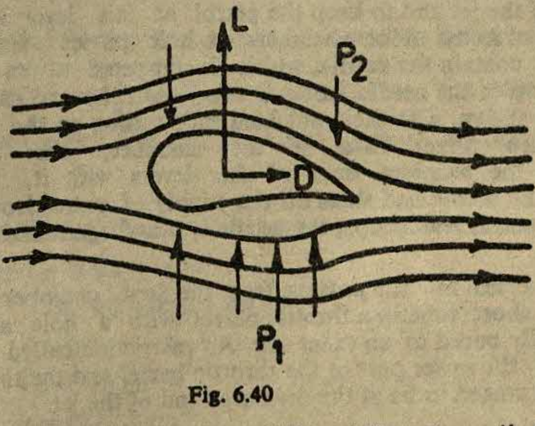


Fig. 6.40

the wing. Aeroplane wings are so designed (*i.e.*, streamlined) that the total distance travelled by air flowing over the wing is *longer* than that of the air flowing under it. Thus, the velocity of air flow above the wing must be higher than the velocity of air flow under the wing. From Bernoulli's Theorem we see that the pressure P_2 above the wing is lower than the corresponding pressure P_1 under the wing. The unbalanced pressure causes a force to act on it. This force can be resolved into two components—the vertical component produces the lift L and the horizontal component provides the drift D.

(4) Carburettor : The function of the carburettor is to deliver to the cylinder of a petrol engine a correctly proportioned mixture of petrol (in vapour form) and air. It is based upon Bernoulli's principle.

Fig. (6.41) shows the principle of a simple type of carburettor, but it must be remembered that the instruments used in practice are much more complicated, especially those in aeroplanes, which are required to function under variable conditions.

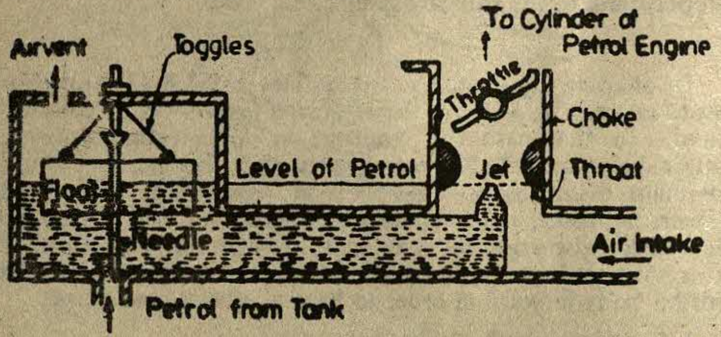


Fig. 6.41

Petrol from the fuel tank enters the float chamber of the carburettor and fills it up to a predetermined level, generally slightly below the level of the jet and to keep the petrol at this level a cylindrical float is fitted in the float chamber. A hole passes axially through the float to contain the needle, which is tappered at its lower end. Near the top of the needle are fixed two small levers, called *toggles* pivoted as shown, with their weighted ends resting on the top of the float. When petrol rises in the chamber, the float rises, forcing up the weighted ends of the levers with it. This action depresses the needle and shuts off the supply of petrol from the tank. When the petrol level drops, the needle is raised again and more petrol enters.

Connected to the passage from the float chamber is the jet, which is a short tube in a throttle barrel with a hole at its upper end carefully bored to an exact size. A constriction called the *choke* is placed in the lower part of the throttle barrel and the throat of the choke is arranged to be at the discharge end of the jet.

When the piston in the petrol engine moves downward, it causes air to enter through the intake to pass through the choke. *Here its velocity is increased by the throat* and hence *its pressure is reduced below the atmospheric pressure* according to *Bernoulli's Theorem.* Since the air above the petrol in the float chamber is at atmospheric pressure, petrol is forced out of the jet. This petrol then meets the fast moving air stream which breaks it up into very fine particles. Thus a uniform mixture of petrol vapour and air passes up the throttle chamber into the engine cylinder. The quantity of mixture passing is controlled by the throttle valve.

Example 6.6 :

A garden sprinkler has 150 small holes each 2.0 mm^2 in area. If water is supplied at the rate of $3{\cdot}0\times10^{-3}\ m^3\ s^{-1}$, what is the average velocity of the spray ?

Solution.

Volume of water per second

from sprinkler = Volume supplied per second

$=3\times10^{-3}m^3s^{-1}$

= total area of sprinkler holes × Average Velocity of spray

$=300\times10^{-6}\times v$

Therefore average velocity

$$\text{of spray } (v)=\frac{3\times10^{-3}}{300\times10^{-6}}$$

$$=10ms^{-1}$$

Example 6.7

Water flows horizontally through a pipe line of varying cross-section. If the pressure of water equals 0·05 *m* of mercury at a point, where the velocity of flow is 28 cms^{-1}, what is the pressure at another point, where the velocity of flow is 0·7 ms^{-1}

Solution.

For a liquid flowing horizontally, Bernoulli's Theorem gives

$\frac{P_1}{\rho}+\frac{1}{2}v_1^2=\frac{P_2}{\rho}+\frac{1}{2}v_2^2$ where P_1, v_1 are the pressure and velocity of the liquid at one point and P_2, v_2 at the other point.

Here

$P_1=5$ cm of Hg $=0\cdot05\times13600\times9\cdot8\ Nm^{-2}$

$P_2=\ ?=h$ (say) *m* of Hg

$=h\times13600\times9\cdot8Nm^{-2}$

$v_1=0\cdot28\ ms^{-1}$

$v_2=0\cdot70\ ms^{-1}$

$\rho=1000\ kgm^{-3}$

$$\frac{0\cdot05\times13600\times9\cdot8}{1000}+\frac{1}{2}(0\cdot28)^2$$

$$=\frac{h\times13600\times9\cdot8}{1000}+\frac{1}{2}(0\cdot70)^2$$

$\therefore h=0\cdot04845$ *m* of Hg.

6.20 LAMINAR FLOW AND VISCOSITY

If a liquid is at rest the pressure is the same at all points in the same level. This is not true when the liquid is in motion. When water flows in a uniform horizontal tube there is a fall in pressure along the tube in the direction of flow. The reason for this fall in pressure is that force is required to maintain motion against friction. This friction is nothing but viscous forces of the liquid.

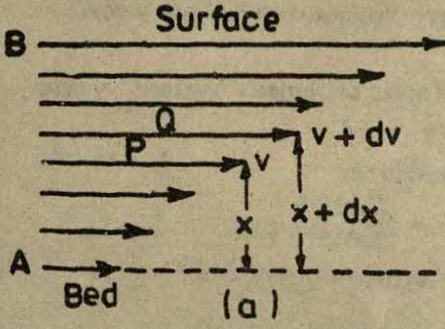


Fig. 6.42

Consider a liquid flowing over a horizontal surface in such a manner that a steady and streamline motion occurs. The velocity of the layer in contact with the surface (bottom) is theoretically zero and as the distance increases towards the upper surface the velocity goes on increasing as shown in Fig. (6.42). The velocity of a layer is thus proportional to its distance from the stationary layer. Such a flow in which a liquid moves in layers is known as *Laminar flow.* In such a flow, one layer of the liquid glides over the other layer of the liquid. Imagine a horizontal plane inside the liquid. The layer just above the plane is moving faster than the layer of the liquid just below the plane. The force acting on the upper layer tends to accelerate the motion of the lower layer, whereas the force acting on the lower layer tends to retard the motion of the upper layer. Thus there are two types of forces which act on the layers in opposite directions and consequently the layer of the liquid is subjected to somewhat of a shearing stress. But according to the definition of liquid it cannot be subjected to shearing stress so in place of shearing strain produced the liquid immediately flows and thus the shearing stress disappears then and there. But force acting on each adjacent layer tends to destroy the relative motion between the layers. This force is called the *dragging force* or the *viscous force.* Since force acts between two layers of the same substance it is also termed as internal friction. Therefore in order to maintain a relative motion between its different layers a force is applied in the direction of flow of liquid to overcome the internal friction or the force of viscosity. This dragging force is the characteristic of a particular liquid. "*So the property by virtue of which a liquid opposes relative motion between its different layers is called viscosity.*"

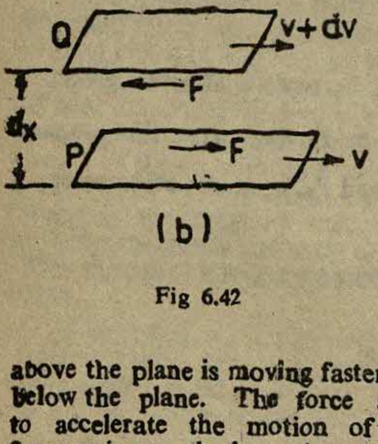


Fig 6.42

All gases and liquids possess the property of viscosity. Gases offer a smaller resistance than liquids do and possess a smaller viscosity. In other words they are more *mobile*. A perfect fluid must be perfectly mobile *i.e.*, it should have no viscosity. There is no such fluid.

Since the internal friction developed in the relative motion depends upon viscosity, this property is utilized in selecting lubricating materials for various machineries. Heavy machineries require high viscous oil whilst light machineries require low viscous oil, for example, a cycle requires oil of higher viscosity for lubrication as compared to a watch.

The quality of foundation pen ink depends largely on its viscosity. The normal circulation of blood through our arteries and veins depends on the viscosity of blood. The shape of the ship or the car is streamlined to minimise the effects of force of viscosity. Thus viscosity plays an important role in our daily life.

Co-efficient of Viscosity

Let the velocities of the layers P and Q situated at x and $x+dx$ in Fig. 6.42 from A be v and $v+dv$ respectively and the two layers dx distance apart are having a velocity gradient equal to $\frac{dv}{dx}$

According to Newton's law the viscous force is proportional to

(*i*) Area A of the layer on which it acts

(*ii*) Velocity gradient $\frac{dv}{dx}$

i.e., $$F \propto A\frac{dv}{dx}$$

or $$\boxed{F=-\eta A \frac{dv}{dx}} \quad \ldots(i)$$

where η is a constant depending upon the nature of the liquid and is known as the co-efficient of viscosity. The negative sign signifies that the viscous force is in a direction opposite to the flow of liquid.

$$\text{If } A=1,\ \frac{dv}{dx}=1 \text{ then } F=\eta$$

"The co-efficient of viscosity of a liquid can be defined as the tangential viscous or dragging force per unit area acting between layers of liquid in which unit velocity gradient is maintained in a direction normal to the layers and is denoted by letter η."

Dimensions :

$$\eta=\frac{\text{Force/area}}{\text{velocity/distance}}=\frac{F/A}{dv/dx}$$

$$=\frac{MLT^{-2}/L^{2}}{LT^{-1}/L}\quad=ML^{-1}\,T^{-1}$$

The dimensions of viscosity are 1 in mass, —1 in length and —1 in time.

Units . The absolute C. G. S. unit of the co-efficient of viscosity is called the "poise" after Poiseuille a pioneer experimenter in this field. From equation (*i*) we get

$$1\ \text{poise}=\frac{1\ dyne/1\ sq\ \ cm}{1\ cm\ \ per\ \ sec/cm}$$

=dyne per *sq. cm.* per unit velocity gradient.

"*A co-efficient of viscosity of 1 poise is one that requires a tangential force of 1 dyne for each square centimetre of surface to maintain a relative velocity of 1 cm/sec between two planes separated by a layer of the liquid 1 cm thick.*" Centipoise (1/100 of a poise) is the unit of viscosity commonly used

The *SI unit* for coefficient of viscosity is $Nm^{-2}\times s$ or *Pas* or $kg\ m^{-1}\ s^{-1}$.

$$\boxed{1\ \text{poise}=10^{-1}\ \text{Pas}}$$

At 20°C, η for water is $1{\cdot}0\times10^{-3}$ Pas or 0·01 poise while for glycerine it is $8{\cdot}2\times10^{-1}$ Pas.

6.21 CRITICAL VELOCITY

It was shown by Osborne Reynold that streamline flow occurs for small values of velocity v upto a *limiting critical velocity* v_c. He observed that there are actually two critical velocities, a lower one at which stream line flow is unstable and turbulence is possible and a higher one above which turbulence is inevitable ; v_c stands for the lower critical velocity. Let the critical velocity v_c of the liquid flowing through a narrow tube depend on the density ρ, the viscosity η of the liquid and the diameter d of the tube, than the relation can be found by method of dimensions as

$$v_c=k\rho^{x}\,\eta^{y}\,d^{z}$$

where k is some constant having no dimensions. The dimensions of v_c are LT^{-1} ; of η, $ML^{-1}T^{-1}$, of d, L and of density ρ, ML^{-3}.

Making a dimensional equation, we get,

$$M^{0}LT^{-1}=(ML^{-3})^{x}\,(ML^{-1}T^{-1})^{y}\,(L)^{z}$$

$$=M^{x+y}\,L^{-y-3x+z}\,T^{-y}$$

Equating the indices of M, L and T, we have,

$$x+y=0$$

$$-y-3x+z=1$$

$$-y=-1$$

whence $x=-1$, $y=1$, and $z=-1$.

Thus the relation is of the form as $v_c=\frac{k\eta}{\rho d}$ where k is a number about 1150 (for water). The usual way in which this statement is put is to combine v, ρ and η to give a dimensionless product $k=\frac{v_c \rho d}{\eta}$ which is here called the Reynold's number for the conditions discussed.

6.22 REYNOLD'S NUMBER

The Reynold's number is expressed by a formula

$$\boxed{k=\frac{v_c\rho d}{\eta}}$$

This can also be written as

$$k=\frac{\rho v_c^2}{\eta v_c/d}=\frac{\text{Inertial force}}{\text{force of viscosity}}$$

This number k which is the ratio of the forces of inertia and forces of viscosity is called ***Reynold's number***. This is so named to honour Osborne Reynold, the discoverer of the law of similarity. The nature of any particular case of flow of viscous liquid may be determined from the value of corresponding Reynold's number. The flow of the fluid is said to be *laminar* when k lies between 0 and 2000. For values of k above 3000, the flow is turbulent. For k between 2000 and 3000 the flow is unstable and many switch over from one type to another.

Reynold number is a pure number and therefore its numerical value is the same in any system of units.

A small value of Reynold's number means that the viscous forces predominate whereas the larger values of it indicate that the forces of inertia predominate. If the tube is narrow, the laws which hold for small values of k will also hold for all velocities which occur in practice whereas if the tube is wide enough, the laws will hold good for very small velocities of very viscous liquids.

When Reynold's number is smaller, the viscous forces are dominating, *i.e.*, in case of highly viscous substances like glue and oil whan the tube is narrow the inertial effects may be neglected as they are small compared with the effects of viscosity and here each particle is in equilibrium under the action of the fall of pressure and the force of friction.

When Reynold's number is very large, the forces due to viscosity are smaller. A liquid at rest remains in equilibrium under two kinds of forces, gravity and difference of pressure. These are also present in the case of liquids in motion. In addition there is viscosity (friction) which is to be regarded as a resistance to change of shape. As it happens the fluids which are most important in actual practice (air and water) have a very small co-efficient of viscosity.

6.2' FLOW OF LIQUIDS THROUGH PIPES (POISEUILLE'S FORMULA) :

When a liquid flows through a capillary tube with streamline motion, then the velocity of the liquid layer along the axis of the tube is maximum and gradually decreases as we move towards the walls where it becomes zero. The volume (V) of the liquid *flowing per second* through the tube depends on the following factors :

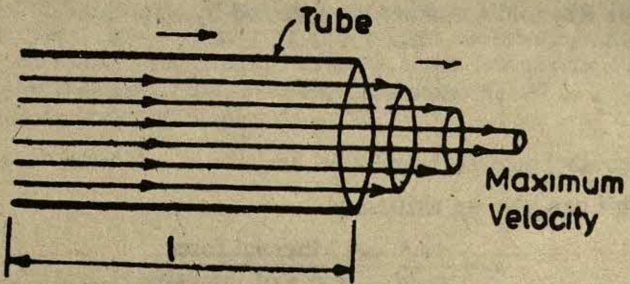


Fig. 6.43

(*i*) the viscosity (η) of the liquid

(*ii*) the radius (r) of the pipe.

(*iii*) the pressure gradient $\left(\frac{P}{l}\right)$ where P is the pressure difference across the length l of the capillary tube

$$\text{Let } V \propto \eta^a$$

$$\propto r^b$$

$$\propto \left(\frac{P}{l}\right)^c$$

$$\text{or } V = K\eta^a r^b \left(\frac{P}{l}\right)^c \qquad \ldots(1)$$

where K is a constant of proportionality and *a*, *b* and *c* are the indices.

Putting the above equation in the dimensional form, we have

$$L^3\ T^{-1}=[ML^{-1}T^{-1}]^a\times[L]^b\times[ML^{-2}T^{-2}]^c$$

Equating the dimensions of M, L & T, we have

$$0=a+c$$

$$3=-a+b-2c$$

$$-1=-a-2c$$

on solving these equations for a, b and c we have

$$\boxed{a=-1}$$

$$\boxed{b=4}$$

and $\boxed{c=1}$

Hence *eq.* (1) (becomes)

$$V=\frac{k\,P\,r^4}{\eta l}$$

By experiment we find that the value of K is given by $\frac{\pi}{8}$. Hence

$$\boxed{V=\frac{\pi P r^4}{8\eta l}} \quad \text{...(2)}$$

This formula is known as *Poiseuille's formula.* We can compare the viscosities of two liquids or we can determine the viscosity of a liquid by using Poiseuille's formula.

If V_1 and V_2 be the volumes of the two liquids flowing out in one secound and η_1 and η_2 are their coefficients of viscosity respectively, then from equation (2) it follows that

$$\boxed{\frac{V_2}{V_1}=\frac{\eta_1}{\eta_2}}$$ if all other factors P, r and l remain unchanged.

The apparatus used is shown in Fig. 6.44 where AB is a tube of narrow bore fixed horizontally into the vessel T known as constant level vessel. The liquid enters the vessel through the inlet tube i and when the level of the liquid rises above the open end of the outlet tube O, the excess of the liquid overflows keeping the level and hence the difference of pressure at the ends of the tube AB constant.

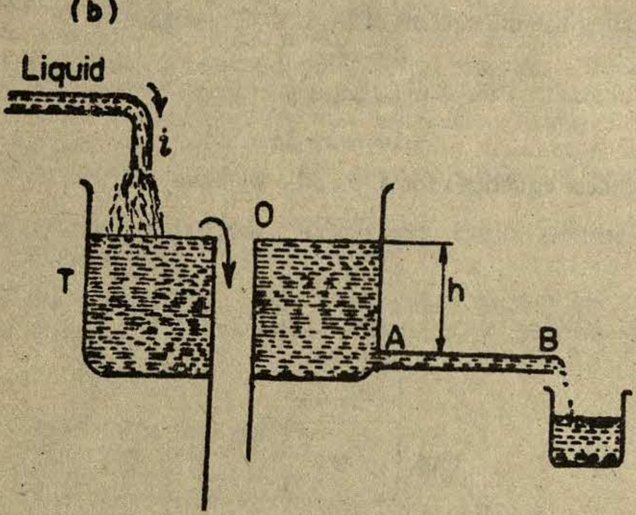


Fig. 6.44

If h_1 is the height of the liquid in the vessel above the axis of the tube, then difference of pressure

$P=h_1\rho_1 g$ where ρ_1 is the density of the liquid. With the second liquid, adjust the height of the overflow pipe O so that the liquid column exerts the same difference of pressure. If ρ_2 is the density of the liquid, then the height h_2 needed to produce the same pressure is given by

$$P=h_2\,\rho_2 g$$

Again measure the volume V_2 of the second liquid which flows out of the tube in the same time, then

$$\frac{\eta_1}{\eta_2}=\frac{V}{V_1}.$$

For determining the co-efficient of viscosity of a given liquid, the pressure $P=h\rho g$ is changed by altering the position of the over flow pipe O. Draw a graph between V and P which will be a straight line upto a certain flow beyond which the graph becomes non-linear due to turbulent motion (Fig. 6.45).

From the graph mean value of $\frac{V}{P}$ is obtained from the *linear portion* and knowing the radius r and length l of the capillary tube, the coefficient' of viscosity (η) is determined using equation

$$\boxed{\eta = \frac{\pi P r^4}{8Vl}}$$

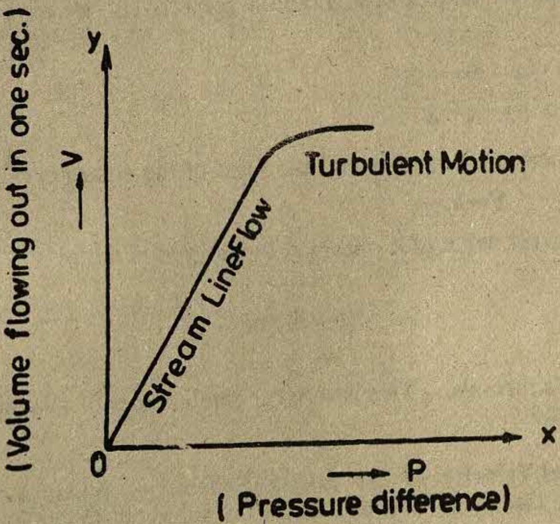


Fig. 6.45

The viscosity of liquids *decreases* with the rise of temperature while that of gases *increases*.

6.24 STOKE'S LAW

It was shown by Stoke that if a sphere of radius '*a*' is moving with uniform velocity v through an infinite, homogenous, incompressible fluid of viscosity η, the retarding force F is given by

$\boxed{F = 6\pi\eta va}$ This is known as *Stoke's Law.*

The above relation can be derived by method of dimensions. Let us consider that the viscous force acting on the sphere depends on the following factors :

(*i*) Radius of the sphere a

(*ii*) Velocity of the sphere v

(*iii*) Viscosity of the fluid through which the body is moving.

Using the dimensional method we have

$$F = ka^x v^y \eta^z$$

The dimensions of F are MLT^{-2}, of a are L^{+1}, of v are LT^{-1} and of η are $ML^{-1}T^{-1}$.

Making a dimensional equation, we get,

$$MLT^{-2}=k^0(L)^x\,(LT^{-1})^y\,(ML^{-1}T^{-1})^z$$

$$=M^z\,L^{x+y-z}\,T^{-y-z}$$

Equating the indices of M,L and T respectively, we have,

$$z=1$$

$$-y-z=-2$$

$$x+y-z=1$$

Hence $x=1$, $y=1$, $z=1$ and the form of the equation is

$$F=ka\eta v$$

The correct value of k obtained by analysis is 6π, so

$$\boxed{F=6\pi\eta va}$$

This is Stoke's formula. This is true for small values of v in a large expanse of fluid.

Terminal Velocity of the falling Sphere :— When a sphere falls vertically through a viscous fluid it is subjected to the following forces.

(*i*) Its weight W acts downwards.

(*ii*) The viscous resistance F acts upwards.

(*iii*) The Archimede's upthrust of bouyancy W_1 equal to the weight of the fluid displaced acts upwards.

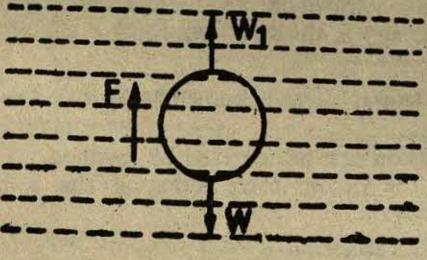


Fig. 6.46 Motion of a sphere through a fluid

The resultant downward force is

$$=W-W_1-F.$$

On account of this resultant force the sphere is accelerated until F has reached such a value that

$$=W-W_1-F=0$$

The sphere has thus attained its *terminal velocity* and *will travel uniformly with this velocity* *i.e.* downward accelaration is reduced to zero. Thus

$$F = W - W_1$$

Let ρ be the density of the material of the sphere and σ the density of the fluid, then,

mass of the sphere $= \frac{4\pi}{3}a^3\rho$ where a = radius of the sphere

and the weight of the sphere $= \frac{4\pi}{3}a^3\rho g$

The weight of the fluid displaced $W_1 = \frac{4\pi}{3}a^3\sigma g$

$\therefore$ Weight of the sphere in the medium is

$$W - W_1 = \frac{4\pi}{3}a^3\rho g - \frac{4\pi}{3}a^3\sigma g$$

$$= \frac{4\pi}{3}a^3(\rho - \sigma)g$$

If v represents the terminal velocity and F the maximum viscous resistance then

$$F = 6\pi\eta va$$

Since $F = W - W_1$

$$6\pi\eta va = \frac{4\pi}{3}a^3(\rho - \sigma)g$$

whence $$\boxed{v = \frac{2}{9}\frac{a^2g(\rho - \sigma)}{\eta}}$$

From the above equation (*i*) it is evident that the terminal velocity of the falling sphere is directly proportional to the square of the radius of the sphere and inversely to the co efficient of viscosity of the fluid. This formula holds only for substances whose Reynold's number is *less than unity i.e., for viscous fluids*. For water droplets in air $v = 1{\cdot}3 \times 10^6\, a^2$ cm./sec. The formula is valid for droplets of radius less than 0·01 mm.

$v = 1{\cdot}3 \times 10^6\,(0.001)^2$ cm sec^{-1}

Such droplets are formed in mist. On account of this, these droplets are independent of gravity and appear to us as floating clouds.

Stoke's method of measuring viscosity of highly viscous liquids :— This is an excellent method for the measurement of the co-efficient of viscosity of highly viscous liquids such as castor oil or glycerine at room temperatures. The equation for the terminal velocity is given as

$$v=\frac{2}{9}\ \frac{a^2g(\rho-\sigma)}{\eta}$$

or

$$\boxed{\eta=\frac{2}{9}a^2g\ \frac{(\rho-\sigma)}{v}} \quad \ldots(ii)$$

Therefore, by knowing v, a, ρ, σ and g, the viscosity can be calculated.

Experimental Method : The experimental liquid is contained in a big vertical jar (Fig 6.47). This is by no means the infinite ocean of fluid as demanded by the theory but the dimensions of the liquid can be considered very large compared with the radius of the sphere which is nearly 2 mm diameter.

The density of the sphere ρ and that of the liquid σ can be determined by some suitable means. Three reference marks P, Q and R are made on the jar by glass marking pencil. The marks should be several centimetres below the free surface of the liquid and the distance PQ and QR are kept equal. The diameter of the sphere is carefully measured by the help of a micrometer screw gauge. The sphere is first moistened with the experimental liquid and then gently set on the surface. The time t_1 and t_2 to cover the distances PQ and QR are measured. These should be very nearly equal. If they differ appreciably the terminal velocity had certainly not been reached at P and we have no means whether or not it was reached even at Q so the whole experiment results in failure. If they are nearly equal then

$$\frac{PQ+QR}{(t_1+t_2)}$$

gives the terminal velocity v and η is determined by substituting the values of various quantities in the equation, (ii) mentioned above.

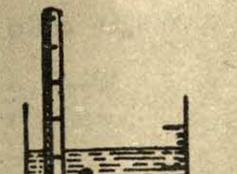


Fig 6.47

Example 6.8 :

An air bubble of radius 5 mm rises through a vat of syrup at a steady speed of 2 mms^{-1}. If the syrup has a density of $1{\cdot}4\times10^3$ kgm^{-3} what is its viscosity ?

Solution :

In this case the terminal velocity is upwards, since $\sigma > \rho$. Indeed, the density of the air inside the bubble may be neglected completely.

Equation

$$v = \frac{2a^2(\rho - \sigma)g}{9\eta} \text{ applies,}$$

with $a = 0.005\ m$

$(\rho - \sigma) = 1.4 \times 10^3\ \text{kgm}^{-3}$

and $v = 0.002\text{ms}^{-1}$

Re-arranging the above equation

$$\eta = \frac{2a^2(\rho - \sigma)g}{9v}$$

$$= \frac{2 \times 25 \times 10^{-6} \times 1{\cdot}4 \times 10^3 \times 9{\cdot}8}{9 \times 2 \times 10^{-3}}\ \text{kg}\ m^{-1}\ s^{-1}$$

$$= 38\ \text{kg}\ m^{-1}\ s^{-1}$$

Example 6.9 :

Determine the radius of a drop of water falling in air, if the terminal velocity of the drop is 0.012 ms^{-1}. Co-efficient of viscosity for air $= 1.8 \times 10^{-3}$ kg $m^{-1}\ s^{-1}$; density of air $= 1.21\ \text{kg m}^{-3}$ and density of water $= 1000$ kg m^{-3}

Solution : When a liquid drop falls with zero acceleration, then

$$6\pi\eta a v = \frac{4}{3}\pi a^3(\rho - \sigma)g$$

$$\text{Radius of the drop} = a = \sqrt{\frac{9\eta v}{2(\rho - \sigma)g}}$$

Here $\eta = 1{\cdot}8 \times 10^{-3}$ kg $m^{-1}\ s^{-1}$

$V = 0{\cdot}012\ ms^{-1}$

$\rho = 1000\ \text{kgm}^{-3}$

$\sigma = 1{\cdot}21\ \text{kgm}^{-3}$

$$a = \sqrt{\frac{9 \times 1{\cdot}8 \times 10^{-3} \times 0{\cdot}012}{2(1000 - 1{\cdot}21) \times 9{\cdot}8}}$$

$$= 9{\cdot}97 \times 10^{-5}\ m.$$

SUMMARY

1. All matter consists of molecules. There are considerable forces between molecules called *molecular forces*. These forces are short ranged and possess many interesting features. Molecular forces are (*i*) attractive at large distances, and (*ii*) repulsive at very close **distances.**

2. Surface Tension is the force produced on the surface molecules of liquids by the attraction of molcules in the interior of the liquid. As a result of this force, liquid surface tend to contract.

3. Capillarity is the phenomenon of rise or fall of a liquid inside a capillary when it is dipped in a liquid The height to which a liquid can rise is given by

$$\sigma = \frac{rhg\rho}{2\cos\theta}. \quad \Rightarrow \quad h = \frac{2\sigma\cos\theta}{rg\rho}$$

4. Pressure on the concave side of the surface film of a liquid is always greater than the pressure on the convex side and is given by $\frac{2\sigma}{r}$ for one surface

5. Critical velocity of a fluid is that velocity below which the flow remains streamline and above which it becomes turbulent It is given by

$$v_c = \frac{K\eta}{\rho d}.$$

where K is Reynold's number—which is a pure number for critical flow

6. The pressure of a fluid moving in a streamline flow falls as its speed increases. This results from the Bernoulli Principle, which states that the sum of the kinetic energy, the potential energy caused by the pressure of the fluid, and the gravitational potential energy of a fluid is constant during streamline flow.

7. Forces of viscosity retard moving fluids in much the same way that frictional forces slow down moving solids. If we follow the motion of any liquid through a tube, we observe that liquid at the centre of the tube moves fastest, and that close to the inside walls of the tube moves most slowly. Looking at the fluid's cross-section, we would see that motion take place along a U-shaped front due to viscosity.

8. The quantity of liquid flowing through a cylindrical pipe is directly proportional to the pressure drop across the liquid in the pipe and proportional to the fourth power of the pipe's radius.

9. A solid sphere that is falling through a viscous fluid eventually reaches a constant "*terminal velocity*" when its weight equals the sum of the viscous force acting on it and the buoyant force.

   The retarding viscous force is given by $F = 6\pi a\eta v$ (Stoke's Law).

QUESTIONS

1. Explain, using a simple molecular theory, why the surface of a liquid behaves in a different manner from the bulk of the liquid.

2. (*a*) What are intermolecular forces ?
   (*b*) Why do molecular forces act only between adjacent molecules ?
   (*c*) Are molecular forces of electrical origin ?
   (*d*) Draw the molecular force graph and explain the various features of molecular forces from it.

3. Explain

   (*a*) In terms of molecular forces why water is drawn up above the horizontal liquid level round a steel needle which is held vertically and partly immersed in water.

   (*b*) Why, under certain circumstances, a steel needle will rest on a water surface ?

   In each case show the relevant forces on a diagram.

4. Giving the necessary theory explain how the rise of water in a capillary tube may be used to determine the surface tension of water.

5. Describe and explain *two* experiments of a different nature to illustrate the phenomenon of surface tension.

   Give a quantitative definition of surface tension and explain what is meant by angle of contact.

6. (*a*) Do molecules ever repel one another ? Distinguish between adhesion and cohesion.

   (*b*) What is the unit of surface tension ? Write down its dimensions.

   (*c*) If a liquid wets a solid surface, what can be said about its angle of contact. Name three liquid-solid surfaces having angle of contact greater than 90°. Can we ever get a perfect horizontal surface of water ? Name the material of the container.

7. Why is the surface of mercury in a glass tube convex and that of water concave ?

8. Deduce an expression for the difference of pressure on the two sides of a spherical drop. How will the problem be altered in dealing with a spherical soap bubble ? *(Delhi Senior Sec. 1980)*

9. (*i*) Explain why water rises against gravity in a capillary tube ?

   (*ii*) Derive a relation between the surface tension σ of a liquid and its ascent h in a capillary tube.

   (*iii*) What will happen if the length of the tube is smaller than h ?

10. Explain the following :

    (*i*) The small space between two parallel circular glass plates is filled with water. Explain why is it much easier to separate the plates by sliding over the other than by a direct pull.

    (*ii*) Why a small drop of mercury is almost perfectly spherical whereas a large drop resting on a glass plate is flat at the top ?

(*iii*) Why does ink rise in a blotting paper ?

(*iv*) Why does a drop of aniline assume a spherical shape when immersed in brine solution of the same density ?

(*v*) Why does a small piece of camphor dance on the surface of water ?

(*vi*) From where does the energy come when a liquid rises against gravity in a capillary tube ?

11. (*a*) Define viscosity, coefficient of viscosity ; poise : and terminal velocity.

(*b*) State Stoke's Law. Derive it from dimensional analysis. Why it is true only for sufficiently low velocities ? [*A.I.S.S.C. 1986*]

(*c*) Explain why a sphere released in a fluid will fall with diminishing acceleration until it attains a constant terminal velocity ?

12. How will you determine the terminal velocity of a lead shot for glycerine and the coefficient of viscosity of glycerine experimentally ? Derive the formula used. [*A.I.S.S.C. 1986*]

13. How will you compare the coefficient of viscosity of two viscous liquids experimentally ? Mention the formula used.

14. An aluminium sphere is suspended by a thread below the surface of a liquid. Show on a sketch the forces acting on the sphere, and explain its equilibrium.

The thread is now cut. Show on a second sketch or explain in words the forces which act on the shpere when it is in motion.

The following figures for x, the total distance travelled by the sphere in the liquid at time t, were obtained.

t (s)	1.0	2.0	3.0	4.0	5.0
x(cm)	3.60	10.3	18.6	27.9	37.4

Draw a graph to display the relation between x and t and explain its form. Find the terminal velocity of the sphere. Give a qualitative account of how you would expect the graph to be modified if the temperature of the liquid were increased.

15. (*a*) Derive the dimensions of coefficient of viscosity. What is the unit of coefficient of viscosity ?

(*b*) Derive by dimensional analysis the following formulae (*i*) $k=\frac{\rho r v_c}{\eta}$ (Reynold's number)

(*ii*) $V=\frac{\pi p r^4}{8\eta l}$ (Poiseuille's formula)

(*c*) Check the dimensional consistency of

((1) $6\pi n a v=\frac{4}{3}\pi a^3 (\rho-\sigma)g$ where a is the radius of a sphere of density ρ falling steadily under gravity through an expanse of liquid of density σ.

16. (*a*) What is meant by streamline flow ; laminar flow and turbulent flow ?

(*b*) What happens to the speed of a fluid through a constriction ?

17. State and prove Bernoulli's Theorem for flow of fluids. Mention some of its applications.

(Delhi Senior Sec. 1980)

18. What is capillarity ? Give some examples of capillarity in daily life.

19. (*a*) Discuss the importance of viscosity in our daily life.

(*b*) Explain the lift of the wing of an aeroplane.

20. (*a*) Explain the following :

(*i*) When an air stream is blown between two sheets of cards close to each other, they attract. Why ?

(*ii*) Why a ping pong ball is kept in equilibrium in mid air by a vertical jet of water ?

(*b*) What happens to the pressure of a fluid when it passes through a region where its speed increases ?

(*c*) By what factor the rate of flow of water will increase if the diameter of the bore of the capillary is doubled ?

(*d*) If a person stands near a fast moving train, why is there a possibility of his falling towards it ?

PROBLEMS

(Delhi Senior Sec. 1980)

1. A capillary tube whose diameter is $0.5\times10^{-3}m$ is placed vertically in a liquid of density $0{\cdot}8\times10^3\ kgm^{-3}$ and surface tension $0{\cdot}03\ Nm^{-1}$. What will be the rise of liquid ? The angle of contact is zero.

($0{\cdot}03m$)

2. The surface tension of a soap solution is $3\times10^{-2}\ Nm^{-1}$. How much work has to be done in making a bubble of radius $0{\cdot}03m$ by blowing ?

($18{\cdot}8\times10^{-4}$ joule)

3. Water rises to a height of 2 *cm* in a capillary. If another capillary has a radius equal to one-third of that of the first one, how high the water will rise in it ? What will happen if first capillary is tilted 60° from the vertical ?

(6 cm ; 4 cm)

4. How much work should be done to convert a water drop of radius 1 mm into 10^6 droplets of the same size at constant temperature ? The surface tension of water is 72×10^{-3} Jm^{-1}

($8{\cdot}96\times10^{-5}$ joule)

5. How much work should be done in increasing the diameter of a soap bubble from 1 mm to 5 mm ? The surface tension of soap solution is 0.03 Nm^{-1}.

($4{\cdot}5\times10^{-6}$ joule)

6. Water is flowing through a pipe of non-uniform cross-section. The speed of water flow is 25cm s^{-1} at a point where the radius of pipe is 0.005 m. Find the speed of water at some other point where the radius of the pipe is 2.0 cm.

(0.0156 ms^{-1})

7. The reading of a pressure meter attached to closed water pipe is 3.5×10^5 Nm^{-2}. On opening the valve of the pipe, the reading of pressure meter reduces to 3.0×10^5 Nm^{-2} ? Calculate the speed of water flowing in the pipe.

(10 ms^{-1})

8. The density and radius of an oil drop are 0.95×10^3 kg m^{-3} and 10^{-4} cm respectively. This drop is falling through air of density 1.3 kg m^{-3} and viscosity 181×10^{-8} poise. Compute the final velocity of the drop.

(11 4×10^{-4} ms^{-1})

9. A water tank has a hole at a distance of 10 *m* from free water surface. Compute the velocity of water through the hole. If the radius of the hole is 1 mm, find the rate of flow of water.

(14 ms^{-1} ; $4{\cdot}4\times10^{-5}$ m^3s^{-1})

10 A pitot tube is fixed in a main of diameter 0.15 m and the difference of pressure indicated by the gauge is 0.04 m of water column ; find the volume of water passing through the main in a minute.

(0.9395 cubic metre)

11. A vertical tube of 4 mm diameter at the bottom has water passing through it. If the pressure is atmospheric at the bottom where the water emerges at the rate of 800 g/min, what is the pressure at a place 25 cm above the bottom, where the diameter is 3 mm ?

(73·25 cm of mercury)

12. Calculate the excess of pressure between the inside and outside of a soap bubble of radius 1 cm. Surface Tension of soap solution is 3.2×10^{-2} Nm^{-1}.

($12{\cdot}8$ Nm^{-2})

13. A U-tube has two limbs of internal radii 0.5 cm and 1 cm respectively. Calculate the difference in the water levels in the two limbs. Surface Tension of water $= 7.2 \times 10^{-2}$ Nm^{-1} and density of water 1000 kg m^{-3}.

($1{\cdot}472 \times 10^{-3}$ m)

14. Calculate the inter molecular binding energy of water molecules. Avogadro's number $= 6 \times 10^{23}$ per mole ; 1 electron volt $= 1{\cdot}6 \times 10^{-19}$ J and Latent heat of vaporization of water $= 2{\cdot}26 \times 10^{6}$ J kg^{-1}

(6.78×10^{-20} J *OR* $0{\cdot}42$ ev)

15. Calculate the speed at which the velocity head of a stream of water is equal to $0{\cdot}50$ m of Hg.

($3{\cdot}1305$ ms^{-1})

16. Calculate the horizontal force required to move a metal plate of area 2×10^{-2}m^{2} with a velocity of $4{\cdot}5 \times 10^{-1}$ ms^{-1} when it rests on a layer of oil $1{\cdot}5 \times 10^{-3}$m thick. Coefficient of viscosity of the oil$=0{\cdot}2$N.Sm^{-2}

($1{\cdot}2$ N)

17. Calculate the terminal velocity of an air bubble of radius 5×10^{-4} m rising in a liquid of viscosity $0{\cdot}15$ N sm^{-2} Density of the liquid $= 900$ kgm^{-3}. Neglect density of air which is small as compared to the density of the liquid.

($3{\cdot}26 \times 10^{-3}$ms^{-1})

18. A glass tube of internal radius 5×10^{-4} *m* is dipped vertically into a vessel containing mercury such that the lower end of the tube is 10^{-2} *m* below the surface of mercury. Calculate the guage pressure of air inside the tube to blow a hemispherical bubble at the lower end of the tube. Surface tension of mercury $= 3{\cdot}5 \times 10^{-1}$ Nm^{-1} and density of mercury $= 1{\cdot}36 \times 10^{4}$ kgm^{-3}.

($14{\cdot}72 \times 10^{3}$ Nm^{-2})

19. A pitot tube is fixed on the wing of an aeroplane to measure the speed of the aeroplane. The tube contains a liquid of density 800 kg m^{-3}. The difference in level between the two limbs is 0.5*m*. Density of air $= 1{\cdot}293$ kg m^{-3}. Calculate the speed of the aeroplane.

($77{\cdot}85$ ms^{-1})

20. A water tank whose sides are vertical is filled upto height H. A small hole is made on one of the walls at a depth of *h* below the water surface. Calculate :

(*i*) the velocity of efflux ;

(*ii*) the range of the emerging stream ; &

(*iii*) for what value of h, this range will be maximum ?

$\{\sqrt{2gh}\,;\ 2\sqrt{h(H-h)}\,;\ H=2h\}$

21. Prove that, if the sphere is released from rest, then the resultant acceleration of the sphere of velocity V, mass m, density ρ and radius a, through a viscous fluid of density σ and coefficient of viscosity η is given by $g\,\dfrac{(\rho-\sigma)}{\rho}$

22. Prove that when two mercury drops are brought into contact to form a single drop, the surface energy of the larger drop is less than the total surface energy of the smaller drops.

[*A.I.S.S.C. 1986*]

[**Hints** : Total surface energy of two drops$=8\pi r^2.\sigma$

$$\frac{4}{3}\pi R^3=\frac{8}{3}\pi r^3$$

$$\boxed{R=2^{1/3}r}$$

Surface energy of a single large drop$=4\pi R^2$

$=4\pi r^2.2^{2/3}\sigma$

Since every physical system tries to attain a state of minimum energy, the drops of mercury coming in contact form one drop.]

Unit—7

ELECTRICITY

7.1 ELECTRIC CURRENT (Drift Velocity)

The practical applications of electricity are numerous and well known *e.g.* it is used for lighting, heating, transportation, communication etc; but they hardly ever make use of the *electrostatic forces between charged bodies. They rely instead upon the effects produced by an electric current flowing through a metal wire that has no net electric charge. When an electrical appliance is plugged into a wall socket, electrons flow in through one pin of the plug, circulate through the wires inside the appliance and leave via the other pin of the plug. As many electrons leave as enter and there is never any net accumulation of charge inside the appliance. Actually, since the wall socket usually supplies 50 cycle alternating current, the electrons surge backward and forward fifty times a second. To make the matter simpler, it will be better to discuss the direct current produced by a storage battery of the kind used to operate the ignition system of an automobile.

Fig (7.1) shows a storage battery with a copper wire of length l connected between its terminals. The battery consists essentially of two metal plates immersed in dilute sulphuric acid. One of the plates is coated with sponge lead and the other is coated with lead dioxide. You learnt in tenth class that when a glass rod is rubbed with silk cloth, the electrons have preference for being in the silk rather than the glass and therefore some electrons migrate from the glass to the silk. Similary, electrons prefer to be in lead rather than lead dioxide, and by a series of elaborate processes electrons are transferred from the lead dioxide plate through the sulphuric acid to the lead plate. The lead dioxide plate, therefore, has a deficiency of electrons, an excess of protons, and a positive charge. It is the positive terminal of the battery. The lead plate acquires an excess of eletrons, is negatively charged, and is the negative terminal of the battery. The

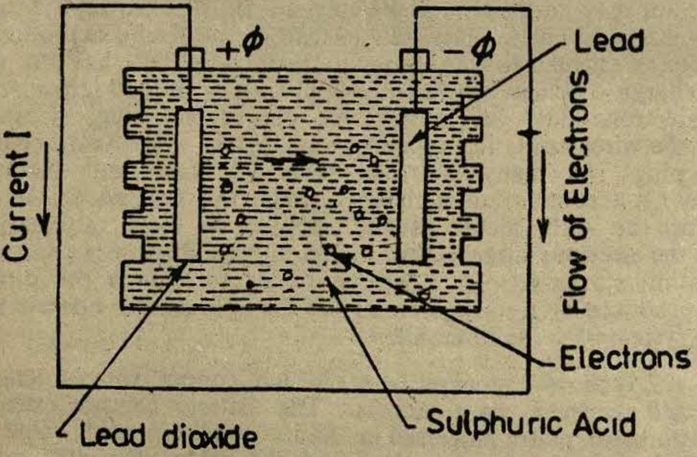
---

* $F = K\frac{q_1 q_2}{r^2} = \frac{q_1 q_2}{4\pi \varepsilon_0 r^2}$ — where F is the force of attraction or repulsion between two charges q_1 and q_2 separated by a distance r and K $= \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N}m^2 \text{ C}^{-2}$ is a constant. $\varepsilon_0 = 8{\cdot}85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} m^{-2}$ is called the *permittivity of vacuum.*

*electric potential $\phi+$ at the positive terminal in higher than the potential $\phi-$ at the negative terminal. The battery operates in such a way that the potential difference V between its terminals is almost constant.

$$V = \phi_{+} - \phi_{-} = 2 \text{ volts, approximately.}$$

V decreases a little if a large current is drawn from the battery and V also steadily decreases with time as the battery runs down because of chemical processes that contaminate its plates. However, these changes are never more than a few percent and we will simplify the discussion by assuming that the current drawn from the battery is small and that the potential difference V between its terminals remains *constant.*



A storage battery with copper wire connected between its terminals
Fig. 7.1

If a copper wire is connected between the terminals, electrons flow continuously from the negative terminal through the wire to the positive terminal and then through the battery back to the negative terminal. In copper, as in other metals, some of the electrons on the outside of the copper atom are very loosely held and are easily shaken free. These electrons do not remain attached to a particular copper atom, but wander freely throughout the whole of the copper wire. They are repelled by the negative terminal and attracted by the positive terminal. This does not mean that the electrons move faster and faster the nearer they get to the positive terminal. They frequently

---

*Electric potential at a point is the amount of work done in bringing a unit positive charge from infinity to the point under consideration. If the work done in bringing *one coulomb* of charge from infinity to a certain point is one *joule*, we say that the potential of the point is 1 *volt*. Electric potential at a point $= K\frac{q}{r}$ where $K = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$, q is the charge and r is the distance.

collide with the copper atoms and although an electron acquires an extra velocity towards the positive terminal in between collisions, this extra velocity is destroyed at each collision. However, in between collisions the electron does get a chance to move toward the positive terminal and the net result is a *slow drift* of the electrons toward this terminal.

In the absence of the storage battery with no current flowing, free electrons move through the copper with very high velocities of the order of magnitude of $10^6 ms^{-1}$ on account of their thermal energy at room temperature. Since these velocities are randomly distributed in all directions, the average velocity of electron given by

$$\vec{U} = \frac{\vec{U}_1 + \vec{U}_2 + \vec{U}_3 + \ldots\ldots + \vec{U}_n}{N}$$

$$= 0$$

where $\vec{U}_1, \vec{U}_2, \vec{U}_3, \ldots\ldots \vec{U}_n$ are velocities of individual electrons and N is the total number of electrons.

In other words, there is no net flow of electrons one way or another. When the storage battery produces a potential difference between the ends of the wire, an * electric field $\vec{E} \left(= \frac{V}{l} \right)$ is established inside it. Due to the electric field the electrons experience coulomb forces in a direction opposite to the field and they undergo an acceleration.

$$\boxed{\vec{a} = \frac{-e\vec{E}}{m}} \qquad \ldots(1)$$

where *e* is the charge on the electron and *m* is the mass of the electron.

The forces are experienced by positive ions also but they are unable to move as they are heavy and tightly bound in the metal. Between two successive collisions an electron thus acquires, in addition to its thermal velocity, a velocity component in a direction opposite to $\vec{E}$. This gain in velocity, however, is very small and is lost in the next collision with an atom.

---

* Electric field is a region in which a charged body experiences force.

Electric field intensity $\vec{E}$ is the force experienced by a unit positive charge placed at a point in an electric field.

$$\vec{F} = \frac{-dV}{dl} = \frac{-\text{Potential difference}}{\text{distance}}$$

If we looked at any arbitrary instant, we would find an electron moving with a velocity $\vec{U}_1+\vec{a}t_1$ where $\vec{U}_1$ is its velocity due to thermal motion and $\vec{a}t_1$ is the velocity acquired under the influence of the external field $\vec{E}$, t_1 being the time that has elapsed since the last collision. The average velocity $\vec{v}$ of all electrons will be given by

$$\vec{v}=\frac{(\vec{U}_1+\vec{a}t_1)+(\vec{U}_2+\vec{a}t_2)+\ldots\ldots(\vec{U}_n+\vec{a}t_n)}{N}$$

$$=\frac{(\vec{U}_1+\vec{U}_2\ldots\ldots\ldots\ldots\vec{U}_n)}{N}+\frac{\vec{a}\,(t_1+t_2+\ldots\ldots+t_n)}{N}$$

or $$=0+a\,\tau \text{ where } \tau=\frac{t_1+t_2+t_3+\ldots\ldots+t_n}{N}$$

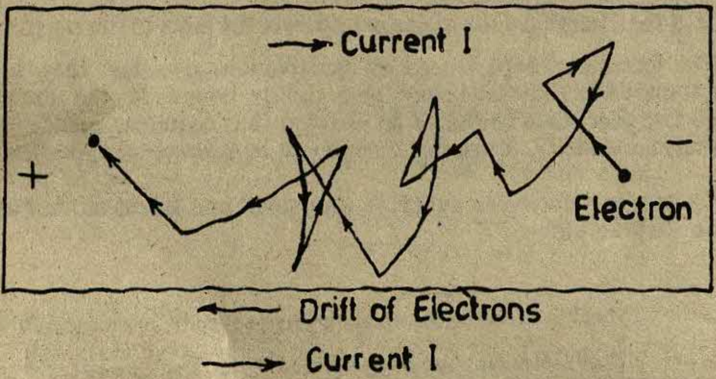
represents the average time elapsed since each electron suffered its last collision and is called the *relaxation time*. It is equal to $10^{-14}s$. It is a characteristic of the given conductors.

Denoting $\vec{v}$ by $\vec{v}_d$ we have

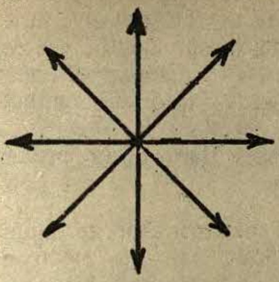
$$\boxed{\vec{v}_d=\vec{a}\tau=\frac{-e\,\vec{E}}{m}\tau} \qquad \ldots(2)$$

where $\vec{v}_d$ is called the *drift velocity* of electrons.

Thus the net result is that when the electric field is applied, the electrons in the copper wire acquire a constant average drift velo-

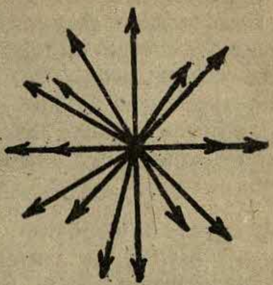


Slow drift towards the positive terminal
Fig. 7.2 (a)



(b)

(In the absence of current, velocities are equally distributed in all directions)
Fig 7.2 (b)



(c)

(In the presence of a current the superposition of a drift velocity $\vec{v}_d$ produces a bias toward the left.).
Fig 7.2 (c)

city $\vec{v}_d$ in a direction opposite to $\vec{E}$ (Fig 7.2). This drift velocity points along the wire toward the positive terminal and is in the same direction and has the same magnitude for all the free electrons. The random chaotic motion therefore acquires a slight bias in this direction and more electrons move toward the positive terminal than away from it. However, $\vec{v}_d$ is always very small compared with the random velocity. A typical value of $\vec{v}_d$ might be 1 mms^{-1} which is a billion times smaller than the random velocity of 10^6 ms^{-1}. But since the number of electrons is very large, even this small drift of electrons adds up to a sizable current.

It may be mentioned here that the drift velocity of electrons may not be confused with the speed of propagation of electrical effects. Although the drift velocity of electrons is low, an electric bulb turns on almost immediately when it is switched on account of the fact that the propagation of an electric impulse takes place with a speed which is 3×10^8 ms^{-1} *i.e.*, the speed of light. It is somewhat like applying a pressure to one end of a long water filled tube. As soon as the pressure is applied, a pressure wave is transmitted rapidly along the tube and when it reaches the other end, the flow of water starts. The water inside the tube also starts moving forward but this speed is much slower than the speed of pressure wave. Similarly in a copper wire free electrons are present everywhere. When a potential difference is applied to it, an electric field gets established throughout the circuit almost with the speed of light. As soon as the electric field is established, the electrons begin to drift under its influence and a current flows in the circuit.

When the electrons flowing through the copper wire arrive at the positive terminal they must be returned to the negative terminal, otherwise the charges on the two terminals would soon be neutralized and the current would cease to flow. The transfer of electrons through the sulphuric acid to the negative terminal against the repulsion of the negative charge on this terminal is a consequence of certain complicated chemical processes that occur inside the battery.

If a piece of wood were connected between the terminals of the storage battery no current would flow. All the electrons in the wood are firmly held to their atoms and are not free to move from one terminal to the other. Wood is an *insulator* because it will not carry an electric current. Copper is a *conductor* of electricity. All metals are conductors.

The Quantitative Definition of Electric Current :

The electric current I is defined as the net rate of flow of charge across any cross-section of a wire.

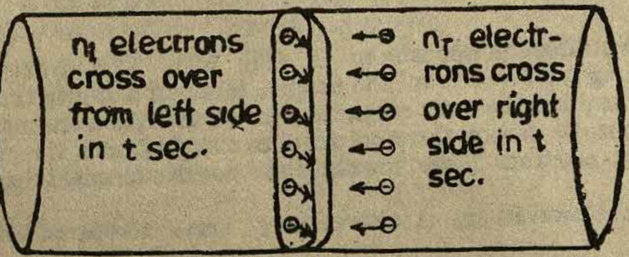

n_l electrons cross over from left side in t sec.
n_r electrons cross over right side in t sec.

Fig (7.3)

Suppose that, in t seconds n_r electrons cross over from right to left and a smaller number n_l cross over from left to right (fig 7.3). There is a net flow of (n_r-n_l) electrons from right to left, corresponding to the flow of a negative charge

$$-q=-(n_r-n_l)\,e$$

from right to left. The current is the charge flowing in one second.

$$\boxed{I=\frac{q}{t}} \qquad ...(3)$$

or

$$\boxed{I=\frac{(n_r-n_l)\,e}{t}} \qquad ...(4)$$

where e is the charge on the electron.

If q and e are in *coulombs* and t in *seconds*, the current I is measured in *amperes*. The current is due to the flow of negatively charged electrons from right to left. This is equivalent to a flow of positive charge from left to right (Fig 7.4) and it is traditional to assign the direction of the current on the assumption that positive charge is flowing. *The direction of the current is therefore always opposite to the direction of net flow of the electrons.*

Fig. 7.4. Flow of positive charge is equivalent to flow of negative charge in the opposite direction.

If the rate of flow of charge does not change with time, the current is said to be steady. In many situations, however, the current may be varying with time as shown in Fig (7.5).

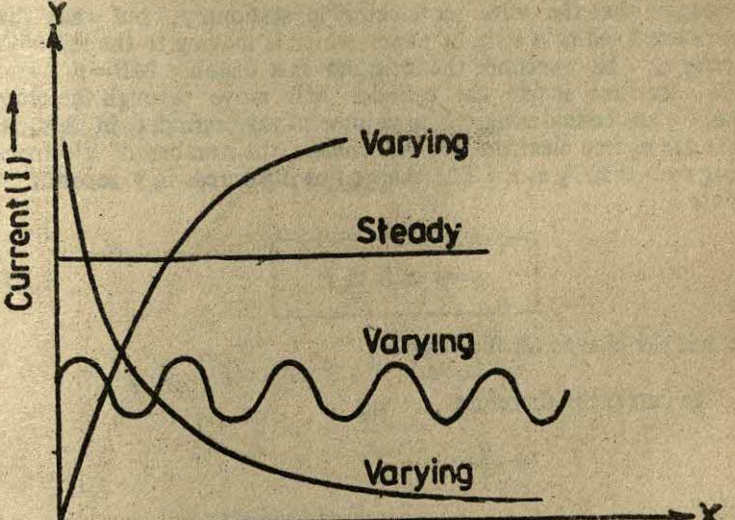


Fig. 7.5

Current in terms of Drift Velocity :

Consider a conductor (copper wire) PQ of length l and of uniform area of cross-section A, across which is applied a potential

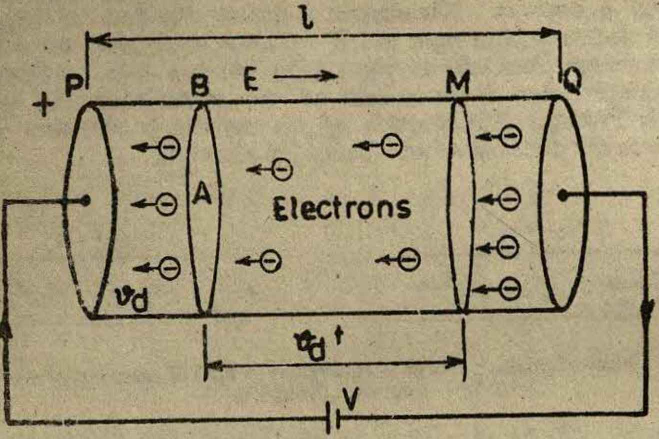


Fig. 7.6 Current in terms of drift velocity

difference V, (Fig 7.6). The electrons start drifting towards left with a velocity v_d. Let us ignore the random motion of the electrons. Also imagine that the wire (conductor) is stationary, but that the electrons are fixed to a strip of paper which is moving to the left with a velocity v_d. In t seconds the strip moves a distance MB$=v_d t$ and all the electrons inside the cylinder MB move through the cross section, we are considering. The volume of this cylinder in A v_d t. If there are n_0 free electrons *per unit volume*, the number of electrons crossing over is n_0 A v_d t. The charge flowing across in t seconds is therefore

$$\boxed{q=e\,n_0\,A\,v_d\,t} \qquad ...(5)$$

where e is the chayge on the electron.

The current is therefore

$$I=\frac{q}{t}$$

$$=\frac{en_0 A v_d t}{t}$$

or

$$\boxed{I=en_0 A v_d} \qquad (6)$$

Since there is a potential difference (V) between the ends of the wire, there is an electric field E inside the wire given by equation

$$E=\frac{V}{l}$$

If the end P is positive with respect to Q, this field points from P to Q.

This field exerts a force on an electron given by

$$F = eE \quad \text{where } F = \text{force exerted}$$

$$F = \frac{eV}{l}$$

Since the electron is negatively charged, this force is in the *opposite* direction to E and points from Q to P. If m is the mass of the electron, it has an acceleration a in the direction Q to P given by

$$a = \frac{F}{m}$$

or

$$a = \frac{eV}{ml}$$

Immediately after a collision with an atom, the electron has no preferred velocity in the direction of flow. If τ sec is the average relaxation time of the electron, then the average drift velocity v_d is given by

$$v_d = a\tau$$

$$v_d = \frac{eV}{ml}\tau \quad \text{...(7)}$$

Putting (7) in (6) we have

$$I = e\, n_0 A\left(\frac{eV}{ml}\tau\right)$$

$$\boxed{I = \frac{e^2 n_0 A\tau}{ml} . V} \quad \text{..(8)}$$

Introducing a new quantity R, defined by

$$R = \frac{ml}{e^2 n_0 A\tau} \quad \text{...(9)}$$

Then

$$\boxed{I = \frac{V}{R}} \quad \text{... (10)}$$

(Ohm's Law)

R depends only on the dimensions of the wire and the properties of the free electrons. For a given wire at a fixed temperature, it is a constant known as the *electrical resistance* of the wire. *Ohm's Law* states that the current is directly proportional to the potential difference between the ends of the wire and inversely proportional to the resistance. From equation (8), it may also be noted, that the current is proportional to the cross-sectional area A of the wire and inversely proportional to its length l. The resistance R is inversely proportional to A and proportional to l. In SI system the unit of electrical resistance is *ohm*(Ω)

$$\boxed{1 \text{ ohm} = \frac{1 \text{ volt}}{1 \text{ ampere}}}$$

We can also write

$$\boxed{R = \rho \frac{l}{A}} \qquad \text{...(11)}$$

where

$$\rho = \frac{m}{e^2 n_0 \tau} \qquad \text{..(12)}$$

is called the *specific resistance* or *resistivity* of material of the conductor. In SI system, the units of specific resistance ρ are *ohm- metre.*

If $l = 1m$

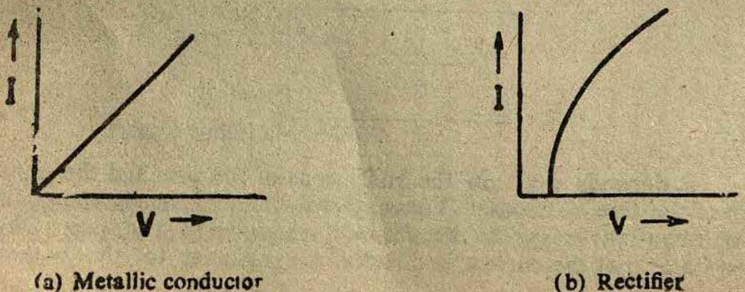
and $A = 1m^2$

then $\boxed{R = \rho}$

Thus *specific resistance of a conductor may be* defined as *the resistance offered by the conductor whose length is one metre and area of cross-section one square metre.*

The reciprocal of resistivity is called *conductivity.* It is expressed in $\text{ohm}^{-1} m^{-1}$.

Ohm's Law is true for metals at a fixed temperature and many other materials, *but it is not universally true.* Our proof is depen-



(a) Metallic conductor (b) Rectifier

Fig. 7.7

dent on the assumptions we have made concerning the behaviour of the flowing electrons. In some materials and in some special devices, the behaviour of the electrons is much more complicated and the current is *not linearly proportional* to potential difference (Fig 7.7). Certain special devices known as *rectifiers* are designed in such a way that the resistance is small when the current flows in one direction, but is very large when it flows in the opposite direction.

Variation of Resistance with Temperature :

As we know, the atoms in a metal occupy fixed positions but they are, however, not stationary but vibrate to and fro about their fixed positions. When a metal gets heated there is an increase in the average kinetic energy of the atoms and they vibrate more vigorously. This leads to more collisions of electrons with atoms and a consequent decrease in the value of relaxation time. Also at higher temperature the thermal speed of electrons is more and this further reduces the time between collisions. A lower value of τ therefore, results in an increase in the resistivity of the conductor. In general the *resistance increases as the* temperature is raised. For pure metals, the curve showing the relation between the resistance and temperature, measured on the gas scale, is almost a *parabola*, but for small changes in temperature, the curve is almost a straight line. Hence, for small changes in temperature, we have

$$R_\theta = R_0 (1 + \alpha\theta) \qquad \text{...(13)}$$

where R_θ = Resistance at θ°C

R_0 = Resistance at 0°C

θ = Rise in temperature

and α = coefficient of increase of resistance with temperature.

For metals α is a *positive* coefficient and for copper its value is 0·0040 per degree. In the case of *alloys*, on the other hand, the rate at which the resistance changes with temperature is *much less*. For example, *manganin*, which is an alloy of copper, nickel, iron and manganese, has $\alpha = 0.00001$ per degree *i.e.*, its resistance practically remains constant for a change in temperature of several degrees. Similar is the case with constantan and eurika For this reason, these alloys are excellent substances to be used for *standard resistance coils.*

Carbon and other semi-conductors display a negative coefficient *i.e.*, their resistance decreases with rise in temperature. α is negative for electrolytes also.

Example 7.1

In a copper wire PQ, a current of one ampere flows in the direction P to Q. Calculate the number of electrons transferred from Q to P in 1·6 seconds. (charge on the electron = 1.6×10^{-19} C)

Solution

Here $I=1$ ampere $=1$ coulomb per sec

charge on the electron $=e=1{\cdot}6\times10^{-19}$ coulomb

Let number of electrons transferred $=n_0$

$$\therefore \text{ charge } q=n_0e$$

$$\text{current} \quad I=\frac{q}{t}=\frac{n_0e}{t}$$

$$\text{or} \qquad I=\frac{n_0\times1{\cdot}6\times10^{-19}}{1{\cdot}6}$$

$$\therefore n_0=10^{19}$$

Example 7.2

In the case of hydrogen atom, an electron moves in an orbit of radius $5\times10^{-11}m$ with a speed of $2{\cdot}2\times10^6\ ms^{-1}$. Calculate the equivalent current. (charge on the electron $=1.6\times10^{-19}$ coulomb)

Solution

Radius of orbit, $r=5\times10^{-11}m$

circumference of the orbit $=2\pi r$

$$=2\times\frac{22}{7}\times5\times10^{-11}m$$

speed of electron $=v=2{\cdot}2\times10^6\ ms^{-1}$

In one second the electron will go round the proton (nucleus) n times, where

$$n=\frac{v}{2\pi r}=\frac{2{\cdot}2\times10^6}{2\times\frac{22}{7}\times5\times10^{-11}}$$

The electric current is defined as the quantity of charge that flows past a fixed point in one second. Although it is the same electron going round and round the proton, the quantity of charge that passes across a fixed point on the orbit in one second is given by

$q=n\,e$ where e is the charge on the electron.

$$\text{current} \quad I=\frac{q}{t}=\frac{ne}{t}$$

$$=\frac{2{\cdot}2\times10^6}{2\times\frac{22}{7}\times5\times10^{-11}}\times\frac{1{\cdot}6\times10^{-19}}{1}$$

$$=1{\cdot}12\times10^{-3}A$$

$$=1{\cdot}12mA$$

Example 7.3

Calculate the drift velocity of electrons in a metal conductor of area of cross-section $10^{-5}\,m^2$ for a current of 20 A. Assume that there are $8{\cdot}4\times10^{28}$ electrons per cubic metre. Charge on the electron $=1{\cdot}6\times10^{-19}$C.

Solution.

We know that

$$I=n_0ev_dA$$

$$\text{Drift Velocity } = v_d = \frac{1}{n_0eA}$$

$$=\frac{20}{8{\cdot}4\times10^{28}\times1{\cdot}6\times10^{-19}\times10^{-5}}$$

$$=1{\cdot}485\times10^{-4}\ ms^{-1}$$

Example 7.4

A copper coil has a resistance of 30 ohms at 20°C. Find its resistance at 60°C. Take α for copper as $4{\cdot}0\times10^{-3}$ per degree.

Solution.

Since

$$R_{20}=R_0\,(1+20\alpha) \text{ where } R_0=\text{resistance at } 0°C.$$

and $$R_{60}=R_0\,(1+60\alpha)$$

$$\therefore \quad \frac{R_{60}}{R_{20}}=\frac{1+60\alpha}{1+20\alpha}$$

$$\text{or } R_{60}=R_{20}\frac{(1+60\alpha)}{(1+20\alpha)}$$

$$=30\frac{(1+60\times4{\cdot}0\times10^{-3})}{(1+20\times4{\cdot}0\times10^{-3})}$$

$$=34{\cdot}5\ \Omega$$

7.2 THERMAL EFFECT OF CURRENT (*Joule's Heat*)

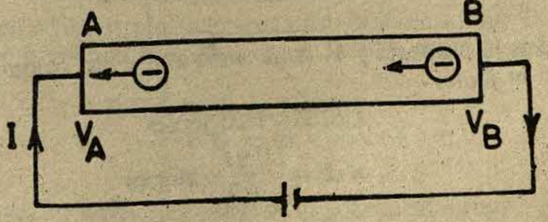


Fig. 7.8

Consider a copper wire AB (Fig. 7.8) through which a current I ows in the direction A to B. Electrons move from the end B at potential V_B to the end A at potential V_A. When the electron is at the end B of the wire, its *potential energy is $-eV_B$ When it reaches A, its potential energy is $-eV_A$, which is more negative. The potential energy of the electron has decreased by

$$e\,[V_A - V_B] = eV \text{ where } V = (V_A - V_B).$$

If the electron were free to move unhindered from B to A, this loss in potential energy would reappear as an increase in kinetic energy of the electron. When the electron is continually colliding with the atoms of the wire, its average kinetic energy is not allowed to increase. In between collisions the electric field accelerates the electron and increases its kinetic energy but the ensuing collision transfers this extra kinetic energy to the copper atoms which are thereby made to vibrate more rapidly. In other words the average kinetic energy of the copper atoms increases. The energy therefore finally appears as *heat* in the copper wire. It is well-known that a wire warms up when an electric current flows through it. The heat generated in this way is known as *joule heat*.

In t seconds a charge

$$-q = -I\,t$$

leaves B and an equal charge arrives at A. The charge leaving B initially had potential energy $-qV_B$, but the charge arriving at A has a potential energy of $-qV_A$. The *loss* in potential energy is therefore

$$q\,(V_A - V_B) = qV$$
$$= ItV$$

This energy appears as heat in the wire.

$$\therefore \quad \boxed{H = IVt} \quad \text{where H = heat developed}$$

Since according to Ohm's Law V=IR where R is the resistance of the wire.

$$\therefore \quad \boxed{H = I^2Rt}$$

If I is in amperes ; R is in ohms and t in seconds, then H is measured in *joules*.

$$\text{i.e., } H = I^2Rt \text{ joules}$$

$$\text{or } H = \frac{I^2Rt}{4{\cdot}2} \text{ calories}$$

$$(\because 1 \text{ cal} = 4{\cdot}2 \text{ J})$$

---

* *Potential Energy* of an electron is the amount of energy that an electron acquires in raising it to a certain potential or in accelerating it through a certain potential difference.

This is known as *Joule's Law of heating*. It states that the amount of heat produced in a conductor is proportional to

(*a*) the square of the current,

(*b*) the resistance of the conductor, and

(*c*) the time for which the current flows.

The rate at which the electrical energy is converted into heat is known as *Electric power* (P).

$$\therefore\quad P=\frac{H}{t}=IV=I^2R \qquad (\because\ V=IR)$$

Thus the power dissipated in a metallic conductor (wire) is proportional to the square of the current in the wire and directly proportional to the wire's resistance.

Thus electric power (P) = current (I) × Potential difference (V). If I is in amperes and V is in volts then electric power P is measured in *watts i.e., joules per second, i.e.,*

$$\therefore\quad \boxed{\text{watts} = \text{volts} \times \text{amperes}}$$

One watt is equal to one joule per second. Sometimes a bigger unit known as *kilowatt* is used.

1 kilowatt = 1000 watts

Although electrical devices are rated according to their power usage, the consumer who uses electricity to operate them actually pays for *electrical energy, which is the product of the power consumed by the device and the time during which it is operating.* This is the rate at which energy is delivered to an appliance and the length of time it operates.

The unit of electric energy commonly used is a *watt-hour. The energy consumed in an electric circuit is one watt-hour if a power of one watt is used for one hour or if a current of one ampere flows through under a potential difference of one volt for one hour.*

watt hours = watts × hours

= volts × amperes × hours.

In practice a bigger unit known as the *kilo-watt-hour* (kWH) is used. It is also known as the Board of Trade unit or simply the *unit.*

$$\begin{aligned}1\ \text{kWH} &= 1000\ \text{watt-hours}\\ &= 1000\ \text{watts} \times 60 \times 60\ \text{sec.}\\ &= 36 \times 10^5\ \text{joules}\end{aligned}$$

The number of units consumed in an electrical circuit.

$$=\frac{\text{watts} \times \text{hours}}{1000}$$

$$=\frac{\text{volts} \times \text{amperes} \times \text{hours}}{1000}$$

The students should carefully note that watt or kilowatt is the unit of *power* and watt-hour or killowatt-hour is the unit of *energy.*

Example 7.5 :

An automobile battery produces a potential difference of 12 volts between its terminals. A head light bulb is to be connected directly across the terminals of the battery and dissipate 40 watts of heat. What current will it draw and what must its resistance be ?

Solution.

$$\text{Electric Power} = \frac{H}{t} = IV$$

$$40 \text{ watts} = (I \text{ amps}) \times (12 \text{ volts})$$

$$\text{or} \quad I = \frac{40}{12} = 3.33 \text{ amp.}$$

The bulb draws 3.33 amp from the battery.

Again,

$$\text{Electric Power} = \frac{V^2}{R}$$

$$40 = \frac{(12)^2}{R}$$

$$R = \frac{144}{40}$$

$$\text{or} \quad R = 3{\cdot}6 \text{ ohms}$$

Example 7.6

A heater is marked 500 watt 220 volts.

Find

(*i*) the current it draws,

(*ii*) its resistance, and

(*iii*) the cost of using it for four hours at 15 paise per unit.

Solution.

Power of the heater = 500 watts

Voltage at which it gives this power = 220 volts

But watts = volts × amperes

$$\text{current} = \frac{\text{watts}}{\text{volts}} = \frac{500}{220} = 2{\cdot}27 \text{ amps.}$$

$$\text{Resistance} = \frac{\text{volts}}{\text{amps}} = \frac{220}{2{\cdot}27} = 96.8 \text{ ohms}$$

$$\text{Total energy consumed} = \frac{\text{watts} \times \text{hours}}{1000}$$

$$=\frac{500\times 4}{1000}$$

$$=2 \text{ units}$$

Total cost $=2\times 15=30$ P.

Thermo-electricity—Seebeck Effect :—If a copper wire and an iron wire are connected together at one end (Fig 7.9) and the

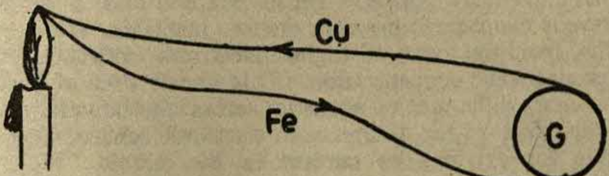


Fig. 7.9

other ends are connected to the terminals of a low resistance galvanometer and the free junction is heated in a Bunsen flame, a *current is produced in the circuit* and this causes a deflection of the galvanometer. The current produced flows *from copper to iron through the hot junction.* Such a current is known as *Thermoelectric current.* It was first discovered by Seebeck in 1821, so this phenomenon is known as *Seebeck Effect.* The arrangement shown in the diagram is known as *Thermocouple.*

It is obvious that an electromotive force is developed in the circuit as a result of the difference of temperature and the *e m.f.* is, therefore, called *Thermo-e m.f.*

In his original experiment Seebeck used copper and Bismuth and heated their junction by holding it in his hand. Later, he noticed a similar behaviour in many other pairs of metals. As a result of his investigations, he found that the value of the thermo-*e.m.f.* is different for different pairs of metals. Below is given a list of metals so arranged that the thermo-electric current flows *across the hotter junction* from the metal occurring earlier to the metal occurring later in the list.

————————across the hotter junction——·——→
Bi, Ni, Co, Pt, Cu, Mn, Pb, Sn, Cr, Au g, Zn, Cd, Fe, Sb.
←——— ———across the colder junction ←— ———

Experiments show that if the amount of thermoelectric *e.m.f.* is greater, the more separated are the metals in this series. Thus the *e.m.f.* developed in a Bi-Sb thermocouple is greater than in a Cu-Fe thermocouple.

The following features of the thermo-electric current are of great interest and practical use :

(*i*) The current produced is very feeble and—the method is never used for generating currents.

(*ii*) The strength of the current produced depends upon the difference of temperatures between the two junctions of the wires and, therefore, the arrangement is used for measuring temperature differences and the actual temperature of the junctions.

Origin of Thermo-e.m.f. We know that all conductors contain a large number of free 'electrons'. Normally the concentration of electrons at the interface in the two metals is not the same as it depends upon the nature of the metal and its temperature Hence, when two dissimilar metals are brought in contact and a difference of temperature is maintained between the two junctions, the electrons tend to diffuse from the metal of higher electronic concentration to that of lower electronic concentration. This transference of electrons produces a contact difference of potential across the boundary which neutralises the effects of the difference in electronic concentration and is responsible for driving the current in the circuit. When the temperature of the two junctions is the same, this contact difference of potential is also identical across the two junctions and so there is no resultant E.M.F. in the circuit and hence no current flows in the circuit. When, however, one junction is heated relatively to the other, the contact potential at the heated junction becomes more than that at the cold junction and the difference acts as the resultant E.M.F. which drives the current through the circuit.

Thermo-e.m.f. and temperature : For a given pair of metals the thermo-*e m.f.* depends (1) on the temperature of the cold junction

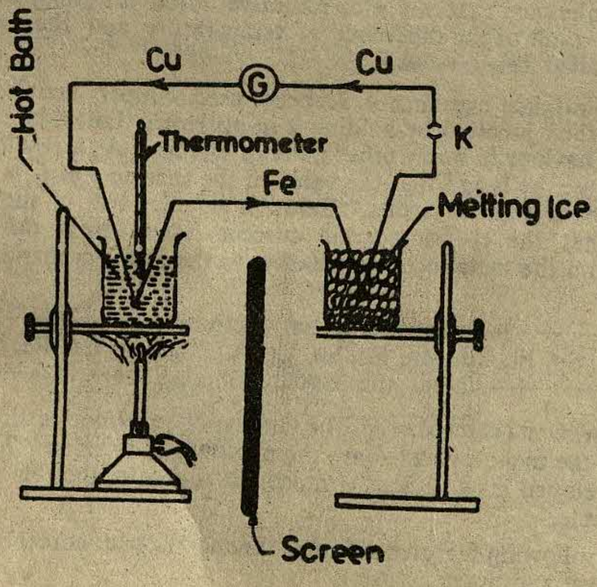


Fig. 7.10

and also (2) on the difference between the temperature of the two junctions. If one junction of a thermocouple consisting of metals A and B, be maintained throughout at 0°C and the other junction heated to various temperatures (Fig 7.10) it is found that *e.m f.* in the circuit varies with the temperature of the hot junction in the manner shown in Fig (7.11). The thermo *e.m.f.* increases in value as the temperature is raised higher and higher till it reaches a maximum at a certain temperature *Tn,* called the *neutral 'temperature.* For *a* Cu-Fe couple the neutral point is about 270°C. After this point the *e.m.f.* diminishes as the temperature of the hot junction is raised higher and higher till at a temperature T_i the *e.m.f.* is zero. This temperature is known as *temperature of inversion* or *temperature of reversal.* When this temperature is raised the e.m.f. in the circuit is reversed. The phenomenon of inversion of e.m.f. was discovered by *Cumming.* For the Copper-Iron couple the temperature of reversal is reached at about 540°C when the cold junction is kept at 0°C. Thus the temperature of inversion is as much above the neutral temperature as much the latter is above that of the cold junction. Moreover the neutral temperature is always the same irrespective of the temperatures of the junctions of the given thermocouple.

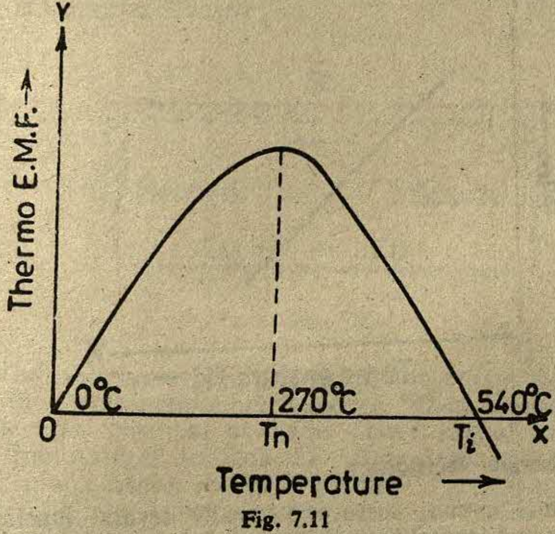


Fig. 7.11

All the above properties follow from the symmetry of the curve about the neutral temperature.

Thermo-electric Power : The curve obtained by plotting *e.m.f.* against temperature of the hotter junction, the colder junction being kept at 0°C ; is a parabola (as shown in Fig. 7.11) and is symmetrical with respect to the ordinate for the maximum e.m.f. The value of the e.m.f. for a given thermocouple is expressed by the relation,

$$E_0^t = at + bt^2$$

where a and b are constants for the given metals and t and 0°C the temperature of the junctions respectively.

The rate of change of the thermo-*e.m f.* with respect to temperature *i e.*, $\frac{dE}{dt}$ is called the *Thermo-electric Power.*

$$\therefore \quad E_0^t = at + bt^2$$

Thermo-electric power at t°C

$$\frac{dE}{dt} = a + 2bt$$

Thus thermoelectric power has a linear relation with temperature and its graph is a straight line called thermo-electric line (Fig. 7.12) which crosses the temperature axis at the neutral point, as at the neutral point, the e.m.f. obtained is maximum and hence

$$\frac{dE}{dt} = 0.$$

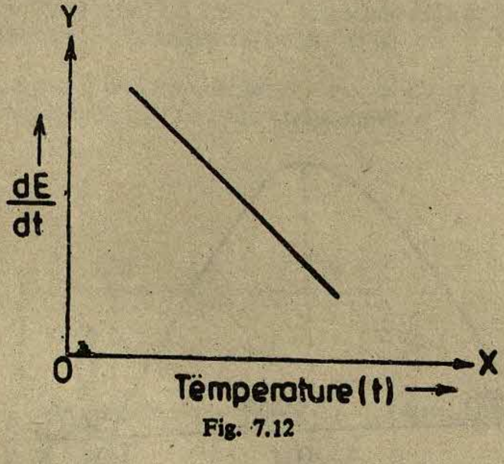


Fig. 7.12

Laws of Thermo-electricity :

In a normal circuit, there are usually several junctions of dissimilar metals and several temperatures. It is essential, therefore, that Laws of addition of Thermo-e m.f's be formulated. These laws are obtained experimentally.

1. **Law of Intermediate Metals** : "If a number of metals are in successive contacts so as to form a chain, the electromotive force between the extreme metals, when placed in direct contact, is the sum of the electro-motive forces between the successive adjacent metals."

If A and N are the extreme metals, and B, C, D,......M, the intermediate metals, then

$$E_A^N = E_A^B + E_B^C + E_C^D + \ldots\ldots\ \ldots + E_M^N$$

provided all the junctions in the series are at the same temperature.

2. Law of Intermediate Temperature :-"For a given thermocouple the electromotive force ${}^nE_1$ for a given temperature difference of t_1 and t_n is the sum of the electromotive forces for any number of successive steps into which the given range of temperature may be divided."

or $${}^nE_1 = {}^2E_1 + {}^3E_2 + {}^4E_3 + \ldots\ldots\ldots\ldots + {}^nE_{n-1}$$

where t_1, t_2, t_3..... are successive intermediate temperatures and ${}^2E_1$, ${}^3E_2$, ${}^4E_3$......are e.m.f.'s for successive temperature differences

***Peltier and Thomson Effects** : In 1834, Peltier observed the inverse of the Seebeck Effect. He found that when a current is passed across the junction between two different metals (Fig 7.13) heat is either generated or absorbed at the junction. This phenomenon is known as *Peltier effect.*

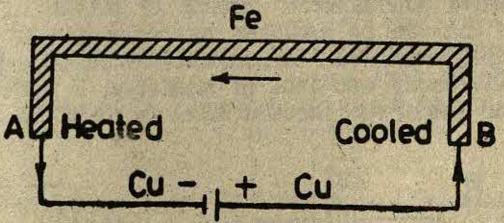


Fig. 7.13

The evolution or absorption of heat energy at a junction is due to the *contact potential differences.* When *Cu* and *Fe* are placed in contact the potential of *Fe* is above that of *Cu*. At B current flows from Copper to Iron *i.e.*, from lower potential to higher potential and hence energy is required for this purpose ; this is absorbed from the junction and hence it is cooled. At A current flows from *Fe* to *Cu*, *i.e.*, from higher potential to lower potential and thus energy is given out so that the junction A is heated.

The Peltier effect should be distinguished from the Joule Effect in which heat is generated (according to Joule's Law) in proportion to (*i*) (current)^2 and (*ii*) resistance. In the Peltier effect, the amount of heating or cooling (*i*) is proportional to the current from the battery and (*ii*) is independent of resistance, While in Peltier effect it depends upon the (*iii*) direction of the current whether a junction is cooled or heated, the Joule Effect is independent of the direction and there is always heating produced depending on the resistance. The Peltier effect is easily detected by using thick bars of metal, the Joule effect in them being negligible on account of their negligible resistance.

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* For bright students only.

William Thomson (Lord Kelvin) found out that if a temperature gradient exists in a conductor this produces an electric potential gradient in the conductor. He found that heat is evolved (other than joule's heating) when a current passes from the hot end to the cold end of a copper rod having a temperature gradient. When the current is reversed heat is absorbed. This reversible effect, first observed and pointed out by Thomson, is known as the *Thomson effect.*

Since the density of the conduction electrons is dependent upon the temperature, there will be a gradient of concentration of these electrons in the same metal if there is a temperature gradient along it. To counteract the tendency of the conduction electrons to diffuse and equalise the concentration, there must be developed an electric pressure or a potential gradient along the metal. This is Thomson effect.

***Measurement of Thermo-electric E.M.F :** The thermo-e.m.f.'s are very small and never exceed 30-40 millivolts. To measure the same, a sensitive potentiometer capable of reading in microvolts is required. This can be arranged by connecting two resistance boxes R and S in series with a potentiometer wire AB which is one metre long and exactly one ohm in resistance, a battery *Ba*, one way key K and a thick wire rheostat R*h* as shown in Fig. 7.14.

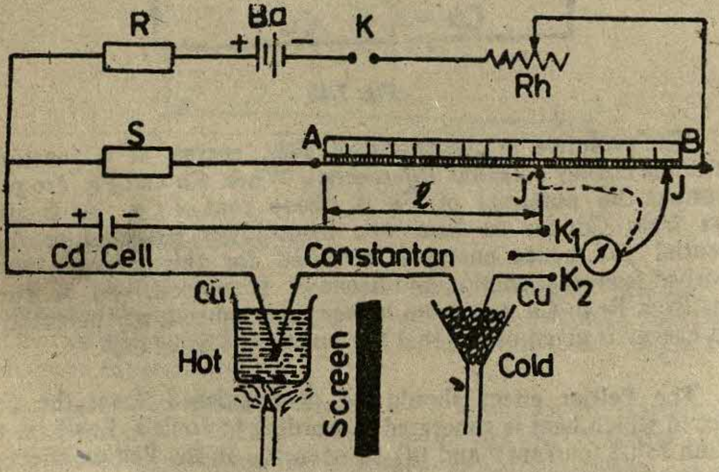

Fig. 7.14

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* For bright students only.

The potentiometer wire is first calibrated to read accurately to one microvolt. For this purpose a standard cadmium cell (E.M.F.-1.0183 volts) is connected along with a galvanometer G to the above arrangement. The positive terminal of the cell is connected to the positive end of the resistance box S and the negative terminal to key k_1 and through galvanometer G to the negative end B of the potentiometer wire (Fig 7.14). A resistance of 1017·3 ohms is introduced in S and the resistance in R is so adjusted that the galvanometer shows no deflection using the rheostat, if necessary. This means the resistance in S and AB equal to 1018·3 ohms, has a fall of 1.0183 volts.

Hence the Potential fall in the wire AB of 1 ohm

$$\doteq\frac{1{\cdot}0183}{1018{\cdot}3}\doteq 10^{-3}\text{ volt.}$$

Since the length of AB = 100 cms, the fall of potential per *mm* is 10^{-6} volt, *i.e.*, one *microvolt.*

To measure the e.m.f. developed in the thermo-couple, the cadmium cell is replaced with the thermocouple, by opening key K_1 and closing key K_2, taking care that the *positive terminal of the thermocouple is connected to the positive end of S* (Fig 7.14). Now the balance point J′ on the potentiometer wire is found when there is no deflection in the galvanometer. It may be necessary to introduce some fresh resistance (a few ohms) in S in order to get the balance point on the wire. In such a case the same amount of resistance must be removed from R so that the total resistance in R and S is always kept constant and the calibration is undisturbed. Supposing that in obtaining J′ as the balance point, in all X ohms have been introduced in S and the length AJ′ in l cms; the e.m.f. of the thermocouple

$$=(X\times 10^{-3}+l\times 10^{-5})\text{ volts.}$$

Temperature Measurement by Thermocouples (*Thermopile*) :

For measuring the intensity of heat radiations and hence the temperature, several junction of dissimilar metals, *e.g.* Bismuth and

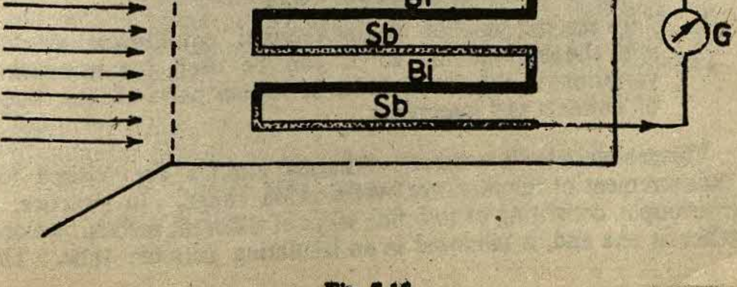


Fig. 7.15

Antimony are joined in series as in (Fig 7.15). One set of junctions is exposed to the radiation while the others are covered with an insulation lid. The exposed ends are blackened so that most of the radiation falling on them is absorbed. This arrangement is called a *thermopile*. This radiation raises the temperature of the exposed ends relative to the opposite ends. The thermal e.m.f. depends on the number of junctions used. A galvanometer G is connected to the thermopile to measure the current which flows. This is proportional to the radiation intensity and temperature. Thus previously calibrated galvanometer can be used to read directly the temperature.

The instrument is so sensitive that it can detect heat radiations from a candle at a distance of 100 metres.

The linear thermopile is used in research for the study of infrared radiations.

The following points should be kept in mind while constructing a sensitive thermopile :

- **(*i*) The metal used should give large thermo-e.m.f.**
- **(*ii*) The conducting wires should be thick to avoid loss of heat by conductor.**
- **(*iii*) The junctions should be as thin as possible.**
- (*iv*) In order to increase the sensitiveness and to keep the deflections steady, it should be mounted in vacuum. In this way loss of heat by conduction and convection is also minimised.
- (*v*) The galvanometer should be of low resistance type.

As a temperature measuring device, the thermocouple has many *merits* :

1. Because of its smallness of its test junction, it is particularly suitable for measuring temperature in very small regions and cavities.
2. **It is accurate as temperatures can be measured to an accuracy of $\frac{1}{10}$th of a degree and the e.m.f. can be measured with an accuracy of about 10^{-6} volt.**
3. Since the test junction attains thermal equilibrium quickly with the surroundings so it can be used for measuring variations in the temperature in various parts of the body of animals and insects etc.

Thermo-electric Pyrometer : Thermal e.m.f.'s are utilised for the measurement of temperature over a wide range. In practice, a thermocouple consisting of two fine wires of different metals, welded together at one end, is mounted in an insulating ceramic tube. The free ends of the wires are led to terminals at the end of the tube, Fig. 7.16. The terminals are connected directly to a moving coil galvanometer calibrated for direct reading of temperature difference. In this form the junctions of thermocouple wires with the galvanometer leads are maintained at room temperature. The variations in room temperature do not affect the readings provided that the hot junction temperature is fairly high. This Pyrometer is used for measuring furnace temperatures. Calibration is achieved by using fixed temperature points.

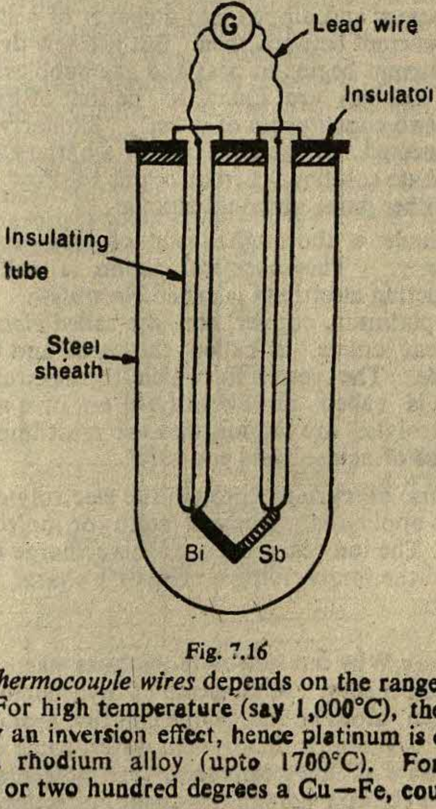


Fig. 7.16

Choice of thermocouple wires depends on the range of temperature required. For high temperature (say 1,000°C), the metals must not melt or show an inversion effect, hence platinum is often coupled with a platinum rhodium alloy (upto 1700°C). For temperature difference of one or two hundred degrees a Cu—Fe, couple is useful.

Example 7.7 :

The thermo-e.m.f. of copper-constantan thermocouple is $50\mu V$ per degree at room temperatures. A galvanometer of resistance 50 ohms and capable of detecting currents of the order of $2\mu A$ is used. Calculate the smallest temperature difference that can be detected with this galvanometer.

Solution.

Current $= I = 2\mu A \quad 2 \times 10^{-6} A$

galvanometer resistance $= G = 50$ ohms

$$V = I \times G = 2 \times 10^{-6} \times 50$$
$$= 10^{-4} \text{ volts}$$

It means that the galvanometer can detect a potential difference of 10^{-4} volts.

$\therefore\ 10^{-4}=50\times10^{-6}\times\theta$ where θ is the smallest
or $\theta=2K$ temperature difference.

7.4. CHEMICAL EFFECTS OF ELECTRIC CURRENT

Electrolysis : If two small strips of platinum are connected to the terminals of a battery and dipped into a beaker of distilled water, it is difficult to pass current between them. But if a few drops of sulphuric acid are added current begins to pass and gas bubbles appear at both plates. These bubbles are found to be that of hydrogen (H) and oxygen (O), the two constituents of water. Similarly if two plates of copper are connected to the terminals of a battery and then dipped into copper sulphate solution, current begins to flow. One plate loses copper and the other gains an equal amount.

Faraday made a thorough study of this process of chemical action by electricity. This chemical action is called *electrolysis.* The liquid conducting electricity is called *electrolyte*. The two plates such as those of platinum, copper, etc., are called *electrodes* ; the one where the current enters, is called the *anode* and the one where it leaves the *cathode*. The vessel in which the electrolyte is contained for electrolysis is called an *electrolytic cell* or a *voltameter*. The majority of electrolytes are liquids, and the most important ones are aqueous solutions of acids, bases and salts.

The carriers of current through the electrolyte are called ions. The term 'ion' is applied to a charged atom or molecule moving in a liquid or gas. The ion carrying a positive charge is called positive ion or *cation* and the ion carrying a negative charge is called a negative ion or *anion*.

Dissociation Theory. Why an electrolyte conducts electricity is explained by the dissociation theory. A molecule of hydrochloric acid consists of one hydrogen atom bound to one chlorine atom. Its chemical formula is HCl. When it dissolves in water, some of its molecules ionize and dissociate. This means that the molecule breaks apart, the hydrogen and chlorine going in opposite directions. At the time of separation the chlorine atom seizes from the hydrogen atom the only electron it possesses The chlorine atom thus gets one extra electron and the hydrogen atom is without any. So the chlorine atom has a negative charge of one unit. Atoms in a charged condition such as these, are called ions. The symbols for these ions are Cl^- and H^+. Ions carry electric charges through an electrolyte.

When a potential difference is applied between the two plates placed in a voltameter, an electric field is set up between them. The positive ions move towards the negative electrode or cathode and the negative ions towards the positive electrode or anode. The ions on reaching the electrodes give their charge to the electrodes and become neutral. These then come out as gases or deposit as a thin layer on the electrodes.

This flow of positive ions in one direction and negative ions in the opposite direction is equivalent to a current flowing in the electrolyte. The direction of this current is the same as that of the positive ions.

Water Voltameter : Fig 7.17 shows a simple form of voltameter for the study of the electrolysis of water. It consists of a glass vessel with two platinum electrodes E_1 and E_2 fused into it. Platinum electrodes are used because they are not acted upon by the sulphuric acid. The vessel contains water mixed with few drops of sulphuric acid. Water is a poor conductor of electricity. Sulphuric acid is added to make it good conductor. Two graduated tubes T_1 and T_2 filled with acidulated water are inverted over the electrodes and held in position as shown. This arrangement is called water voltameter. When current is passed through it, water is decomposed into oxygen and hydrogen. Oxygen is collected at the anode in tube T_1 and hydrogen is collected at the cathode in tube T_2.

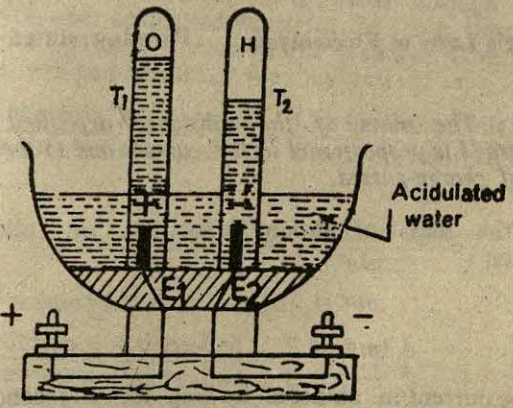


Fig 7.17

Theory : Sulphuric acid when dissolved in water dissociates into hydrogen and sulphate ions as follows :

$$H_2SO_4 \rightarrow 2H^+ + SO^-_4{}^-$$

The hydrogen ion (H^+) carrying a unit positive charge moves towards the cathode and the sulphate ion ($SO^-_4{}^-$) carrying two units of negative charge moves towards the anode. The hydrogen ions give their charge to the electrode and collect at the top inside the tube T_2. The sulphate ions too give up their charge at the cathode. They being unstable combine with water to form sulphuric acid as below:

$$2SO_4 + 2H_2O \rightarrow 2H_2SO_4 + O_2$$

The oxygen so liberated collects at the anode in the tube T_1

Copper Voltameter : A copper voltameter consists of a glass vessel containing copper sulphate solution as electrolyte and two copper plates as electrode Fig 7.18. When copper sulphate is dissolved in water it splits up into Cu^{++} and SO_4^{--} ions. The copper ions carry two units of positive charge and the sulphate ions two units of negative charge.

$$CuSO_4 \rightarrow Cu^{++} + SO_4^{--}$$

When the electrodes are connected to a battery through an ammeter and a rheostat the Cu^{++} ions move towards the negative plate and after delivering their charge become neutral atoms. The SO_4^{--} ions go to the positive plate and combine with the copper atoms of the electrode to form $CuSO_4$. The $CuSO_4$ so formed goes into solution and the concentration of the solution is maintained. Thus the cathode gains as much copper as the anode loses.

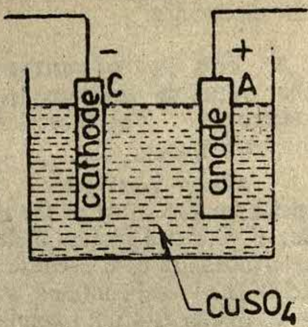


Fig 7.18

Faraday's Laws of Electrolysis : Faraday stated two laws of electrolysis :

Law I : *The mass of the substance deposited or collected at either electrode is proportional to the current and to the time i.e., to the quantity of charge passed.*

If m is the mass of an ion deposited or collected at the electrode, then

$$m \propto It$$

or

$$m \propto a.$$

where I is the current in amperes flowing for t seconds and q is the total charge in coulombs passing through the electrode.

$$\boxed{m = zIt = zq} \qquad \ldots(1)$$

where z is the constant of proportionality and is called the *electro chemical equivalent* (E. C. E.) of the substance. Its value depends upon the nature of the substance. Thus we *may define E. C. E. as the mass in grams liberated by one coulomb of electricity passing through an electrolyte.* Hence

$$\text{E.C.E.} = \frac{\text{Mass of the substance deposited}}{\text{Current} \times \text{Time}} \qquad \ldots(2)$$

The value of E.C.E. of a few substances is given below :

Element	E. C. E. (g/coulomb)
Silver	0.0011180
Copper	0.0003294
Gold	0.0006812
N	0.0003041
Oxygen	0.00008291
Hydrogen	0.00001046

Law II *The masses of different substances liberated by the same quantity of electricity are proportional to their chemical equivalent.*

According to this law if a current is passed through a solution of $CuSO_4$, $AgNO_3$ and acidulated water for the same time, then for every one gram of hydrogen liberated, 108 grams of silver and 31·5 grams of copper will be liberated. We may write it as

$$\frac{\text{Mass of substance A deposited}}{\text{Mass of substance B deposited}} = \frac{\text{Chemical Equivalent of A}}{\text{Chemical Equivalent of B}} \quad ...(3).$$

$$\left(\text{Note : Chemical Equivalent} = \frac{\text{Atomic weight}}{\text{Valency}}\right)$$

From the relation (2) above we get

$$\frac{\text{Mass of substance A deposited}}{\text{Mass of substance B deposited}} = \frac{\text{E.C.E. of A}}{\text{E.C.E. of B}}$$

$$\text{or} \frac{m_A}{m_B} = \frac{E_A}{E_B}$$

Thus relation (3) becomes

$$\frac{\text{Mass of A deposited}}{\text{Mass of B deposited}} = \frac{\text{Chemical equivalent of A}}{\text{Chemical equivalent of B}}$$

$$= \frac{\text{E.C.E. of A}}{\text{E.C.E. of B}}$$

Experimental Verification : To verify the first law experimentally take a copper voltameter and connect a battery, an ammeter, a key and a rheostat in series with it as shown in Fig 7.19. Pass a known current for a certain length of time. By weighing the cathode before and after passing the current, calculate the amount of copper deposited.

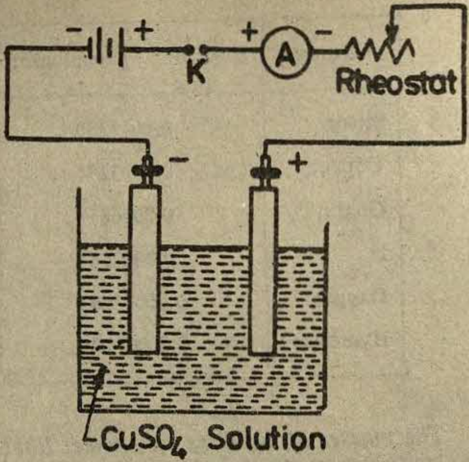


Fig. 7.19

Repeat the experiment with double the current but for the same time. It will be found that the mass of copper deposited is double than that in the first case. Repeat the experiment once again, passing the same current as in the first case but for double the time. It will be found that by doubling the time also, the amount of copper deposited becomes double. This verifies the first law of Faraday.

To verify the second law, take three voltameters containing acidulated water, copper sulphate solution and silver nitrate solution respectively. Join the three in series with a battery, ammeter and a rheostat as shown in Fig. 7.20. Pass a suitable current through them for sufficient time. As they are all in series the same current will pass through them for the same time. Measure the volumes

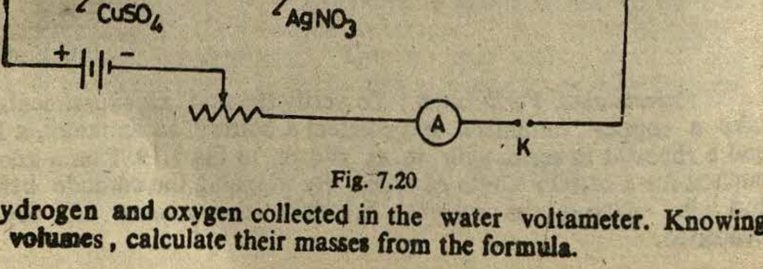


Fig. 7.20

of hydrogen and oxygen collected in the water voltameter. Knowing the volumes, calculate their masses from the formula.

Mass of gas = Volume of gas at N.T.P. × Density of gas at N.T.P. By weighing the cathodes of the $CuSO_4$ and $AgNO_3$ voltameters before and after the passage of current. Calculate the amount of Cu and Ag deposited in the two cases.

It is found that for one gram of hydrogen liberated, 8 grams of oxygen, 108 grams of silver and 31·5 grams of copper are liberated by the current. But these numbers give the chemical equivalents of these substances respectively. Hence the second law is also proved.

Faraday : According to Second Law of Electrolysis

$$\frac{m_A}{m_B}=\frac{E_A}{E_B}$$

From the First Law of Electrolysis

$$m_A = Z_A q$$

$$\text{and } m_B = Z_B q$$

Combining the two laws, we have

$$\frac{E_A}{E_B}=\frac{Z_A}{Z_B}$$

or

$$\frac{E_A}{Z_A}=\frac{E_B}{Z_B}$$

In general,

$$\boxed{\frac{E}{Z}=\text{constant}=F}$$

where F is known as *Faraday*.

From First Law of Electrolysis,

$$m = Zq$$

$$Z = \frac{m}{q}$$

$$F = \frac{Eq}{m}$$

If $m = E$, then

$$\boxed{F = q}$$

Hence a Faraday is the quantity of electricity required to liberate one gm equivalent of the substance.

One Faraday represents 96500 coulomb of electricity.

$$\text{Charge on a } \textit{monovalent ion} = e = \frac{F}{N} = \frac{96500}{6{\cdot}023 \times 10^{23}}$$

$$= 1{\cdot}6 \times 10^{-19}\text{C}$$

where N is Avogadro's Number.

(For monovalent ion, Atomic weight = Chemical eq.wt.)

Example 7.8 :

For what time must a current of 2·5 amp. pass through a solution of zinc sulphate to deposit 1 gm of zinc ? (E.C.E. of zinc = ·0003387 g/coulomb)

Solution : We have

$$m = zIt$$

Here

$$m = 1 \text{ g}$$

$$z = {\cdot}0003387 \text{ g/coulomb}$$

$$I = 2{\cdot}5 \text{ A}$$

$$t = \frac{m}{zI} = \frac{1}{{\cdot}0003387 \times 2{\cdot}5}$$

$$= 1181 \text{ seconds}$$

$$= 19 \text{ minutes and } 41 \text{ seconds}$$

Example 7.9 :

A current of 2 A passing for half an hour through a water voltameter liberated 423 cc of hydrogen measured at 13°C and 80 cm pressure. Calculate the E C.E. of hydrogen. (1 litre of hydrogen at N.T.P. weighs 0·089 g.)

Solution :

$$\text{Volume of hydrogen collected at N.T.P.} = \frac{423 \times 273 \times 80}{286 \times 76} \text{ cc}$$

$$\text{Mass of hydrogen} = \frac{423 \times 273 \times 80 \times 0{\cdot}089}{286 \times 76 \times 1000} \text{ g}$$

$$\text{Quantity of electricity passed} = 2 \times 60 \times 30 = 3\,600 \text{ coulombs}$$

$$\therefore \text{E.C.E. of Hydrogen} = \frac{423 \times 273 \times 80 \times 0.089}{286 \times 76 \times 1000 \times 3600}$$

$$= 0{\cdot}0000105 \text{ g/coulomb.}$$

Measurement of Current by Silver Voltameter : Since the mass of substance deposited in any given time is proportional to the current, it is possible to use a voltameter for measuring current. The standard voltameter is due to Rayleigh. It is a silver voltameter Fig. 7.21 in which a disc of silver hangs in a solution of silver nitrate contained in a clean platinum bowl. The bowl serves as the cathode and the disc as the anode. When a current is passed silver from the solution of silver nitrate decomposes according to the reaction.

$$AgNO_3 \rightarrow Ag^+ + \bar{NO_3}$$

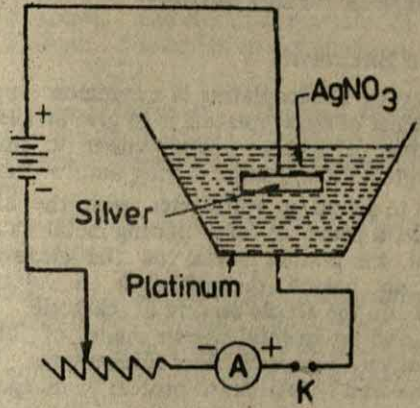


Fig 7.21

The silver so liberated goes to the bowl and is deposited there. The nitrate ions go to the anode and react with it forming. $AgNO_3$ and thus the strength of the solution is maintained constant. If the bowl is weighed before and after passing the current, the mass of silver deposited can be calculated. Knowing the time for which the current is passed and E.C.E. of silver, the current in the circuit can be calculated from the formula

$$I = \frac{m}{Zt}$$

If an ammeter is connected in series in the circuit the accuracy of the instrument can be checked by this method. Measurement of current by a voltameter is a very slow process but is very accurate.

The electro chemical equivalent of silver has been determined with great accuracy, hence this is used to define the unit quantity of electricity. According to international agreement a coulomb is defined as the quantity of electricity which liberates 0·0001118 g of silver by electrolysis.

Example 7.10 :

How many amperes are required to deposit on the cathode 5 gm of gold per hour. E.C.E. of gold =0·000681 g/ coulomb If an ammeter connected in series reads 2·0 A ., find the error.

Solution :

$$\text{Mass deposited} = \text{E.C.E.} \times \text{number of coulombs used}$$

$$= \text{E.C.E.} \times (\text{amperes} \times \text{seconds})$$

$$5\ \text{g} = 0{\cdot}000681\ \text{g/coulomb} \times I \times 3600\ \text{s}$$

or

$$I = \frac{5}{0{\cdot}000681 \times 3600} = 2{\cdot}04\ \text{A}$$

Error in the ammeter reading $= 0{\cdot}04$ A

Some Applications of Electrolysis

1. *Electroplating* : Electroplating is a common application of electrolysis. The object of electroplating is to give articles made of one metal a coating of another metal, either to improve their appearance or to protect them against rusting and tarnishing.

The method is to set up a voltameter with the article to be plated as the cathode, a piece of the plating metal as anode and a solution of a salt of the plating metal as the electrolyte. When a suitable current is passed through such a voltameter, the metal is deposited on the article serving as cathode. In this way table forks, spoons, and ornamental objects made of inferior metal can be coated with silver to improve their appearance. Iron articles are coated with nickel and chromium to protect them against rusting. They are usually first coated with copper, as nickel and chromium stick better to copper than to iron. 'Galvanized' iron sheets have a coating of zinc.

2. *Electrotyping* : Electrotyping is commonly used in printing books. A wax impression of the type is made and the surface of wax covered with a layer of graphite or other material to make it conducting. This conducting wax impression is then made the cathode in a copper voltameter and a layer of copper of sufficient thickness is deposited on the wax. The copper replica of the type is then heavily backed with an alloy of lead, tin, and antimony and is used for printing. Metals other than copper may be used in the same manner.

3. *Purification of Metals* : **Electrolysis has been found** very useful in purifying metals such as copper. Large anodes of impure copper are placed in large vessels containing a solution of $CuSO_4$ with 3·2% copper by weight and 16 free sulphuric acid. The copper anodes contain many impurities, such as lead, arsenic, iron, and possibly selenium and some gold. By using a suitable current and keeping the potential difference between anode and cathode little above 0·34 volt, all the materials but gold go into solution, the gold falling to the bottom. If the potential difference is less than 0·34 volt, only copper deposits on the cathode, the other metals remain in the solution. Zinc, cadmium and other metals may be separated in a similar manner.

4. *Decomposition of Salts* : In 1806 Sir Humphry Davy first obtained small quantities of metallic sodium and potassium by

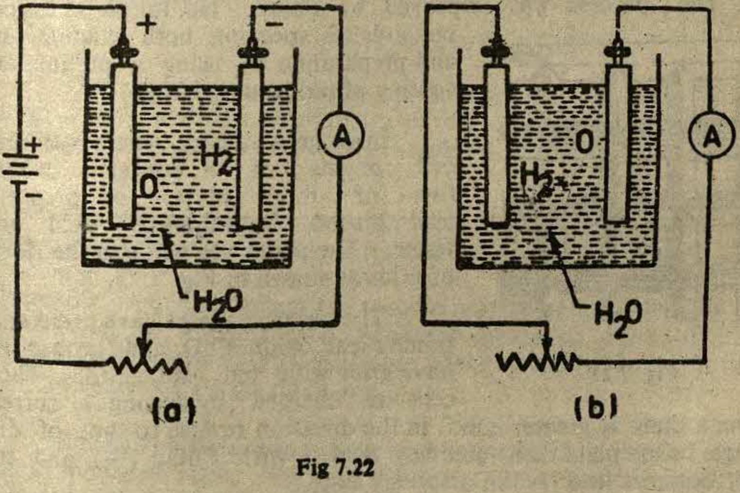
electrolysis. Today most of the supply of these metals is produced by electrolysis of sodium or potassium hydroxide with iron electrodes.

5. *Electropolishing* : Electropolishing is the reverse of electroplating and is of great interest to metallurgists. If an object, such as a stainless steel spoon, is placed in an electrolytic bath and made the anode, with copper or lead electrode as the cathode, then with a suitable electrolyte, metal is removed from the anode by the action of electric current. The elevated portions of the anode are preferentially dissolved. The result on the anode is similar to that of polishing.

6. *Manufacture of Chemical* : The manufacture of oxygen and hydrogen has been already mentioned in water voltameter. A very commonly used chemical, caustic soda, is prepared by the electrolysis of NaCl solution.

7. *Measurement of Current and E.C E.* : This has already been describedin details above.

Storage Cells : The storage cell or accumulator is an application of the principles of electrolysis. The fundamental principle of the accumulator may be demonstrated by means of two lead plates (8 cm × 6 cm.) immersed in dilute sulphuric acid and fitted as a water voltameter Fig 7.22 (a).



(a) (b)

Fig 7.22

When current is passed hydrogen is collected at the cathode and oxygen at the anode. All the oxygen liberated does not appear as gas. A portion of it combines with the lead on the surface of the anode, forming a dark grey peroxide of lead (PbO_2). Let the current be passed for sometime so that an appreciable quantity of lead peroxide is formed. Remove the battery from the circuit and reverse the connections of the voltameter as in Fig 7.22 (b). When the circuit is completed the ammeter will show a small current. Obviously this passes through the voltameter in the *opposite direction to that of the charging current*. Thus an e.m.f. which acts in the opposite direction is set up. *This opposing e.m.f. which is produced in the electrolyte due to absorption of ions at the electrodes is known as* **back e m.f.** and the phenomenon is known as *polarisation*. The plate which was the anode when being charged becomes the positive plate of the charged voltameter. During the discharge, an electrolytic process is going on. The hydrogen travels with the current and moves towards lead peroxide. The hydrogen then reacts with lead peroxide, removes oxygen from it and thus changes it to lead again. As soon as this process is completed the current stops. With repeated use the lead plates become efficient.

In the process of charging, the energy of electric current is converted into chemical action. In the process of discharging chemical action is converted into electric current.

The original type of accumulator consisted of two sheets of lead rolled up together and separated by felt or similar material. Plante, in 1860, found that frequent charging and discharging improved the efficiency of the plates because the amount of surface area increased with the formation of spongy or porous lead. Large surface area can absorb more hydrogen and hence can give current for a much longer time.

The process was improved by Faure. He found chemical methods of speeding both manufacture and preparation by using a preliminary coating of red lead (litharge).

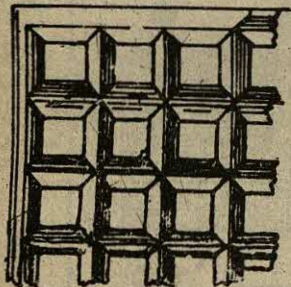


Fig. 7.23

In its present form the lead storage cell consists of a container with two or more plates of prepared lead placed in sulphuric acid and water. The plates are made in the form of grids as shown in Fig. 7.23.

The negative plates have grids containing lead oxide, PbO, and the positive have grids with red lead, Pb_3O_4. These cells are "charged" by driving a current through them at greater e.m.f. in the direction reverse to that of discharge. One plate then becomes coated with PbO_2 (+) and the other becomes lead (–) in a spongy state.

The reactions which take place during the *discharge* of the cell are as follows :

At the positive plate :

$$PbO_2+H_2=PbO+H_2O$$

$$PbO+H_2SO_4=PbSO_4+H_2O$$

At the negative plate :

$$Pb+O=PbO$$

$$PbO+H_2SO_4=PbSO_4+H_2O$$

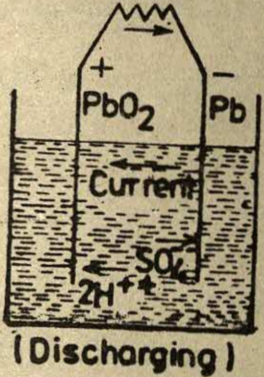


Fig. 7.24 Discharging

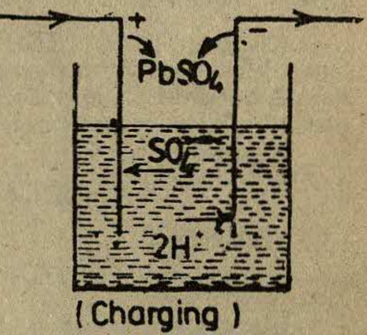


Fig. 7:25 Charging

The reactions which take place during *charging* of the cell are as follows :

At anoue :

$$PbSO_4+O+H_2O=PbO_2+H_2SO_4$$

At cathode :

$$PbSO_4+H_2=Pb+H_2SO_4$$

During the passage of the current through the cell from the negative to the positive plate , sulphuric acid is electrolysed. The hydrogen travels with the current and is liberated at the positive plate.

During charging the quantity of sulphuric acid in the liquid of the cell is increased. Hence its specific gravity increases. During discharge acid is taken from the liquid, hence its specific gravity decreases. For this reason the specific gravity of the liquid is often used for testing whether a cell is charged or is in need of furthur charging **Relativedensityof**electrolyte rises to about 1·3 on full charge and decreases to 1·1 on discharge.

Storage batteries need to be kept in use by charging and recharging at moderate rates. They must be kept clean and supplied with distilled water as it evaporates or is decomposed during charging. The lead cell has a low internal resistance and a steady e.m.f of 2·1 volts which decreases when the battery approaches complete discharge.

This type of cell is called *lead accumulator* because the positive and negative plates are of lead.

Edison Accumulator : Edison tried to make a storage cell of less weight and longer life than the lead cell. An Edison cell consists of a steel vessel containing a solution of 20% KOH in distilled water. The plates are made of steel grids. The positive plate is filled with nickel hydroxide and flaked nickel and the negative plate contains iron hydroxide having a trace of mercuric oxide which decreases its internal resistance.

Charging Process : KOH molecules dissociate into K^+ and OH^- ions. When a current is passed OH^- ions move towards the positive plate and the K^+ ions move towards the negative plate. At the plates the following reactions take place :

Anode :

$$2Ni(OH)_2+2OH \rightarrow 2Ni(OH)_3$$

Cathode :

$$Fe(OH)_2+2K \rightarrow Fe+2KOH$$

Discharging Process : During the discharging of the cell the following reactions take place :

Anode :

$$2Ni\,(OH)_3+2K \rightarrow 2Ni(OH)_2+2KOH$$

Cathode :

$$Fe+2OH \rightarrow Fe(OH)$$

Edison accumulators are light, strong and durable. They may be left uncharged for long periods without damage. Even freezing does not destroy them. It is not damaged by overcharging or over-discharging. It is 'fool proof' but costly. Its e.m.f. is about 1·2 volts but is not as steady as that of the lead cell.

Cells like Daniell and Leclanche are called primary cells. In these cells the energy is produced by chemical reactions. As aganist these the storage cells are called *secondary cells*. Here the energy is first given to the cell during the process of charging. This energy is stored in the cell and is given out when the cell is discharge.

7.5. MAGNETIC EFFECTS OF ELECTRIC CURRENT :

So far we have been developing the idea that the universe is a collection of discrete particles moving through empty, featureless space. We have also assumed that, in the ultimate analysis, the behaviour of the universe must be described in terms of the motion of these particles, and that this motion can be explained and predicted if we know certain laws that tell us the acceleration given to a particle by its interactions with the other particles. You have already studied the first two of these laws, Newton's Law of universal gravitation and Coulomb's Law for electrostatic forces in the Xth class. The next step is to consider a third type of force which exists between *two moving charges* and ***which is the origin of magnetism.*** Unfortunately the laws governing magnetic forces cannot be expressed as simple as the laws of gravitation and electrostatics. In the early nineteenth century when the laws of magnetism were being discovered, the discovery was considerably aided by the new attitude to the nature of the universe embodied in the concept of electric and magnetic fields. Towards the end of the century, Clerk Maxwell was able to explain all the known phenomena of electro-magnetism by a set of simple, elegant equations describing the behaviour of the electric and magnetic fields. Simplicity and success are two powerful advocates for any scientific theory, so we must now turn our attention towards this new idea of a *field.*

Our previous attitude implies that two bodies at a great distance from one another, with only empty, featureless space between them can nevertheless influence one another's motion. This is called *action-at-a distance* and the two bodies are said to *interact* with each other. Thus the moon and the earth interact with one another even though the space between them is apparently empty (Fig 7.26). Another kind of interaction called electric interaction occurs between two charged bodies and the force that one charged body exerts upon the other is given by *Coulomb's Law*. We are also aware of the force that one magnet exerts on another. This is called *magnetic interaction.* The interaction between electric and magnetic forces is known as *electromagnetic interaction.*

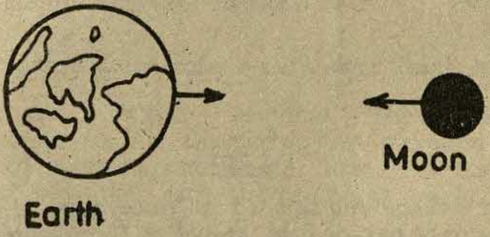


Fig 7.26

This action-at-a-distance worried many scientists and philosophers, who felt that two bodies could interact only if they were in direct contact with one another *e.g.*, pushing a body, pulling a door etc. How do the gravitational, the magnetic and the electric forces transmit themselves from one object to another ? The interaction between two bodies through intervening space can be understood by introducing the *concept of field.* Let us illustrate it by taking the examples of the force between two charges.

Electric Field and its Properties : Just as the gravitational field at any point in space is defined as the gravitational force per unit mass on a small body placed at the point; similarly the *electric field is the electric force per unit charge.* If a charge q is placed at a point where the electric field is E, the electric force F on it is q E. If q is a positive charge, F and E are in the same direction, but if q is negative they point in opposite direction.

The electric interaction between two charges can be understood by saying that one of the charges sets up an electric field in the surrounding space, this field acts on the other charge and produces the observed force. Since the force is mutual, we may regard any one of the charges to be a source of the field which acts upon the other. The electric field thus acts as ***intermediary*** for interaction between the two charged bodies.

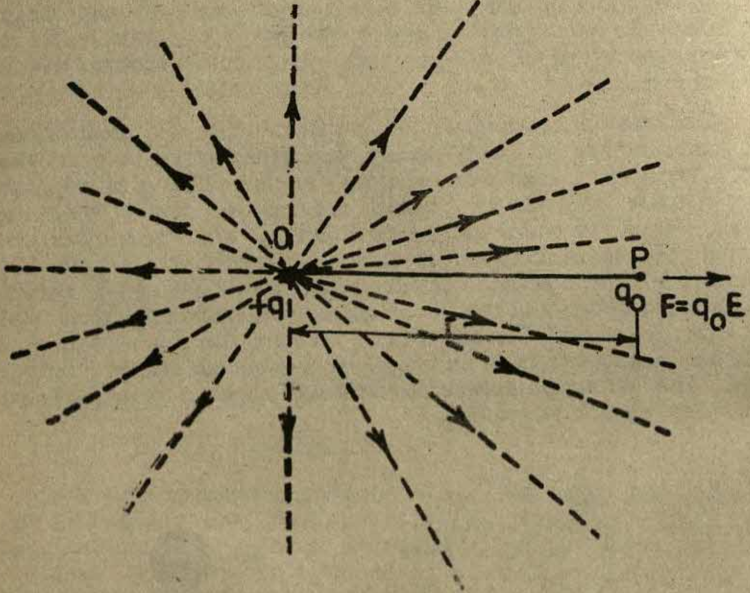


Fig. 7.27

Consider a charge $+q$ situated at some point O (Fig. 7.27) in space. It creates an electric field around it (represented pictorially by *electric field lines*). These lines are such that the direction of $\vec{E}$ at any point is tangential to the field line passing through that point. Electric field lines always diverge from a positive charge and converge upon a negative charge. We call q, the source charge. If another charge q_0, which we call the *test charge*, is placed at some point P at a distance r from the source charge q in this field, it experiences a force $\vec{F}$ given by

$\vec{F}=q_0\vec{E}$ where $\vec{E}$ is the electric field intensity at point P.

Now the electrostatic force on q_0 is

$$\vec{F}=\frac{q\,q_0}{4\pi\epsilon_0 r^2}\,\vec{r} \qquad \text{(Coulomb's Law)}$$

where $\vec{r}$ is a unit vector from q to q_0 and $\epsilon_0 = 8.851 \times 10^{-12} C^2 Nm^{-2}$ is the permittivity of free space.

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \ Nm^2C^{-2}$$

is a constant.

Hence electric field $\vec{E}$, due to q_0 at the point P is, therefore, given by

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{q}{4\pi\epsilon_0 r^2} \vec{r}$$

In the above discussion the source of the field was a stationary charge. *It is found that a moving charge is also a source of electric field* If the charge q is moving then the force on a stationary charge q_0 is still given by $\vec{F} = q_0\vec{E}$ where $\vec{E}$ is the electric field due to the moving charge q at the site of q_0 at the instant of observation. For *small* velocities of q, the field is the same as due to a stationary charge q. For *high velocities* (of the order of velocity of light *i.e.*, $3 \times 10^8 ms^{-1}$), the field becomes somewhat complicated.

An electric field $\vec{E}$ associates a *vector* with each point in space. The units of $\vec{E}$ are newton / coulomb. The electric field obeys *principle of superposition*, *i.e.*, the electric field produced at a certain point in space when there are number of sources is the *vector sum* of the electric fields that each source charge would individually produce if it alone were present.

The force equation $\vec{F} = q_0\vec{E}$ enables us to define the electric field at a point without reference to its sources. If a test charge q_0 placed at rest at a point in space experiences a force, we say an electric field exists at that point. The strength of the field is the force per unit charge on the test charge q_0 and its direction is the same as that of the force acting on a *positive* test charge.

An important property of the electric field is that the force that it exerts on a test charge is *independent of the velocity of test charge* *i.e.*, if there is an electric field $\vec{E}$ at a point, the force exerted on a test charge q_0 at that point will be $q_0 \vec{E}$, irrespective of the fact whether the charge q_0 is at rest or in motion.

Force Between Moving Charges (Definition of Magnetic Field) :

Fig. (7.28) illustrate a simple experiment to demonstrate the forces between electric currents. The vertical wires W_1 and W are freely hinged at the top and their lower ends dip into mercury pools. This leaves the wires to move sideways, but also, since mercury is a metal and conducts electricity, enables electric currents to be fed into the wires from the two storage batteries shown. When the storage batteries are connected so that the two currents flow in the *same* direction, the *wires move toward one another.* When the currents flow in *opposite* directions, the *wires move apart.* Thus *parallel currents moving in the same direction attract* one another. *Parallel currents moving in opposite directions* repel one another. It is convenient to imagine, that one current produces some kind of field which exerts a force on the other current. Let us investigate the nature of this field.

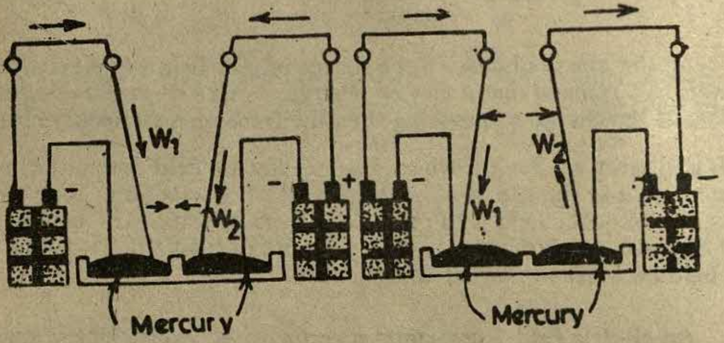


Fig. 7.28

In the above experiment the wires always contain as many protons as electrons and do not accumulate any net charge. The forces between the wires are not the electrostatic forces (Coulomb's Law), they are a *consequence of the fact that electrons are moving inside the wires.* Hence the field cannot be electrostatic. Moreover if we remove wire W_2 and place a charged conductor there, it experiences no force. *This field, therefore, does not interact with stationary charges.*

Further, if there is no current in the wire W_2, the mutual force of attraction or repulsion also vanishes, although current in the wire W_1 and its field are still there. *It follows, therefore, that the field due to the current in the wire* W_1 *acts on the wire* W_2 *only when there is a current through it i.e., when there is a flow of charge through the wire* W_2.

To test whether the field acts only upon the charges moving in a conductor or it also acts on charges moving freely, we replace the wire W_2 with an evacuated cathode ray tube to which a high voltage is applied between its two electrodes (Fig. 7.29). As the negatively charged electrons make their way from the negative electrode to the positive one carrying the electric current, the electron beam *bends.* In other words a force acts on the moving charges. *The magnitude of deflection varies with the direction of motion of the charges in the beam.* It is found that there is a direction for which the deflection is *zero* and at right angles to it, the deflection is a *maximum*. *Also the deflection is found to be proportional to the speed of the electrons.*

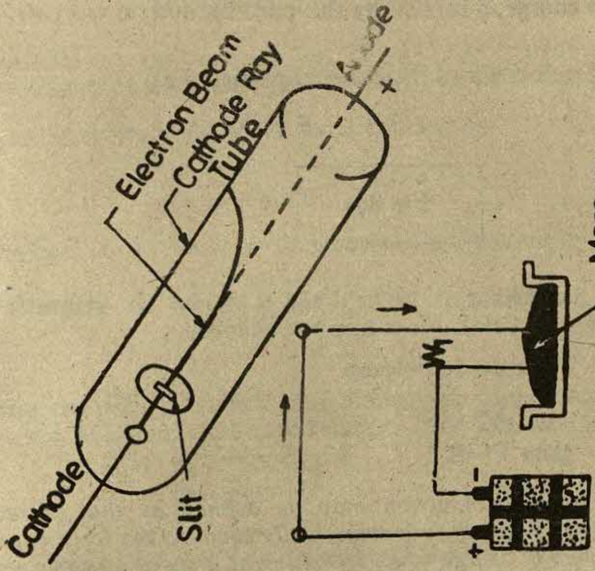


Fig. 7.29

From these observations from the above mentioned experiment we come to the conclusion that *when two charged particles are both in motion, they exert on one another a new kind of force that depends upon their velocities and is zero if either velocity is zero.* So we have a field which exerts, on a moving charge, a force, which is velocity dependent. We call it the **magnetic field**. *This new field (or force) is responsible for the magnetic properties of materials such as iron bar magnets and also for the magnetic effects of electric currents.*

If we were to proceed as we have done previously, we would consider two charges q_1 and q_2 a distance r apart moving with velocities $\vec{v}_1$ and $\vec{v}_2$. We would then write down a law, similar to Coulomb's Law of electrostatic forces expressing the force in terms of the charges, their velocities and their distance apart. Unfortunately, it is not possible to formulate such a law in any simple form. Even a reasonable approximation to the law is very cumbersome. At this point, the concept of the *electromagnetic field* makes the discussion much simpler.

We can put the observations concerning the force on a charge in a magnetic field into mathematical form. Let us imagine that a *positively* charged particle is travelling at right angles to the field, it will be found experimentally that the charged particle experiences a force which has its *maximum value.* The magnitude of this maximum force, $\vec{F}$ depends on

(*i*) the charge, q carried by the particle, and

(*ii*) the velocity $\vec{v}$ of the charged particle *i e.,*

$$i.e. \qquad F \propto qv$$

or

$$\boxed{\vec{F} = B q \vec{v}} \qquad \ldots(i)$$

where $\vec{B}$ is a constant of the field and is known as **magnetic induction** (or magnetic field). It is a *vector* quantity.

When $q=1$ coulomb

$v=1\ ms^{-1}$

then $F=B$

Thus *magnetic induction may be defined as that characteristic of the magnetic field which is numerically equal to the force experienced by one coulomb of charge while travelling at a speed of one metre per second at right angles to the field.*

From equation (*i*) we find

$$B = \frac{F}{qv}$$

Thus the unit of B is N/C ms^{-1} or N/A-m. In S.I. system, the name *tesla* (T) has been given to this unit in honour of an American Engineer Nikola Tesla.

A magnetic field of one tesla will give rise to a force of one newton on a charge of one coulomb while moving at one metre per second perpendicular to the field. In the C.G.S system the unit of B is *gauss* as a field of tesla is a very strong magnetic field. This unit is most commonly used and it is named after the German mathematician and physicist John Karl Friedrich Gauss who achieved a great deal of understanding of the earth's magnetism.

$$1 \text{ gauss} = 10^{-4} \text{T}$$

or

$$\boxed{1\text{T} = 10^{4} \text{ gauss}} \qquad \ldots(ii)$$

When the charged particle travels *along the direction* of the magnetic field instead of at right angles to it as explained above, the magnetic force on it is observed to be zero. But if the charged particle is travelling neither parallel nor perpendicular to the magnetic field then in this situation what about the force? In this general case, let the angle between the velocity $\vec{v}$ of the particle and the magnetic field $\vec{B}$ be θ. Then resolving the magnetic field $\vec{B}$ into two rectangular components $\vec{B}\cos\theta$ parallel to the velocity and $\vec{B}\sin\theta$ perpendicular to the velocity, we find that the former gives no force on the charged particle while the latter exerts a maximum force $\vec{F}$ given by

$$\vec{F} = qv\,B\sin\theta$$

Therefore, the net force in the general case is given by

$$\boxed{\vec{F} = qv\,B\sin\theta} \qquad \ldots(iii)$$

which is directed perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. In terms of vector notation, the above relation (*iii*) can be written as

$$\boxed{\vec{F} = q\,(\vec{v} \times \vec{B})} \qquad \ldots(iv)$$

which gives magnitude as well as direction of the force on the *positive* charge. The relationship between $\vec{F}$, $\vec{v}$ and $\vec{B}$ is shown in Fig 7.30. It is governed by the usual rule for a vector cross product;

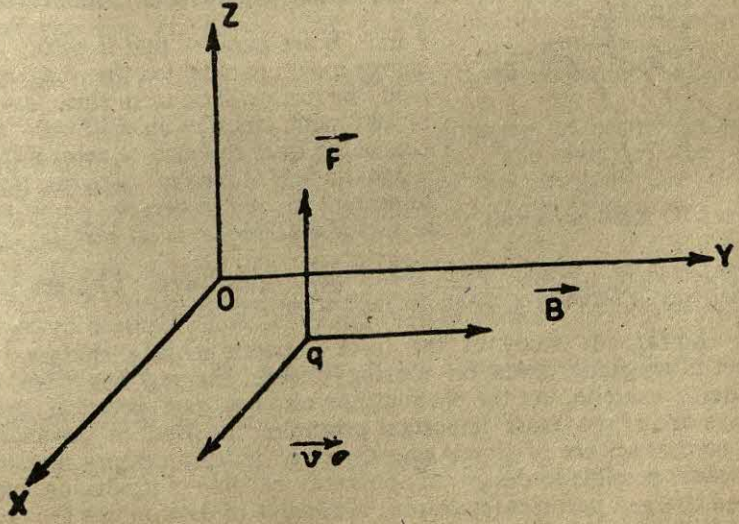


Fig. 7.30

the direction of $\vec{F}$ will be the one in which the *right handed screw* will advance by a rotation of $\vec{v}$ towards $\vec{B}$ through the smaller angle between them. Thus if $\vec{v}$ and $\vec{B}$ lie in the X—Y plane (Fig 7.30), $\vec{F}$ will point out in the Z—direction. *The direction of $\vec{B}$ is the direction of the magnetic field lines.*

For a beam of electrons q is *negative* and it follows from equation (*iv*) that $\vec{F}$ has the *opposite direction* to that of $(\vec{v} \times \vec{B})$.

Some students may wish an alternative rule to use for finding the direction of the force on a charge moving through a magnetic field. Several are available, but, right-hand screw rule is usually the most easy to use once you learn how to approach it. However, you might wish to resort to the following :

Open your right hand and place your right thumb along the direction in which a *positive* charge would move, that is, along the direction of the velocity $\vec{v}$, and place your fingers in the direction of magnetic field $\vec{B}$; then your palm will be pushing in the direction he force on the charge as shown in Fig 7.31.

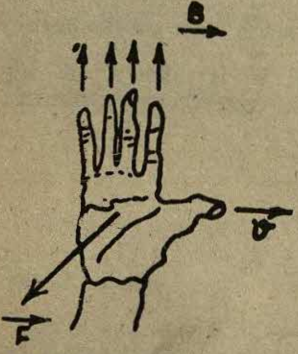


Fig. 7.31 **Right Hand Rule**

From equation (*iii*) above, it follows that when $\vec{B}$ and $\vec{v}$ are at right angles, $\theta=90°$ and $\sin\theta=1$. The force will then be a maximum, in agreement with experiment. Likewise, if $\vec{v}$ and $\vec{B}$ are parallel, that is when the charge moves parallel to the magnetic field, the force will be zero since $\theta=0°$ or $180°$ and $\sin 0°=\sin 180°=0$. Of course the force can also be zero if the magnetic field is zero or perhaps more important to remember, *the force will be zero if the charge is at rest in the magnetic field. When $\vec{v}=0$, then $\vec{F}$ must equal zero.* Charges only experience a force in a magnetic field when they move.

So far, our study of the force between moving charges (or effect of magnetic fields on moving electric charges) has been an academic exercise, but the phenomenon also has vast practical uses. It was one of the most important principles involved in advancing the nascent science of atomic physics early in this century because it made possible to design of apparatus capable of producing high speed charged particles (cyclotron) or capable of determining the ratio of the charge to the mass of electrons (J.J. Thomson's method) etc.

7.6. MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

Consider a charged particle carrying a *positive* charge q to be travelling with a velocity $\vec{v}$ in a direction *perpendicular* to a *uniform* magnetic field $\vec{B}$ (uniform means that $\vec{B}$ has the same magnitude and direction at all points throughout the region of space inside which the particle moves). In Fig. 7.32 a uniform magnetic field exists which is perpendicular to the plane of the paper but *pointing* into *the paper*. The charged particle, therefore, experiences a force $\vec{F} = qvB$ in a direction that is perpendicular to both $\vec{B}$ an v. When a particle moves in a direction perpendicular to the force acting on it, *the force does no work*. The energy of the particle, therefore, remains constant. Since the energy in this case is entirely kinetic energy, the *magnitude of the velocity must remain constant*. Only the direction of velocity will change. The force, and therefore the acceleration, are always perpendicular to the velocity. *These are exactly the conditions for uniform circular motion.* Thus the charged particle follows a circular path of radius r and has centripetal acceleration $\frac{v^2}{r}$. If its mass is m, then according to Newton's Second Law of Motion,

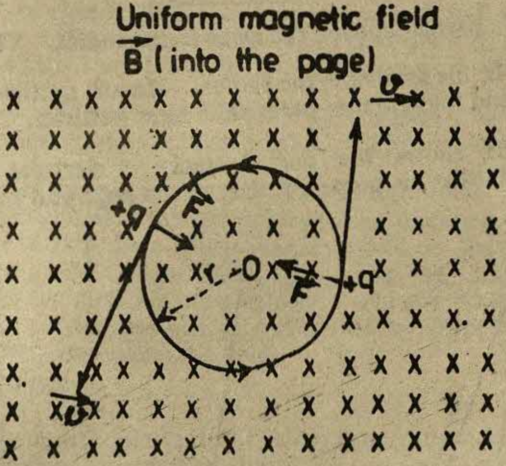


Fig. 7.32. Motion of a charged particle in a uniform magnetic field

$$\text{centripetal force} = \text{mass} \times \text{acceleration}$$

$$= \frac{mv^2}{r}$$

or
$$\boxed{q v B = \frac{mv^2}{r}} \qquad \ldots(v)$$

Equation (v) can be arranged to give the radius

$$\boxed{r = \frac{mv}{qB}} \qquad \ldots(vi)$$

The faster the particle moves, the large is the radius. The larger the applied field $\vec{B}$, the smaller is the radius. Heavy particles are more difficult to bend into circular paths than light particles.

Fig. 7.33 shows the curved tracks of electrons moving in a magnetic field that is perpendicular to the page and *points towards*

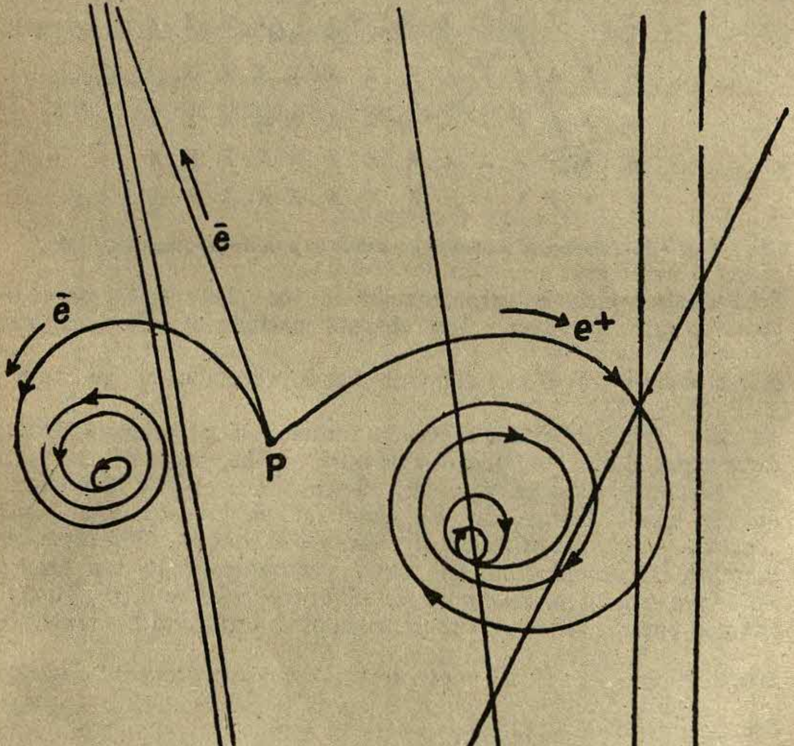


Fig. 7.33

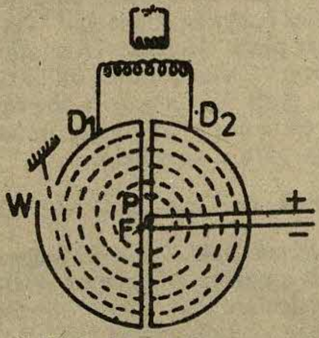
the reader. At P a certain process suddenly produces two negatively charged electrons (e^-) and a positron (e^+) which is identical with an electron except that it is positively charged. The electron e^- which moves off to the left follows a circular path in the magnetic field but slows down as it loses enegy in collisions. As its velocity decreases, the radius of circular path decreases (equation *vi*) and so it spirals inwards. The positron e^+ which moves off to the right has the opposite charge and its path is curved in the opposite direction. The electron e^- in the centre has a very high initial velocity and so the radius of its circular path is very large.

Let us reiterate one aspect of our discussion to clarify the significance of the electric field $\vec{E}$ *and the magnetic field* $\vec{B}$ at any point (say P) in space. If a charge q is placed at that point P, the electric field enables us to calculate that part of the force on it which is independent of its velocity using $\vec{F}=q\vec{E}$. The magnetic field $\vec{B}$ enables us to calculate that part of the force which depends on the velocity of the charge using equation $\vec{F}=q\,(\vec{v}\times\vec{B})$ and right hand screw rule. If the magnetic field acts at right angles to the direction of travel of the charge, no work is done by the magnetic field on the charge and hence the speed of the charge cannot change but the magnetic field can only change the *direction* of its velocity. An electric field on the other hand, can accelerate a charge and the electric force always gives rise to force which acts along the field.

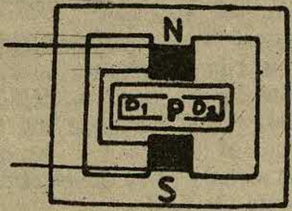
7.7 CYCLOTRON

This is the most important of the high speed particle generators and was first devised in 1930 by Lawrence, in California, who was awarded the Nobel Prize in 1939. Here the particle is accelerated in several steps using the same source of potential difference ; each step being so synchronised that it produces an accelerating influence on the particle at each stage. Thus the necessity of very high potential difference to start with is dispensed with and the final velocity of the projected particle corresponds to a potential difference of several million volts.

Cyclotron (Fig 7.34) consists of a flat, Cylindrical copper chamber split into two parts D_1, D_2 each part called a "Dee" by a diametrical cut. These are then placed in vacuum chamber about 90 cm in diameter and 16 cm high, which is mounted horizontally in a vertical magnetic field produced by the pole-pieces N and S from a huge electromagnet. The whole thing is cooled in a water tank in which cold water is circulating. A rapidly alternating potential difference of about 10 000 volts at a frequency of the order of 10 000 kilocycles per sec is maintained between the dees with the help of a transformer and a high frequency valve oscillator, the apparatus being similar to a short-wave-transmitter.A heated filament is placed in the gap between the electrodes, and on passing an electric current, emits thermions which ionise some of the gas introduced at low pressure into the chamber. Thus hydrogen gas yields protons, deuterium gas produces deuterons and helium gas produces α-particles.



(a) Top View



(b) Side View

Fig 7.34

Suppose a positive ion is formed at P when D_1 is positive and D_2 is negative, then it will be accelerated under the action of the electric field and travel into the negative dee D_2. Once inside the dee, the ions move with a constant speed under the action of the magnetic field alone and follow a semi-circular path of small radius r given by

$$Bqv = \frac{mv^2}{r}$$

where $\vec{B}$ is the intensity of the magnetic field, q the charge on the positive ion, $\vec{v}$ its velocity inside the D, m its mass and r is the radius of the circular path inside the dee.

$$\therefore \qquad \frac{v}{r} = \frac{Bq}{m} = w$$

where w is the angular velocity of the ion. The time t required to describe a semi-circle with this angular velocity is

$$t = \frac{\pi}{w} = \frac{\pi m}{Bq}$$

and is constant so long as the mass of the ion m remains constant *i.e.,* so long as the velocity of the ions does not closely approach the velocity of light and the relativistic increase of mass is not appreciable.

Now, if by the time the ion emerges out at the other end of the dee D_2, the potentials applied to the dees get reversed, so that D_2 is now positive and D_1 negetive, the ion will again get accelerated in the gap and enter the dee D_1 with greater velocity, and hence under the influence of the magnetic field again describes another semi-circular path but of larger radius. This process is continued, the ion gaining in speed each time they pass from one dee to another until they finally emerge out with very large velocity. It is, therefore, thus essential that the applied potential difference should also have the semi-periodic time given below :

Angular frequency of the ion $=w=2\pi f$ where f is the frequency and T is the time period.

$$=\frac{2\pi}{T}$$

$$\therefore \qquad f=\frac{w}{2\pi}=\frac{Bq}{2\pi m}$$

and Time period of the ion= $T=\frac{1}{f}=\frac{2\pi m}{Bq}$

$\therefore$ Semi periodic time of the applied P.D.$=\frac{1}{2}T=\frac{\pi m}{Bq}$

$\frac{\pi m}{Bq}$ does have dimensions of time *i.e.* $\frac{M^1L^1T^{-1}}{M^1L^1T^{-2}}=T$

and thus be in resonance with the periodicity of the ion.

The largest cyclotron is housed in William H. Crocker Radiaation Laboratories at Berkeley in California, and has a 1·524 *m* diameter of the vacuum chamber and powerful electromagnets. This accelerates deuterons which emerge with a kinetic energy of 24 Mev (1 Mev$=1{\cdot}6\times10^{-13}$J). Such a beam has a velocity of 2×10^5 km sec. and 600 trillion particles come out every sec ; giving an enormous energy. If the beam is ejected into the air, it penetrates over 1·53 *m*. The large consumption of power and the high cost of providing powerful radio frequency systems make the 1·524 *m* size about the practical limit of the cyclotron, so far as energies are concerned.

These high energy projectiles are used for bombarding nuclei, or for producing other high energy particles such as neutrons by the action of fast moving particles (from the cyclotron) on beryllium.

With the invention of the cyclotron, it is thus possible to produce high velocity protons, deuterons and α-particles which are powerful bullets which can smash the atomic nuclei of other atoms. These have been mostly used in artificial transmutation of elements and for producing induced radio-activity in various elements which have found wide applications in radio-therapy as tracer elements.

7.8 J. J. THOMSON'S EXPERIMENT FOR SPECIFIC CHARGE $\left(\frac{e}{m}\right)$ of Cathode Rays : *(Discovery of Electrons)*

On reviewing the properties of cathode rays in 1897, Prof. J. J. Thomson at the Cavendish laboratory Cambridge proposed the rather startling hypothesis that cathode rays were streams of negatively charged particles produced at the cathode itself and moving with high velocity. He was the pioneer worker who devised an experiment by which the ratio of charge (*e*) to mass (*m*) (*i.e.*, specific charge $\left(\frac{e}{m}\right)$ of such particles could be determined. *He employed the principle of crossed magnetic and electric fields.*

Description of Apparatus :

The apparatus used for the determination of $\frac{e}{m}$ has been shown in Fig. 7.35. It consists of highly evacuated discharge tube in which the cathode rays are produced. C is the cathode which consists of a small aluminium disc and A is the anode. When a high potential is applied between C and A, cathode rays are emitted from cathode surface normally and fall upon the cylindrical anode D. In D there is a small hole so that a pencil of cathode rays passes on to E containing similar small hole. The narrow pencil of cathode rays finally strikes the surface of the discharge tube at P normally and produces fluorescence due to the fluorescent material pasted at this end of the tube.

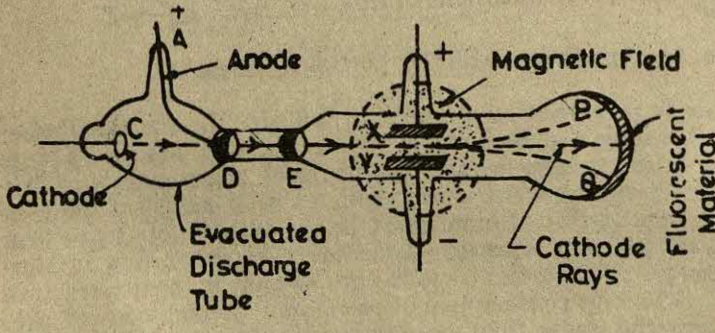


Fig. 7.35 Thomson's Method of determining *e*/*m*

X and Y are two parallel plates between which suitable potential difference can be applied and thus *an electric field is produced between X and Y at right angles to the plates and in the plane of the paper.* A strong uniform magnetic field is applied within the same space (shown by dotted area) in a *direction perpendicular to the plane* *of the paper*. This is done by placing the tube between the pole pieces of a powerful* electromagnet.

Theory :

Let $\vec{B}$ tesla be the intensity of the magnetic field perpendicular to the direction of motion of the cathode rays. If 'e' be charge in coulombs on each electron and $\vec{v}$' its velocity in metres per sec, it will be acted upon by a force Bev due to the magnetic field *i.e.*, F=Bev. According to right-handed screw rule this force will act in a *direction perpendicular to the motion of electrons* and magnetic field. Hence path of electrons becomes circular right from the moment it enters the magnetic field.

If r be the radius of circular path the centripetal force $\frac{mv^2}{r}$ will act on the electron. This force is obtained from force Bev acting on electron. Hence

$$Bev = \frac{m\,v^2}{r}$$

$$\boxed{\frac{e}{m} = \frac{v}{Br}} \qquad \text{—}(i)$$

If we can know r and $\vec{v}$, $\frac{e}{m}$ can be determined.

Determination of r :— After emerging from magnetic field at G the electron will travel in straight line and will strike the screen at Q. Now therefore scintillation will be seen at Q.

In the Fig. 7.36 path of electron KGQ is shown. Electron travels along the arc of a circle from K to G whose centre is C and radius is r.

Let PQ=d and $\angle$ POQ= $\angle$KCF=θ

From $\triangle$ POQ, $\tan\theta = \frac{PQ}{OP} = \frac{d}{OP}$

From $\triangle$ KCF, $\tan\theta = \frac{KF}{KC} = \frac{KF}{r}$

---

* Powerful electromagnet consists of two circular coils situated outside the tube which are separated by a distance equal to their radius. Such coils are known as *Helmholtz Coils* and when a current is sent through them a magnetic field is produced in the central region which is almost uniform and whose magnitude can be estimated from the radius and number of turns in the coils and the size of the current flowing in them.

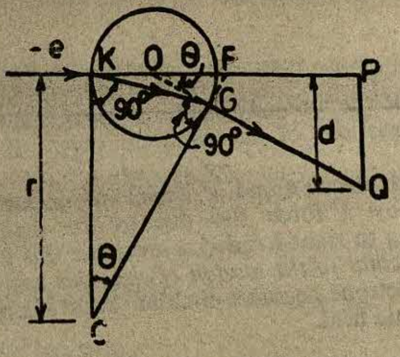


Fig 7.36

$$\therefore \quad \frac{d}{OP}=\frac{KF}{r}$$

$$\boxed{r=\frac{KF\times OP}{d}} \quad \ldots(ii)$$

Here KF is the length of magnetic field which can be taken equal to length of X or Y plate. OP is length from centre of magnetic field to P. d=PQ is the deflection. All these quantities in eq. (*ii*) can be measured from the experimental set up. Thus from above eq. (*ii*) 'r' is calculated.

Determination of v :

Now an electric field $\vec{E}$ newtons/coulomb is applied at right angles to both the magnetic field and the motion of the electrons, so that it produces a deflection of rays in the direction opposite to that produced by the magnetic field. Its value is so adjusted that deflection becomes zero *i.e.*, the fluorescent spot comes back to P. In such a case the force on the electron due to the electric field is $\vec{E}e$ and is balanced by the deflecting force Bev due to the magnetic field.

$$\text{Hence } Bev=Ee$$

$$\text{or} \quad \boxed{v=\frac{E}{B}} \quad \ldots(iii)$$

Determination of $\frac{e}{m}$

Putting equation (*iii*) in equation (*i*), we have

$$\boxed{\frac{e}{m}=\frac{E}{B^2}\cdot\frac{1}{r}} \quad \ldots(iv)$$

where Electric field $E = \frac{\text{P.D. between the plates XY}}{\text{Distance between the plates}}$

Thus substituting the values of r, E and B in equation (*iv*), e/m can be determined. The current accepted value of $\frac{e}{m}$ is $1{\cdot}76 \times 10^{11}$ coulomb/kg.

Discussion of Results :

(*i*) In this pioneering work, Thomson obtained a value of $\frac{e}{m}$ which was of the order of magnitude of $1{\cdot}76 \times 10^{11}$ C/kg This value comes very close to that of modern accurate measurements.

(*ii*) Thomson used a cold discharge tube as a source of cathode rays instead of hot filament and found that the cathode rays had the same ratio of $\frac{e}{m}$ *regardless of the gas used in the tube or the metals of the electrodes* thus demonstrating *that cathode rays are to be found wide spread through nature and they were the lightest particles and were all alike and that they could be obtained from matter which is ordinarily electrically neutral. Since matter is made up of atoms, this implies that cathode rays exist in atoms. In other words, ordinary neutral matter is made up of electrical constituents. He named these cathode rays as electrons.*

J.J. Thomson's startling Hypothesis-cathode rays are negatively charged particles common to a large number of elements.

(*iii*) The results showed that the value of velocity of cathode ray particles obtained varies from $\frac{1}{5}$ to $\frac{1}{10}$th velocity of light ; hence very high. The velocity of cathode ray particles evidently depends upon a number of conditions, chiefly on the pressure, and potential difference of the discharge tube.

(*iv*) The deflection suffered by the cathode ray particle was too small The electric and magnetic fields were not strictly uniform and the particles were emitted with different velocities and hence a *dispersion effect* was observed in both the electric deflections.

(*v*) It will be apparent too that the apparatus used has the basic features of the *oscilloscope* so widely used to-day.

Example 7.11

The voltage across the electrodes of a cathode ray gun is 500 volts. Calculate (*i*) the energy gained by the electrons (*ii*) increase in the speed of the electrons and (*iii*) increase in the momentum of the electrons.

Solution.

Here voltage, $V = 500$ volts

charge on the electron, $e = 1.6 \times 10^{-19}$ coulomb

and mass of the electron $= m = 9{\cdot}1 \times 10^{-31}$ kg

(*i*) Energy gained by the electrons

$$= Ve$$
$$= 500 \times 1.6 \times 10^{-19}$$
$$= 8 \times 10^{-17} \text{ J}$$

(*ii*) Let U be the speed attained by the electrons.

Then

$$\frac{1}{2} m U^2 = eV$$

or
$$U^2 = \frac{2eV}{m}$$

or
$$U = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1{\cdot}6 \times 10^{-19} \times 500}{9{\cdot}1 \times 10^{-31}}}$$

$$= \frac{4}{3} \times 10^{7} \, ms^{-1} = 1{\cdot}33 \times 10^{7} \, ms^{-1}$$

(*iii*) Momentum of the electrons $= mU$

$$= 9{\cdot}1 \times 10^{-31} \times \frac{4}{3} \times 10^{7}$$

$$= 1.2 \times 10^{-24} \text{ kg } ms^{-1}$$

Example 7.12

An electron beam of problem (7.11) passes through a parallel plate condenser. The electric intensity between the plates is 20 volts cm^{-1}n, the distance between the plates is 5 cm and the length is 0 cm. Calculate the deflection angle of the beam.

(mass of the electron $= 9 \times 10^{-31}$ kg)

Solution :

Here

Electric Intensity E = 20 volts/cm = 2000 volts/m

Force acting perpendicular to the path of the electrons and acting along the

positive plate $= F = Ee = 2000 \times 1{\cdot}6 \times 10^{-19}$

$$= 3{\cdot}2 \times 10^{-16} \text{N}$$

Acceleration $a = \frac{F}{m} = \frac{3.2 \times 10^{-16}}{9 \times 10^{-31}} = \frac{32}{9} \times 10^{14}\, ms^{-2}$

Time taken by an electron to pass through

the plates $= t = \frac{\text{distance}}{\text{speed}} = \frac{0.1}{4/3 \times 10^{7}}$

$$= \frac{3}{4} \times 10^{-8} \text{ sec}$$

∴ Displacement suffered by an electron perpendicular to its path is

$$Y = \frac{1}{2} at^2$$

$$= \frac{1}{2}\left(\frac{32}{9} \times 10^{14}\right)\left(\frac{3}{4} \times 10^{-8}\right)^2$$

$$= 10^{-2} m$$

∴ Deflection $\theta = \frac{Y}{\text{Length of the plate}}$

$$= \frac{10^{-2}}{0{\cdot}1}$$

$$= 10^{-1} \text{ rad}$$

7.9 SOURCE OF THE MAGNETIC FIELD

We have read earlier that a current carrying wire produces a magnetic field. If the current is switched off, the magnetic field disappears. It immediately suggests that magnetic effects are a consequence of the behaviour of moving charges. *In other words a moving charge is a source of magnetic field.* When there is no current in the wire, the electrons are in thermal motion. Though each electron acts as the source of the magnetic field, these fields cancel out since electrons are moving in a random manner. There is no net magnetic field as there is no net flow of charge in any direction.

In an ultimate analysis all magnetic phenomena can be related to the interactions between charged fundamental particles.

We now turn to the case of the magnetic effects of electric currents in metal wires, which was historically important to the development of the theory of electromagnetism and which is still very important because of its engineering applications to electrical machinery and electrical devices. As already mentioned these magnetic effects are a consequence of the presence of free-electrons *drifting steadily along the wire.* A great simplification is introduced by the fact that the drift velocity is always very much smaller than the speed of light. Also, the number of protons in the wire is equal to the number of electrons and they *cancel out each other's electric field.* However, although a proton is moving chaotically in all directions, it stays in the vicinity of one point and has *no net drift velocity* to produce a magnetic field which would cancel the magnetic field of the drifting electrons. We can therefore, concentrate on the magnetic field $\vec{B}$ and assume that the electric field $\vec{E}$ is zero.

7.10 BIOT-SAVART LAW (*Magnetic field due to some current distributions*) :

With the help of Fig. (7.37) we shall consider the problem of calculating the magnetic field $\vec{B}$ at a point P due to a current I flowing in a wire bent into any arbitrary shape. Concentrate upon a ***short segment*** CD of the wire which is so short that it can be considered to be straight. If we can calculate the contribution of this segment to the value of magnetic field at P, then the wire can be completely divided up into such short lengths and the contributions from all of them added vectorially to give the total field $\vec{B}$ at P. Suppose that dl is the length of CD and r is the distance from CD to P. The current I is due to free electrons drifting with a velocity $\vec{v}_d$ in the opposite direction to I. In accordance with our convention, let us reverse this drift velocity and pretend that the drifting charges are positive and move in the same direction as the current. Let θ be the angle between the direction of the reversed velocity and the line joining the electron to the point P. Since CD is so very short, θ is the same for all the electrons and is the same as the angle between CD and the line joining the centre of CD to P.

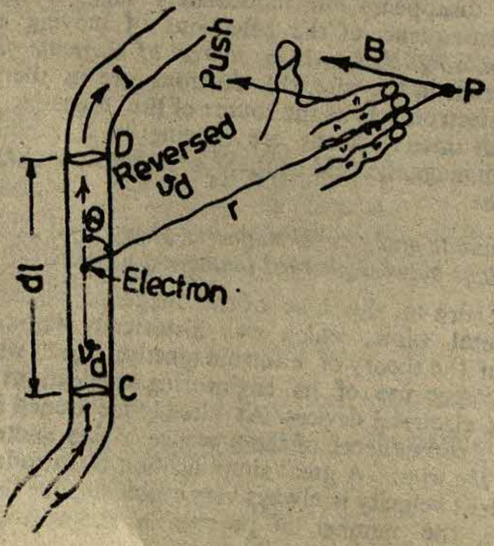


Fig 7.37 Biot-Savart Law

The magnetic field $\vec{dB}$ at the point P due to the small current element CD of length dl is given by

$$\boxed{\vec{dB}=\frac{\mu_0}{4\pi}I\quad\frac{\vec{dl}\times\vec{r}}{r^3}}\qquad ..(1)$$

This law was proposed by Biot and Savart in the early nineteenth century to provide a consistent explanation of numerous experiments on the magnetic effects of currents, long before the discovery of the electron by J.J. Thomson in 1897. This law was arrived at *empirically.*

Here μ_0 is a constant called the *permeability* of the vacuum (or free space) and its value $\mu_0=4\pi\times10^{-7}\ TmA^{-1}$

The *magnitude* of the field is given by

$$\boxed{dB=\frac{\mu_0}{4\pi}\quad\frac{I\,dl\sin\theta}{r^2}}\qquad ...(2)$$

The *direction* of $\vec{dB}$ is that of vector $\vec{dl}\times\vec{r}$ *i.e.*, is given *by right handed screw rules.* In the figure (7.37) $\vec{dB}$ at the point P is directed into the page at right angle to the plane of the figure.

The direction of $\vec{dB}$ can also be obtained by the following *right hand rule* :

"Hold the right hand flat. Point the thumb in the direction of the current through CD. Point the fingers in the direction from CD to the point P. Then $\vec{dB}$ is in the direction in which the hand would push".

Even when the Biot Savart Law has been used to calculate the contribution from each segment of the wire, there remains the problem of adding together these contributions. This is usually a difficult mathematical problem. It is easy to make use of this law to get the field in case of regular geometrical distributions.

Field at the Centre of a Circular Coil.

Consider a loop of wire of radius r carrying current I (Fig. 7.38). The loop lies in the plane of paper. We wish to calculate the magnetic field at the centre of the loop. Consider the element CD $=dl$ of the loop. The magnetic field $\vec{dB}$ at the centre O due to this current element is given by Biot-Savart Law

$$\vec{dB} = \frac{\mu_0}{4\pi} I . \frac{\vec{dl}\times\vec{r}}{r^3}$$

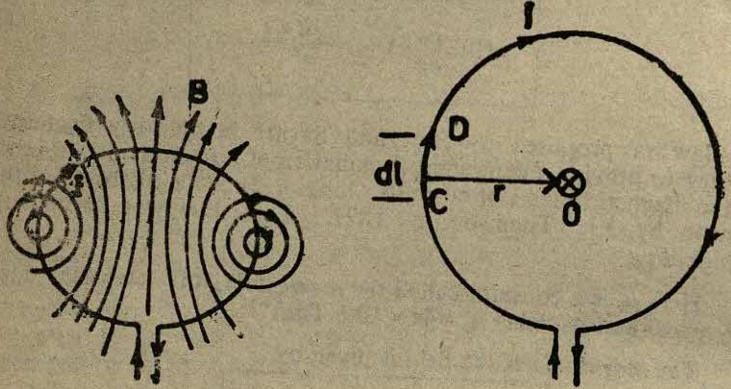


Fig 7.38 Field at the centre of a cirular coil carrying current

The *direction* of the field is perpendicular to the plane of the loop and points into the paper as shown by ⊗ in the figure (7.38). Since the angle between $\vec{dl}$ and $\vec{r}$ is 90°, the *magnitude* of $\vec{dB}$ using equation (2) is given by

$$\vec{dB} = \frac{\mu_0 I dl}{4\pi r^2} \qquad \text{...(3)}$$

If we divide the loop into a large number of such elements we find that the total field at the centre of the loop O is given by

$$\vec{B} = \int \vec{dB} = \int \frac{\mu_0 I dl}{4\pi r^2} = \frac{\mu_0 I}{4\pi r^2}\int dl$$

Now $\int dl$ is the total length of the loop wire and hence $\int dl = 2\pi r$ (circumference of the loop)

$$\therefore \vec{B} = \frac{\mu_0 I}{4\pi r^2} \cdot 2\pi r = \frac{\mu_0 I}{2r} \qquad \text{...(4)}$$

If instead of a single loop, there is a coil of n turns, all wound over one another, then

$$\boxed{\vec{B} = \frac{\mu_0 I n}{2r}} \qquad \text{...(5)}$$

The magnetic field lines due to a circular coil carrying current are shown in fig 7.38 (b). The field is *fairly uniform* in the interior of the coil. The direction of the magnetic field can be found by using the following rule :

"Curl the fingers of the right hand in the direction of the current. The stretched thumb then points in the direction of the field".

Magnetic Field on the Axis of a Circular Coil

Consider a circular coil of radius r carrying current I. Let P be a point on the axis of the circular coil at a distance x from its centre O where the magnetic field $\vec{B}$ is to be obtained. Let us divide the coil into a large number of short elements of length dl. The contribution $d\vec{B}$ due to one such element say at A at a distance y from P is given by

$$d\vec{B} = \frac{\mu_0 I dl}{4\pi y^2} \ (\because \theta = 90^\circ)$$

(Biot—Savart Law)

The direction of $d\vec{B}$ is along PQ according to right handed screw rule. If $\angle APO = \theta$, then the rectangular components of $d\vec{B}$ are $d\vec{B} \cos\theta$ and $d\vec{B} \sin\theta$. Similarly by resolving $d\vec{B}$ at P due to each other short current elements into its rectangular components, it is clear from the figure (7.39) that the components $d\vec{B} \cos\theta$ cancel each other and the components $d\vec{B} \sin\theta$ get added in the same direction.

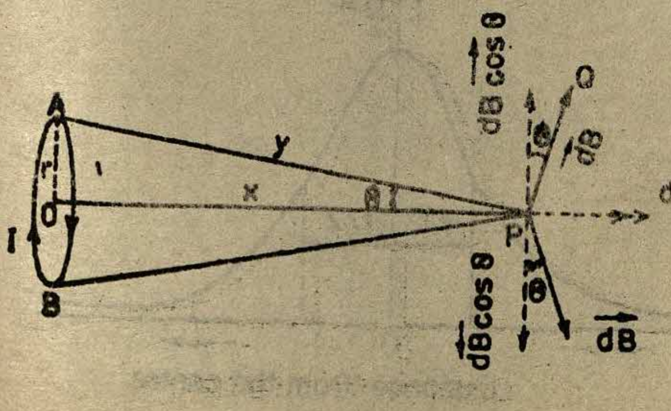


Fig 7.39

Hence the total magnetic

$$\text{field at } P=\vec{B}=\Sigma d\vec{B}\sin\theta$$

$$\text{or } \vec{B}=\sum\frac{\mu_0\, I dl}{4\pi y^2}\,.\,\sin\theta$$

$$=\frac{\mu_0\, I\sin\theta}{4\pi\, y^2}\sum dl$$

$$=\frac{\mu_0\, I}{4\pi y^2}\,.\,\frac{r}{y}\,.\,2\pi r$$

$$\because\ (\sin\theta=\frac{r}{y})$$
$$\text{and } \Sigma dl=2\pi r$$

$$=\frac{\mu_0\, r^2.\, I}{2y^3}$$

$$=\frac{\mu_0\, r^2\, I}{2(r^2+x^2)^{3/2}}$$

$$[\because\ y=(r^2+x^2)^{\frac{1}{2}}\ ]$$

If there are n turns in the coil, then

$$\boxed{\vec{B}=\frac{\mu_0 n r^2\, I}{2(x^2+r^2)^{3/2}}} \qquad \text{...(6)}$$

Equation (6) shows that the field B falls rapidly with the distance as shown in Fig (7.40).

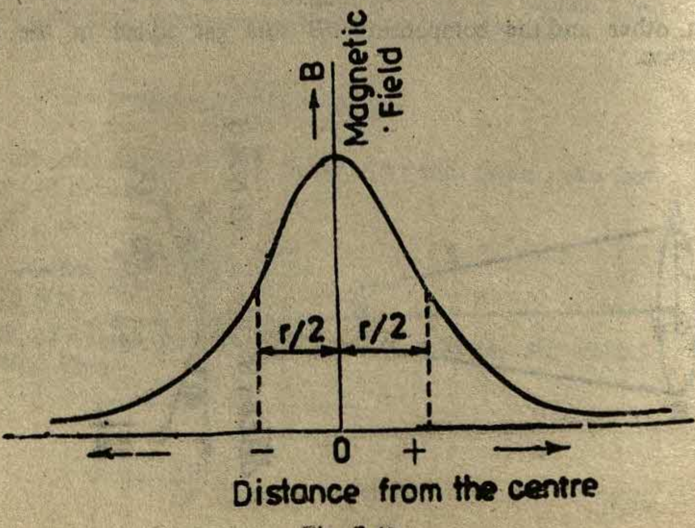


Fig. 7.40

Magnetic field due to a Solenoid :

Solenoid is a long coil carrying current. It can be shown that the magnetic field is *fairly uniform* in the interior of the Solenoid. For a very long solenoid, the field is given by

$$\boxed{\vec{B} = \mu_0 I n} \quad ...(7)$$

where I is the current and *n* is the *number of turns per unit length.*

Magnetic field lines produced by a current in a Solenoid are shown in Fig (7.41).

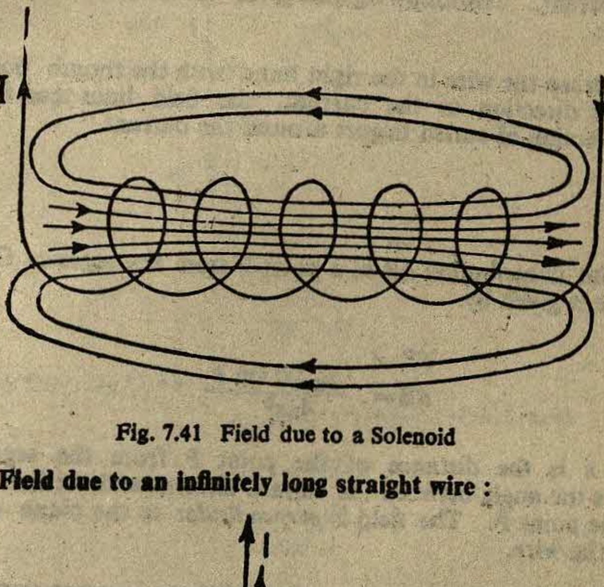


Fig. 7.41 Field due to a Solenoid

Magnetic Field due to an infinitely long straight wire :

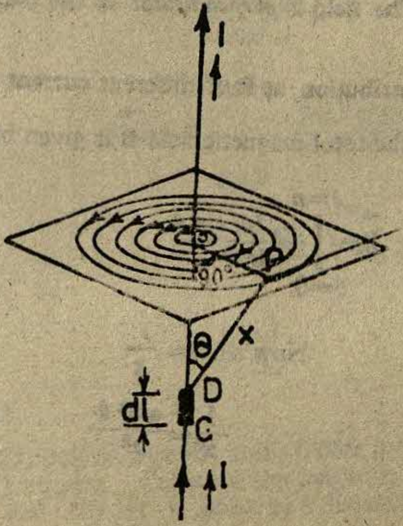


Fig. 7.42 Field due to a current in a straight wire

Fig (7.42) shows the magnetic field lines due to a current I flowing in a straight wire. The field is symmetrical about the current and its value at a point P, distant r from the current is given by

$$\boxed{\vec{B}=\frac{\mu_0 I}{2\pi r}} \qquad \ldots(8)$$

The magnetic field lines are circular and lie in a plane perpendicular to the current. The following rule gives the direction of the magnetic field :

> "Grasp the wire in the right hand with the thumb pointing in the direction of the current, the field lines then follow the direction of curled fingers around the current."

***Proof :**

The contribution $d\vec{B}$ at a point P from any segment CD $= dl$ of the wire is given by

$$d\vec{B}=\frac{\mu_0 I dl \sin\theta}{4\pi x^2}$$

where x is the distance of the point P from the segment CD and θ is the angle between the current element and the line joining it with the point P. The field is *perpendicular* to the plane containing P and the wire.

Since the contribution at P of different current elements have the same direction, the total magnetic field $\vec{B}$ is given by

$$\vec{B}=\int_{\theta=0}^{\theta=\pi}\frac{\mu_0 I dl \sin\theta}{4\pi x^2}$$

$$\text{Now } \sin\theta=\frac{r}{x}$$

$$\therefore \qquad \frac{1}{x^2}=\frac{\sin^2\theta}{r^2}$$

---

*Only for bright students

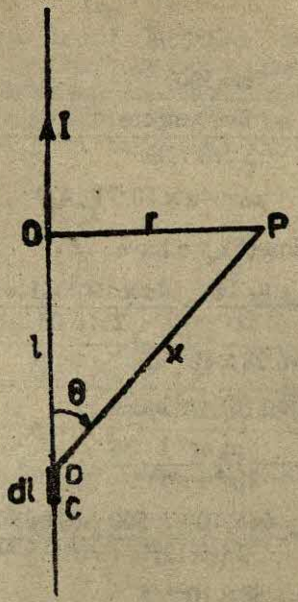


Fig. 7.43

Also from fig (7.43)

$$l = r\cot\theta \quad \text{where } OD = l \text{ (say)}$$

$$\therefore\; dl = -r\,\text{cosec}^2\,\theta\; d\theta$$

$$= \frac{-r\,d\theta}{\sin^2\theta}$$

$$\text{Hence } B = \int_{\theta=0}^{\theta=\pi} \frac{-\mu_0 I r\,d\theta}{\sin^2\theta} \cdot \frac{\sin\theta \cdot \sin^2\theta}{4\pi r^2}$$

$$= \frac{-\mu_0 I}{4\pi r}\int_0^{\pi} \sin\theta\, d\theta$$

$$= \frac{\mu_0 I}{2\pi r}$$

Example 7.13

A circular coil of wire of radius 0·05*m* having 500 turns carries a current of 1 ampere. Calculate the magnetic field (*i*) at the centre of the coil and (*ii*) at a point at a distance of 0·12 m from the centre along the axis of the coil

Solution :

Here $r=0{\cdot}05m$

$n=500$

$I=l$ ampere

$x=0{\cdot}12m$

$\mu_0=4\pi\times10^{-7}T_mA^{-1}$

(*i*) Using equation (5), we have

$$B=\frac{\mu_0 In}{2r}=\frac{4\pi\times10^{-7}\times1\times500}{2\times0{\cdot}05}$$

$$=6{\cdot}28\times10^{-3}\ T$$

(*ii*) Using equation (6) we have

$$B=\frac{\mu_0 nr^2 1}{2(x^2+r^2)^{3/2}}$$

$$=\frac{4\pi\times10^{-7}\times500+(0{\cdot}05)^2\times1}{2\,[(0.12)^2+(0{\cdot}05)^2]^{3/2}}$$

$$=3{\cdot}58\times10^{-4}\ T$$

Example 7.14

Find the magnetic field in air ·01 *m* away from an infinitely long wire that carries a current of 2 A.

Solution.

Here $r=.01m$

$I=2A$

$\mu_0=4\pi\times10^{-7}\ TmA^{-1}$

Using equation (8) we have

$$B=\frac{\mu_0 I}{2\pi r}$$

$$=\frac{4\pi\times10^{-7}\times2}{2\pi\times{\cdot}01}$$

$$=4\times10^{-5}\ T$$

7.11 FORCE ON A CURRENT CARRYING CONDUCTOR

An electric current is nothing but the flow of free electrons. When a current carrying conductor is placed in a magnetic field, the free electrons inside the conductor which are in motion experience forces. Hence a net force is transmitted to the conductor as a whole.

Let a current I flow through a conductor DC of arbitrary shape placed in a uniform magnetic field $\vec{B}$ (Fig 7.44). In order to calculate the force exerted by the magnetic field on the current carrying conductor, divide the conductor into segments that are so short, that they can be considered to be straight. The force on each segment is calculated by the method given below and the forces are then added vectorially.

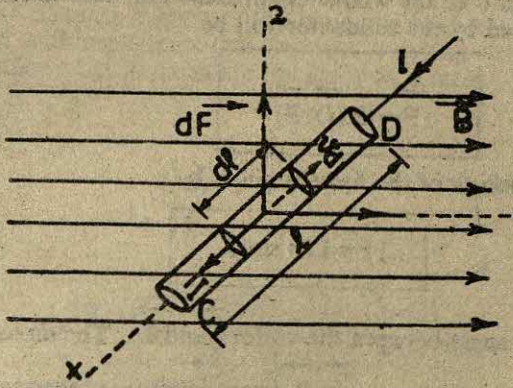


Fig. 7.44 Force on a current carrying conductor

The current I is due to electrons drifting with velocity $\vec{v}_d$ in the opposite direction to I. The magnitude of the current is given by

$$I = en_0 A\, v_d$$

where n_0=number of free electrons per unit volume.
A=area of cross-section of the conductor and
e=charge on each free electron

Now. each free electron experiences a force

$$= (-e)\,(\vec{v}_d \times \vec{B})$$

In a segment of length dl,

the number of free electrons $= n_0\, A dl$

The total force experienced by the segment dl is given by

$$d\vec{F} = n_0 A dl\,(-e)\,(\vec{v}_d \times \vec{B})$$

$$= -n_0\, Ae\,(dl\, \vec{v}_d \times \vec{B})$$

If we represent the segment of length by vector dl pointing in the direction of current, then $\vec{v}_d$ and dl point in opposite directions and the term $\vec{dl}\, v_d$ may be replaced by $-v_d\, \vec{dl}$. Hence

$$\begin{aligned} d\vec{F} &= -n_0\, Ae\,(-v_d\, \vec{dl} \times \vec{B}) \\ &= n_0\, Ae\, v_d\, (\vec{dl} \times \vec{B}) \\ &= I\ \ (\vec{dl} \times \vec{B}) \end{aligned}$$

If $\vec{B}$ is uniform over the whole length DC=l of the conductor, the force experienced by the conductor will be

$$\boxed{\vec{F} = I\,(\vec{l} \times \vec{B})} \qquad ...(i)$$

The ***magnitude*** of the force is given by

$$\boxed{F = I\,lB \sin\theta} \qquad ...(ii)$$

where θ is the angle between the vector $\vec{l}$ and $\vec{B}$. The *direction* of the force is given by the vector $\vec{l} \times \vec{B}$ where $\vec{l}$ vector is taken ***in the direction of the current***. If the condutor is held at right angles to the field then $\theta = 90°$ and hence the magnitude of the force is given by

$$\boxed{F = IlB \text{ newtons}} \qquad ...(iii)$$

where I is in amperes, l is in metres and B in tesla. The force is ***perpendicular*** to both, the conductor and the field. The relationship between the three vectors $\vec{F}$, $\vec{I}$ and $\vec{B}$ may be described by "***Fleming's Left Hand Rule***" (Fig 7.45) which is stated below :

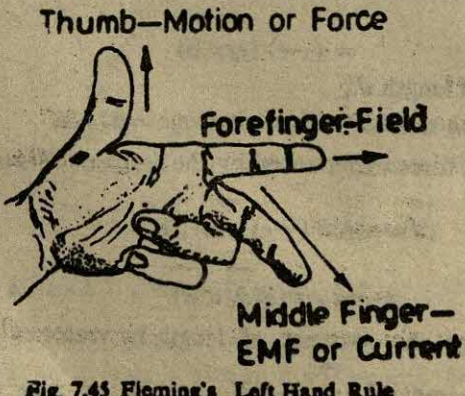


Fig. 7.45 Fleming's Left Hand Rule

"Hold the thumb, the first finger and the middle finger of the left hand mutually at right angle to each other. If the first finger points in the direction of the magnetic field, the middle finger in the direction of the current, then the thumb will indicate the direction of motion of the conductor."

7.12 FORCE BETWEEN PARALLEL WIRES CARRYING CURRENT :

Two very long straight parallel wires carry currents I_1 and I_2 and are a distance r apart. The magnetic field of one exerts force on the other and hence these two current carrying wires exert forceson one another. Let us calculate the force on a length l_2 of the wire with current I_2

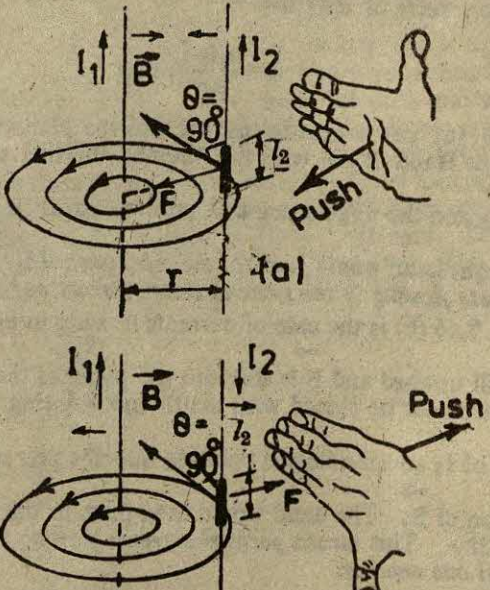


Fig. 7.46

Fig. 7.46 (*a*) is the case of two conductors carrying current in same direction. The magnetic field lines due to I_1 are circles that go round anti-clockwise for an observer looking in the direction of I_1.

The magnetic field $\vec{B}$ produced by I_1 at a point on the other wire is

$$\vec{B} = \frac{\mu_0 I_1}{2\pi r} \text{ where } \mu_0 = 4\pi \times 10^{-7} \text{ TmA}^{-1}$$

This field is perpendicular to I_1 and to the radius r drawn perpendicularly to both wires. Since $\vec{B}$ is also perpendicular to I_2' $\theta=90°$: $\sin\theta=1$. Hence the force exerted by the field $\vec{B}$ on a length l_2 of the wire with current I_2 is given by

$$\vec{F}=I_2\, l_2\, B$$

$$\boxed{\vec{F}=\frac{\mu_0 I_1 I_2 l_2}{2\pi r}} \quad \text{...(I)}$$

$$\text{Force per metre of length}=\frac{F}{l_2}$$

$$=\frac{\mu_0 I_1 I_2}{2\pi r}\, Nm^{-1} \quad \text{...(II)}$$

In figure 7.46 (a) the magnetic field is into the plane of the paper. Applying Right Hand Rule with the thumb pointing *upward* in the direction of $\vec{I_2}$ and the fingers pointing into the paper in the direction of $\vec{B}$, the right hand *would push to the left, toward* I_1. *This means parallel currents flowing in the same direction attract one another.*

Figure 7.46 (b) is the case of currents flowing in *opposite* directions. $\vec{I_1}$ is still upward and $\vec{B}$ is still into the plane of the paper. The right hand must now be placed with the thumb pointing *downward* in the direction of $\vec{I_2}$ and the fingers pointing into the plane of the paper in the direction of $\vec{B}$. The hand would then push to the right ; so the force is *repulsive*. This means *parallel currents flowing in the opposite direction repel one another.*

Definition of Ampere :

As derived above the force per metre between two infinitely long current carrying conductors is given by

$$F=\frac{\mu_0 I_1 I_2}{2\pi r}$$

where I_1 and I_2 are the currents and r is the separation between the conductors and $\mu_0=4\pi\times10^{-7}\ TmA^{-1}$.

When $I_1=I_2=1$ ampere and $r=1$ metre

then

$$F=\frac{4\pi\, 10^{-7}\times1\times1}{2\pi\times1}=2\times10^{-7} \text{ newton}$$

This is the basis of the definition of ampere in SI system.

> *"The ampere is the strength of that steady current which when flowing in two parallel infinitely long conductors of negligible cross-section placed in vacuum at a distance of one metre from each other, produces between those two conductors a force of 2×10^{-7} newton per metre length."*

This is a precise and reproducible laboratory method.

7.13 MOVING COIL GALVANOMETER :

A galvanometer in which the magnet remains stationary but the coil rotates is called a moving coil galvanometer.

It is commonly used for accurate measurement of current. It consists of a small rectangular coil *aedf* having a large number of turns of fine insulated copper wire wound on a light metallic frame (Fig 7.47). The coil is suspended from a torsion head T by a fine strip of *phosphor-bronze alloy* in-between the pole pieces N and S of a strong permanent horse-shoe magnet. The lower end of the coil is connected to a fine spiral spring C of phosphor-bronze. The other end of the spring is connected to a terminal T_2. A mirror M is fixed along the phosphor bronze strip by means of which the deflection of the coil is measured with a lamp and scale arrangement. To level the instrument, levelling screws are provided at the base. The torsion head T is connected to the terminal T_1 through the case.

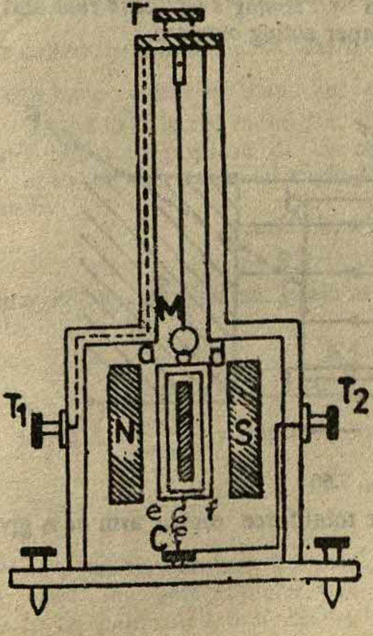


Fig. 7.47

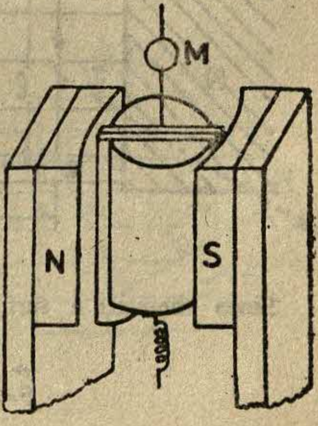


Fig. 7.48

The pole-pieces of the magnet are given a *concave cylindrical shape* and a cylinder of soft iron is fixed midway between them as shown in **Fig (7.48)**. The instrument is levelled so that the coil lies symmetrical with respect to the cylindrical core and is not touching any part.

The lines of force starting from the north pole crowd into the soft iron and end at the south pole. *The direction of the lines of force within the narrow air gap coincides approximately with the radii of the pole* pieces. Such a magnetic field is called **radial field**. In this field the plane of the coil always remains *parallel* to the lines of force for all its positions as shown in Fig (7.49).

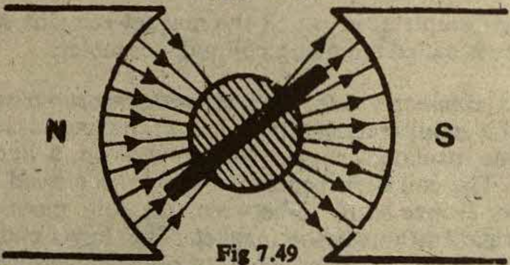


Fig 7.49

Theory (*Torque on a coil*) : Let n be the number of turns in the coil, l its length, b its breadth and I the current flowing through it.

As the wire carries a current I and is *at right angle* to the direction of a uniform magnetic field $\vec{B}$, it experiences a force equal to IlB. The direction of this force is given by *Fleming's left hand rule* and is perpendicular to the plane of the paper acting *outward*.

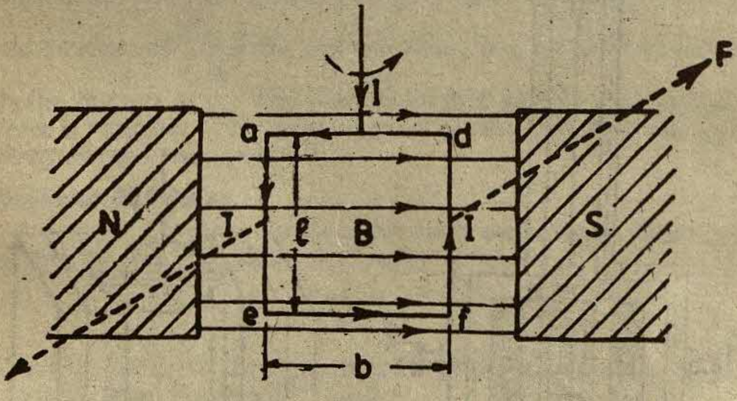


Fig. 7.50

Since there are n turns the total force on the arm ae is given

$$\vec{F} = nIlB$$

The total force on the arm *fd* has the same value but acts in the opposite direction, *i.e.*, *inward* in a plane perpendicular to the plane of the paper.

There will be no magnetic force on *ad* and *ef* since the sides carrying current are *parallel* to the magnetic field.

Hence there are two equal, opposite and parallel forces acting at different points of the coil forming a couple.

Torque acting on the coil $=\tau=nIlB\times b$

But $l\times b=A$ the area of the coil

Torque acting on the coil $=\tau=nIBA$ ...(*i*)

This torque tends to rotate the coil and its rotation produces a twist in the suspension wire. The untwisting tendency of the suspension gives rise to a restoring torque tending to bring the coil back to its original position. This restoring torque is proportional to the deflection.

If θ is the deflection produced in the coil before it comes to rest and k is the torque per unit angular twist, then

Restoring torque $=k\theta$ ...(*ii*)

Hence in equilibrium

Deflecting torque = Restoring torque

$$nBIA=k\theta$$

$$I=\frac{k\theta}{nAB}=K\theta$$

Thus the current is proportional to the deflection. The constant K is called the *galvanometer constant* and is equal to $\frac{k}{nAB}$. Thus we can have a *linear scale* and hence the importance of radial field.

If the field is not radial *i.e.*, if the field is due to *flat poles* then in any deflected position of the coil, when it is so oriented that the normal to the coil makes an angle θ with the field lines, the torque is given by

$$\tau=(BIl_n)\times b\cos\theta$$

$$\tau=BIA\ n\cos\theta$$

Hence in the equilibrium position

$n\,BIA\cos\theta=k\theta$

$$\text{or } I=\frac{k\theta}{n\,BA\cos\theta}=\frac{K\theta}{\cos\theta}$$

$$\text{where } K=\frac{k}{n\,BA}=\text{const.}$$

$$\text{Thus } I\propto\frac{\theta}{\cos\theta}$$

Hence we cannot have a linear scale and the galvanometer will have to be calibrated first before using it for measuring current.

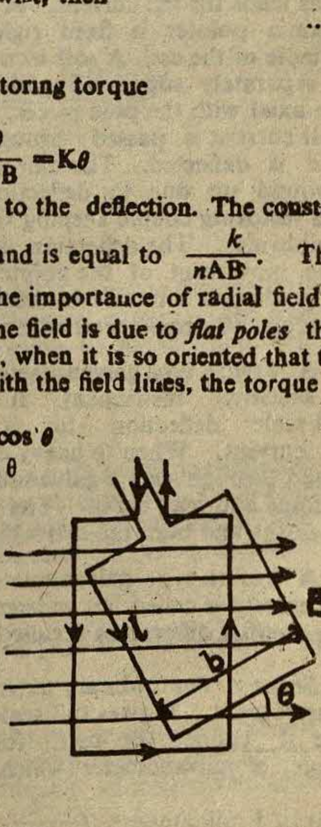


Fig. 7.30 (b)

Pointer type galvanometer : The moving coil galvanometer already described is called the reflecting type or D'Arsonval type galvanometer. In it the deflection is measured by the lamp and scale arrangement which is not very convenient. Moreover it is not easily portable.

Weston modified the design in such a way that the deflection could be read by a pointer and the instrument became small and portable as shown in Fig 7.51. This is known as ***Weston type galvanometer*** and is commonly used in the laboratories.

It consists of a coil of insulated thin copper wire wound over a metallic frame pivoted lightly between two jewel bearings. The coil is kept in position between the concave pole pieces of a strong permanent magnet by two phosphor bronze hair springs. The two springs are soldered to the two ends of the coil and serve as leads for the current. A light aluminium pointer is fixed rigidly to the spindle of the coil. A soft iron cylinder is separately supported within the coil co-axial with the pole pieces. When a small current is passed through the coil, it is deflected. The hair springs are wound up due to deflection and exert a restoring couple keeping the coil in equilibrium. The deflection is read by the movement of the pointer on a scale graduated with zero in the middle and usually 30 divisions on either side.

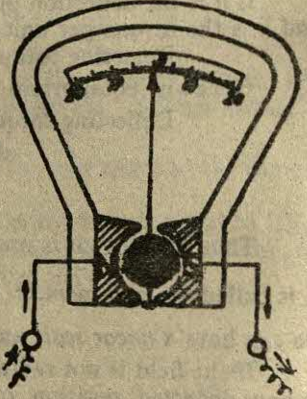


Fig 7.51

A Weston type galvanometer is a very sensitive instrument. It gives a full-scale deflection for a very small current. When a heavy current is passed through such a galvanometer, the pointer is deflected with a great force and may break The heavy current may produce excessive heating and burn the coil which consists of a very fine wire.

A Weston type galvanometer modified for the measurement of heavy current is called an *ammeter* and that for the measurement of large potential differences is called a *voltmeter*.

Shunt : An ordinary moving coil galvanometer is a very sensitive instrument. It gives full scale deflection for a very small current. Hence it cannot be used for measuring large currents. For example, a galvanometer which gives a full scale deflection for a current of 1 milliampere ($\frac{1}{1000}$ th of an ampere) cannot be used to measure a current of one ampere as such a heavy current will produce excessive heating which causes damage to the coil.

To measure heavy currents with such a galvanometer a *shunt* is applied across its terminals. *A shunt is a resistance placed in parallel with a galvanometer coil to allow only a fraction of* the total current to pass through the galvanometer.

Let G be the resistance of the galvanometer, I_g the current that can pass through it for full scale deflection and I the maximum current which is desired to be measured with it. If S is the resistance of the shunt placed in parallel with the galvanometer, then the current through the shunt

$$I_s = I - I_g$$

As the current divides inversely in the ratio of resistances

$$\frac{I_g}{I_s} = \frac{I_g}{I - I_g} = \frac{S}{G}$$

$$\text{or } S = \frac{G \times I_g}{I - I_g}$$

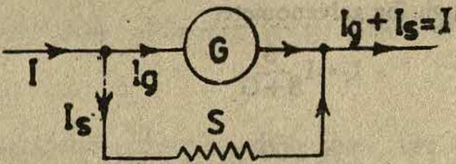


Fig 7.52

It is clear from this expression that the current flowing through the galvanometer is only a fraction of the total current. The fraction of this current can be adjusted to any value by suitably adjusting the shunt resistance.

Example 7.15

A cell of negligible resistance and *e.m f.* 2 volts is connected to the galvanometer of resistance 999 ohms shunted with a resistance of 1 ohm Find the strength of currents passing through the cell and the galvanometer when the shunt is (a) disconnected, (b) connected.

Solution.

Here $E = 2$ volts ; $G = 999$ ohms and

$S = 1$ ohm

As the internal resistance of the cell is negligible there is no fall of potential inside the cell. Hence whatever be the current the potential difference between the terminals of the galvanometer or shunt is 2 volts.

1. When the shunt is disconnected, then

Current through the circuit or cell $I=\frac{2}{999}=0{\cdot}002$ A

The same current will also flow through the galvanometer.

2. When the shunt is connected, the current divides itself into two parts ; I_g passes through the galvanometer and I_s through the shunt.

The combined resistance R of the galvanometer and shunt is given by

$$\frac{1}{R}=\frac{1}{999}+1=\frac{1000}{999}\ \text{ohms}$$

$$\therefore \qquad R=\frac{999}{1000}\ \text{ohms}$$

Current through the circuit or cell

$$I=\frac{E}{R}=2\times\frac{1\,000}{999}=2{\cdot}002\ \text{A}$$

Current through the galvanometer

$$I_g=I\frac{S}{S+G}$$

$$=2{\cdot}002\times\frac{1}{1+999}$$

$$={\cdot}002\ \text{A}$$

Conversion of a moving coil galvanometer into an ammeter ;—A galvanometer due to its high sensitivity has a very low range, so that it gives a full scale deflection even for a very small value of the current. To measure heavy currents with a galvanometer a suitable shunt resistance is connected with it in parallel so that only a fractional part of the total current passes through the galvanometer and the rest through the shunt. Such shunted galvanometers are called *ammeters* as shown in Fig. 7.53. The shunt is connected inside the instrument between the ends of the coil. The value of the shunt resistance depends upon the range of the instrument.

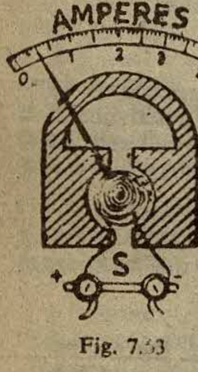


Fig. 7.53

Let G be the resistance of the galvanometer and I_g be the maximum current required to produce full scale deflection. If I is the maximum current which is to be measured Fig. 7.54, then

$$I_g=I\frac{S}{S+G}$$

or $$S=\frac{G.I_g}{I-I_g}$$

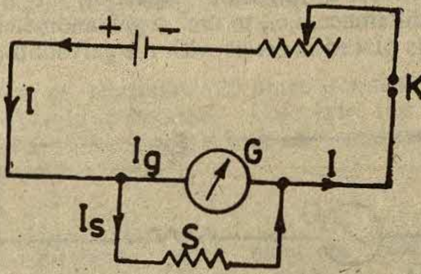


Fig. 7.54

The combined resistance of the shunt and the galvanometer will be less than the resistance of any one of these. Since the resistance of shunt is very low, the resistance of the ammeter will be even lower. Thus an ammeter is a very low resistance galvanometer.

For measurement of current it is connected in series in the circuit.

Example 7.16

A galvanometer of resistance 100 ohms can take a current of 1 milliampere for a full scale deflection. Calculate the resistance of the shunt if a current of one ampere is to be measured by it. What is the resistance of the ammeter so formed ?

Solution.

Here $I_g=1$ milliampere$=\cdot 001$ ampere

Shunt resistance $$S=\frac{G\times I_g}{I-I_g}$$

$$S=\frac{100\times\cdot 001}{1-\cdot 001}=\frac{\cdot 1}{\cdot 999}$$

$$=\frac{100}{999}=\cdot 1001 \text{ ohm.}$$

Let the combined resistance, when shunt is applied, be R, then

$$\frac{1}{R}=\frac{1}{G}+\frac{1}{S}$$

$$=\frac{1}{100}+\frac{999}{100}=\frac{1000}{100}=10$$

$$R=\frac{1}{10}=0\ 10 \text{ ohm}$$

Conversion of a moving coil galvanometer into a voltmeter :—A galvanometer gives a full scale deflection for a very small current and also has a small resistance. Hence only a small potential difference applied to it produces a full scale deflection in it. To measure a large potential difference, *i.e.*, to use a galvanometer as a voltmeter a high resistance is placed in series with the galvanometer coil as shown in Fig. 7.55.

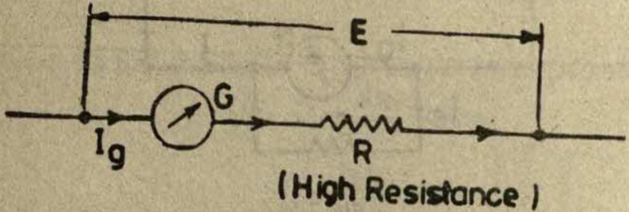


Fig. 7.55

If I_g is the maximum current that can pass through the galvanometer and G its resistance, then the resistance R to be connected in series to measure a maximum potential difference of E volts is given by

$$I_g = \frac{E}{R+G}$$

or

$$R+G = \frac{E}{I_g}$$

$$R = \frac{E}{I_g} - G$$

The galvanometer along with this resistance connected in series will work as a voltmeter of range O to E volts. The resistance R is connected inside the instrument as shown in Fig.7 .56. The total resistance when R is connected in *series* with G becomes R+G and since R is very high, a voltmeter is a high resistance galvanometer.

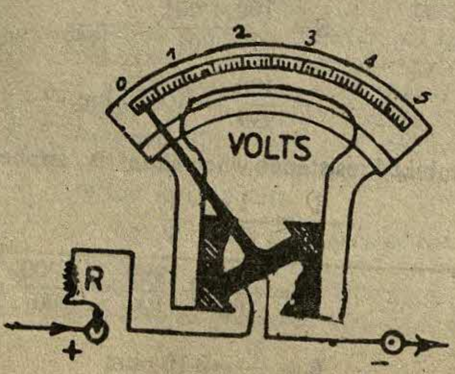


Fig. 7.56

It is always connected in *parallel* with the two points across which the potential difference is to be measured. As it has a very high resistance, a very small current flows through it and current in the main circuit remains practically unchanged.

Example 7.17

A galvanometer of resistance 20 ohms requires a current of 5 milliamperes for full scale deflection. Calculate the resistance that must be placed in series with it so that it can measure a maximum potential difference of 200 volts.

Solution.

Let R be the resistance which must be placed in series to measure a potential difference of E volts, then

$$I_g = \frac{E}{R+G}$$

or

$$R = \frac{E}{I_g} - G$$

$$= \frac{200}{{\cdot}005} - 20$$

$$= 40000 - 20 = 39980 \text{ ohms}$$

Example 7.18

It is required to convert a galvanometer of current range 15 milliamperes and voltage range 750 milli-volts into (*a*) an ammeter of range 25 amperes and (*b*) a voltmeter of range 150 volts. Calculate the values of the necessary resistances and show how they are connected.

Solution.

Current for full scale deflection $I_g = 15$ mA

$= {\cdot}015$ A

Potential differences for full scale deflection $= 750$ milli-volts

$= {\cdot}75$ volt

Resistance of the galvanometer $G = \frac{{\cdot}75}{{\cdot}015} = 50$ ohms

(*i*) Range for conversion into an ammeter $= 25$ A

$$\text{Shunt resistance } S = \frac{GI_g}{I - I_g} = \frac{50 \times {\cdot}015}{25 - {\cdot}015}$$

$$S = \frac{50 \times {\cdot}015}{25 - 0{\cdot}015} = 0{\cdot}03\ \Omega$$

(*ii*) Range of conversion into a voltmeter $E = 150$ volts

$$\text{Series resistance } R = \frac{E}{I_g} - G = \frac{150}{{\cdot}015} - 50$$

$$= 9950 \text{ ohms}$$

SUMMARY

1. An electric current is produced whenever charged particles flow from one place to another. The current is defined as the amount of electric charge that flows across a cross-section of conducting material per unit time. The usual unit of current is the ampere, which is equal to one coulomb per second.

2. A lasting current requires the maintenance of a potential difference, normally with an electric cell (or storage battery or similar means of generation), a closed circuit and carriers of charge that are free to move through the circuit. In metals, the carriers of charges are free electrons.

3. The individual members of each type of charge carrier-ion or electron move with some average velocity when carrying an electric current. We call this a the drift velocity $\vec{v}_d$.

4. The result of frictional forces that slowdown moving charge carriers is known as electrical resistance. According to Ohm's Law, resistance is directly proportional to voltage and inversely proportional to current.

5. The resistance of any piece of material is related to its length, its cross-sectional area, and the material's resistivity. The latter is an inherent property related to the nature, purity, and temperature of the material.

6. Electrical resistance leads to loss of electrical energy in the form of heat (Joule's Heating Law).

7. The production of electric current or e.m. f. due to the difference in the temperatures of the junctions of two dissimilar metals is known as Seebeck effect. A thermocouple thermometer can measure temperature of a point where as thermocouple Pyrometer can be used to determine temperature of furnace and very hot bodies.

8. Free elements are liberated from an electrolytic solution by the passage of an electric current (electrolysis). The electrolysis is carried in a voltameter and the quantitative laws of electrolysis were formulated by Faraday.

9. A secondary cell is not a direct source of e.m.f. It requires charging from an external source of electricity. It accumulates energy in the chemical form.

10. When two charged particles are both in motion, they exert on one another a new kind of force that depends upon their velocities and is zero if either velocity is zero. This new force (known as the magnetic force) is responsible for the magnetic properties of materials such as magnets and also for the magnetic effects of electric currents.

11. The interaction between two charged particles, both in motion, through intervening space can be understood by introducing the concept of field-action at a distance.

12. The magnetic interaction is not a new kind of interaction. Magnetic forces can be explained in terms of electric forces acting on moving charges.

13. Magnetic fields influence moving electric charges, acting on them with a force that is proportional to the size of the moving charge, its speed, and the strength of the field.

14. The direction of the magnetic force on a moving charge is determined by using the right hand screw rule.

15. Electric currents produce magnetic fields whose strength at any point is related to the size of the current and inversely proportional to the square of the distance between the current and the point. (Biot-Savart Law).

16. "Current loop" which consist of almost complete circles of wire through which current is moving, behave magnetically much like bar magnets,

17. When a conductor carrying current is placed in a magnetic field, it experiences a force. The direction of motion of the conductor is given by Fleming's Left Hand Rule.

18. Instruments and devices as diverse as galvanometers are made possible through the movement of current loops in magnetic fields.

19. A moving coil galvanometer can be converted into an ammeter of suitable range by connecting certain low resistance in *parallel* to the galvanometer.

Similarly, a [moving coil galvanometer can be converted into a voltmeter of suitable range by connecting a certain high resistance *in series* with it.

QUESTIONS

1. Develop an analogy between the flow of electricity through a circuit and the flow of water through a pipe. For every quantity relevant to the flow of the electricity find a corresponding quantity for the flow of the water. What are the analogs of Ohm's Law and Joule's Heating Law ?

2. Although the drift velocity of the electrons is very slow, an electric light bulb turns on almost immediately when the switch is closed. Explain this.

   (**Hint** : The analogy with the flow of water through a pipe may prove helpful).

3. Is the resistance of a 150 watt light bulb greater or less than that of a 60 watt bulb ? Remember that the voltage applied across either is 220 volt.

4. Is equation, Heat developed per sec=current × Potential difference valid for a conductor which does not obey Ohm's Law ?

5. Is the distance travelled by an electron in between collisions equal to $\vec{v}_d t$ where v_d is the drift velocity and t is the time that elapses before the next collision ?

6. If a metal were composed of positively charged electrons (positrons) and negatively charged protons (antiprotons), would there be any way of discovering this by studying the phenomena described in this chapter ?

7. A dense beam of electrons all moving in the same direction has a tendency to spread out as it advances. Two overlapping beams of protons and electrons of equal density moving in opposite directions have a tendency to shrink. **Explain this**.

8. **In which of the following cases is there no magnetic force between two electrons ?**

   (*a*) **When they move directly toward one another.**

   (*b*) **When they move directly away from one another.**

   (*c*) **When they have the same velocity in a direction perpendicular to the line joining them.**

   (*d*) **When they have equal and opposite velocities in a direction perpendicular to the line joining them ?**

9. **If an electron moves horizontally toward the north in a horizontal magnetic field pointing toward the west, in which direction is the magnetic force on it ?**

10. **You are looking down on a proton which is moving horizontally in a uniform vertical magnetic field pointing at you. Do you see it going round its circular path in a clockwise or counterclockwise direction ?**

11. **Imagine that you have invented an apparatus that can project an electron with any velocity in any direction and can then measure the force on it with very great accuracy. Explain precisely how this apparatus could be used to determine electric fileld $\vec{E}$ and magnetic field $\vec{B}$ at any point in space.**

12. **Try to discover a case in which the magnetic forces between two moving charges do not obey Newton's Third Law of Motion.**

13. **A student trying to find the directions of the forces between two moving charges always uses his left hand by mistake. Does he get the right answer ?**

14. If the earth's magnetism is assumed to be due to a large circular loop of current in the interior of the earth, what is the plane of this loop and what is the direction of the current around it ?

15. Make rough sketches of the pattern of magnetic field lines produced.

    (*a*) by a circular coil carrying current in the clockwise direction

    (*b*) by two parallel infinitely long straight wires carrying equal currents (*i*) in the same direction, (*ii*) in opposite directions.

16. Does an infinitely long vertical straight wire carrying a current exert a net magnetic force on an infinitely long horizontal straight wire carrying a current ? The wires do not necessarily intersect. Prove your answer mathematically.

17. (*a*) State Ohm's Law and hence define the resistance of a conductor. What is its unit and define it. What are the limitations of Ohm's Law ?

    (*b*) Define resistivity and conductivity of a material. What are their units ?

    (*c*) What are the factors upon which the resistance of a conductor depends ?

18. Obtain from first principle an expression for the heat developed by the passage of an electric current through a resistance. Define the terms watt and kilo-watt hour.

19. (*a*) State Faraday Laws of Electrolysis and describe experiments to verify them.

    (*b*) Define E.C.E. of an element. What is its unit. How is it related to chemical equivalent weight ?

    (*c*) Define a Faraday.

    Compare and contrast the conduction of electricity in an electrolyte and in a metallic wire.

20. What is a voltameter ? Describe a silver voltameter and explain how with its help a current can be accurately determined ?

21. (*a*) What is back e.m.f. in an electrolyte ?

    (*b*) Describe the construction of a secondary cell (Lead Accumulator). Give an account of the chemical reactions that take place at each plate during charging and discharging.

22. (*a*) What is Seebeck effect ? What is thermo e.m.f. ? What is a thermocouple ?

    (*b*) Two wires, one of copper and the other of iron are joined together at one end, the other end being connected to a low resistance galvanometer. Describe the indications of the galvanometer as the iron copper junction is gradually heated to redness. How does thermo e.m.f. vary in this case ? What is neutral temperature and inversion temperature ?

23. (*a*) Describe a thermopile. How can it be used for measuring the temperature of a hot body ? **List two advantages of using a thermocouple as a temperature measuring device. →** ***(Delhi Senior Sec. 1980)***

24. (*a*) The equation $F = qv\,B \sin\theta$ involves the magnitudes of three vectors, $\vec{F}$, $\vec{v}$ and $\vec{B}$. Of the three vectors which pairs are always right angles ? Which pairs may have any angle between them ?

    (*b*) What is the unit of $\vec{B}$? How is it related to gauss ? Define this unit.

25. What is a cyclotron ? Describe its construction and working ? What is its purpose ?

26. (*a*) Describe how you would proceed to determine $\frac{e}{m}$ of an electron by J.J. Thomson method ?

    (*b*) What is J.J. Thomson's startling hypothesis ?

    (*c*) What is the value of $\frac{e}{m}$ of an electron ?

27. (*a*) State the various ways in which magnetic force differs from an electrostatic force ?

    (*b*) State Biot-Savart Law. Apply it to find the field (i) at the centre of a circular coil carrying current, and (*ii*) on the axis of a circular coil carrying current.

    [*A.I.S.S.C. 1986*]

28. Prove that the mechanical force on a conductor carrying current placed in a magnetic field is given by $\vec{F} = B\,I\,l \sin\theta$ where the symbols have their usual meaning and hence define magnetic field.

29. Calculate the force between two infinitely long straight conductors carrying current and hence define ampere.

30. (*a*) Give various rules to determine the direction of magnetic field produced by current carrying conductor.

 (*b*) A particle having charge $(-q)$ is introduced in a magnetic field of magnetic induction $\vec{B}$. If the particle travels with velocity $\vec{v}$ in a direction perpendicular to $\vec{B}$, show that its path will be circular and hence calculate its radius.

31. Demonstrate by a simple experiment that a force acts on a current carrying conductor. State (*a*) Right Hand Rule and (*b*) Left Hand Rule to determine the direction of this force.

32. (*a*) What is a galvanometer ?

 (*b*) Give the construction, principle, theory and working of a moving coil galvanometer. What is the importance of the radial field ?

33. (*a*) What is a shunt ? Why is it used with galvanometers ?

 (*b*) Deduce the relation between the currents flowing in the galvanometer circuit and the shunt circuit.

 (*c*) Why should a voltmeter have a resistance and an ammeter a low resistance ?

34. How a moving coil galvanometer can be converted into (*i*) an ammeter and (*ii*) a voltmeter ?

35. (*a*) Two identical galvanometers are to be converted into an ammeter and a milliammeter. Whose shunt will have more resistance and why ?

 (*b*) Whose resistance is less—of voltmeter or of millivoltmeter ?

 (*c*) If a student by mistake connects voltmeter in series with or an ammeter in parallel of a circuit, what will happen ?

36. After connecting an ammeter in series with and a voltmeter in parallel of a resistance wire, a current is passed through the wire. After sometime, the wire is heated up, but not the ammeter. Why ?

37. (*a*) Is any work done by a magnetic field on a moving charge.

 (*b*) H potentiel difference V is applied to a conductor of length l, diameter d. How are the electric field $\vec{E}$, the drift velocity $\vec{v_d}$ and the resistance R affected when

 (*i*) V is doubled (*ii*) l is doubled (*iii*) d is doubled.

(c) Two parallel wires carrying current in the same direction attract each other while two beams of electrons travelling in the same direction repel each other. Why ?

[**Hint** : A current in the wire produces only a magnetic field while an electron beam is a source of both, an electric and a magnetic field).

38. A particle of mass m and charge q goes round in a circle with a speed V in a magnetic field $\vec{B}$. Show that its momentum is proportional to the radius r.

[A.I.S.S.C. 1986]

$$\left[\textbf{Hints :}\quad qv\text{B}=\frac{mv^2}{r}\right].$$

PROBLEMS

1. The type of wall socket commonly found in the house is capable of delivering a current of 5 amperes. If this current flows through a copper wire with a diameter of 0·1 cm, what is the drift velocity of the electrons ? It is assumed that the copper atom has one loosely bound electron and charge on the electron is $1{\cdot}6\times10^{-19}$ coloumb. The atomic mass of copper is 64. Avogadro's number $=5{\cdot}0\times10^{23}$ and 1 cm^3 of copper has a mass of 9·0 gm. If the wire is 1 m long, calculate the time taken for an electron to drift from one end to the other.
($4{\cdot}74\times10^{-4}\ \text{ms}^{-1}$: $2{\cdot}11\times10^3$ sec)

2. If a current of 10^{-15} A flows through a wire, what is the net number of electrons passing any point in the wire per second ?
($6{\cdot}25\times10^3$ electrons per sec)

3. If a resistance of 120 ohm is connected across the terminals of a 6 volt battery, what current flows in it ? How much heat is developed in it per second ?
(0·05 A, 0·3 watt)

4. A copper wire has a resistance of 10 ohms. It is melted down and made into a new wire with only one half the original length, but with a correspondingly larger diameter. What is its new resistance ?
(2·5 ohms)

5. A copper wire 1 m long with a diameter of 0·1 cm has a resistance of 0·02 ohm. Calculate τ, the average time between collisions of an electon with the copper atoms. About how far does an electron travel between collisions ?

Compare this distance with the average distance between adjacent copper atoms and comment on the comparison. Charge on the electron $= 1{\cdot}6 \times 10^{-19}$ Coulomb

($3{\cdot}2 \times 10^{-13}$ sec; Assuming a thermal velocity of about 10^{8} cm/sec, an electron travels about 3×10^{-5} cm between collisions. The distance between adjacent copper atoms is about 2×10^{-8} cm)

6. A proton has a velocity of $1{\cdot}5 \times 10^{7}$ ms^{-1} due north in a magnetic field of 5×10^{5} gauss pointing vertically upward. What are the magnitude and direction of the magnetic force on it ? Charge on the electron $= 1.6 \times 10^{-19}$ Coulomb.

($1{\cdot}2 \times 10^{-10}$ N, horizontal, toward the east)

7. An electron has a velocity of 1.5×10^{6} ms^{-1} vertically upward. What are the magnitude and direction of the magnetic field which it produces at a point 10^{-8} m due north of itself. Charge on the electron $= 1{\cdot}6 \times 10^{-19}$ coulomb.

($2{\cdot}4 \times 10^{-4}$T ; horizontal toward the east)

8. A straight segment of wire 0·001 m long carries a current of $\frac{10}{3}$ ampere. What magnetic field does it produce at a distance of 1.5 m in a direction perpendicular to itself ?

($1{\cdot}48 \times 10^{-10}$ T)

9. A straight horizontal wire 35 cm long carrying a current of 5 ampere is at an angle of 60° to a horizontal magnetic field of 8×10^{-2} T. What are the magnitude and direction of the magnetic force on it ?

($1{\cdot}2 \times 10^{-1}$ N ; horizontal toward the east)

10. Two long straight wires are parallel and 20 cm apart. They carry current of $\frac{5}{3}$ A and $\frac{20}{3}$ A respectively.

What is the magnetic force on 1 m of either wire ?

(11×10^{-6} N)

11. A pointer galvanometer of 10 ohms resistance has a 50 divisions scale and indicates 1 micro-ampere per division Show how it can be used (a) as a milliammeter of range 50 milliamps (b) as a voltmeter of range 5 volts.

(S=0·01001 ohm; R=99990 ohms)
in parallel in series

12. A house is fitted with 8 lamps ; 5 of 40 watts and 3 of 100 watt each ; 3 fans each taking 0·25 A and an electric iron having a heating element of 44 ohms. If all the appliances are worked on 220 volts supply for 5 hours a day, find the bill for the month of April at 20 P a unit.

(Rs. 52·95)

13. The electron in the hydrogen atom circle round the proton with a speed of $2{\cdot}18\times10^{6}\ ms^{-1}$ in an orbit of radius $5{\cdot}3\times10^{-11}\ m$. What magnetic field does it produce at the proton ? Also determine the frequency and the current in the orbit.

(12·4 T ; $6{\cdot}5\times10^{15}$ Hz ; $1{\cdot}05\times10^{-3}$ A)

14. A charge of 4.0×10^{5} coulombs passes through an electrolytic solution of copper sulphate. Calculate

(*i*) the number of Cu^{++} ions liberated from the electrolyte.

(*ii*) the number of electrons flowing in the outer circuit.

(charge on one electron $=1{\cdot}6\times10^{-19}$ coulomb)

($1{\cdot}25\times10^{24}$; $2{\cdot}5\times10^{24}$)

15. If the current flowing be 1·25 amperes, calculate the sufficient time to coat a 0·1 mm thick copper layer on a circular metallic plate of radius 2·5 cm. The density of copper is 8.9 g/cc and E.C.E. of copper $=3{\cdot}3\times10^{-7}$ kg coulomb.

(8476·19 sec)

16. 90 coulomb charge passes in 1 hour 15 minutes in a silver wire of diameter 1 mm. Silver contains $5{\cdot}8\times10^{28}$ electrons per cubic metre (*i*) what is current in the wire ? (*ii*) calculate drift speed of electrons in the wire.

(charge on the electron $=1{\cdot}6\times10^{-19}$ coulomb)

(2×10^{-2} ampere; $2{\cdot}75\times10^{-6}\ ms^{-1}$)

17. The resistance of a piece of aluminium wire 100 *m* long, is found to be 0.855 ohm. The cross sectional area of the wire is $3{\cdot}31\times10^{-2}$ sq cm. Calculate the specific resistance of aluminium.

($2{\cdot}83\times10^{-6}$ ohm × cm)

18. A 450 watt electric press is made to operate at 225 volts. Find (*i*) its resistance (*ii*) rate of heat produced in joule per sec (*iii*) power of the press, if voltage drops to 200 volts.

(112·5 ohm; 450 watt; 355·55 watts)

19 An electric kettle has two thermal coils. When a current is passed in one of them, water in kettle boils in 6 minutes and when current is passed in other one, the water boils in

8 minutes. If both the coils are joined in (*i*) series, (*ii*) parallel, how long it will take in boiling the water ?

(3·43 minutes ; 14 minutes)

20. How long will it take to rise the temperature of air in a room 4 meters × 5 meters × 3 meters through 10°C by means of an electric heater taking 5 amperes from 220 volts mains assuming no heat loss ? Density of air = 0·001293 g/cc $C_p = 0{\cdot}24$ and $J = 4{\cdot}2$ joules /calorie.

(11·8 minutes)

21. A platinum wire has resistance of 10 ohms at 0°C. If its temperature coefficient of resistance is $\frac{1}{273}$ /C°C°, what is its resistance at 273 C°

(20 ohms)

22. The thermo-e.m.f. of a thermocouple is 40μ V per degree at room temperature. A galvanometer of 50 ohms resistance capable of detecting currents as low as 10^{-5} amperes is connected with one such thermocouple. Calculate the smallest temperature difference that can be detected by such a thermocouple.

(12·5 degree)

23. The least count of a moving coil galvanometer is 0·05 degree and the maximum deflection permissible is 30°. Find the minimum and maximum currents which can be detected by the galvanometer. Given that (a) coil is 0·1 × 0·5 *m* (b) number of turns of wire is 100 (*c*) magnetic field is 0·25 tesla and (d) restoring torque is 10^{-5} Nm/degree.

(4×10^{-7} A ; 24×10^{-5} A)

24. A cyclotron in which the magnetic field is 1·4 T is used to accelerate protons. How rapidly should the electric field between the dees be reversed ? Also calculate the cyclotron frequency. Charge on the proton is 1.6×10^{-19} coulomb. Mass of a Proton $= 1{\cdot}67 \times 10^{-27}$ kg.

($2{\cdot}343 \times 10^{-8}$ sec; $2{\cdot}133 \times 10^{7}$ Hz

25. An electron beam is passing through a magnetic field of 2×10^{-3} T and an electric field of 34×10^{4} volts meter both acting at the same place. If the path of electron beam remains undeviated calculate the speed of electrons. If the electric field be removed, what will be the radius of curvature of the path of the electron beam ?

($1{\cdot}7 \times 10^{8}$ ms^{-1}; $47{\cdot}8 \times 10^{-3}$ *m*)

26. A coil of 20 cm radius has 15 turns and carries a current of 3·0 A. Find the magnetic field at the centre of the coil ($\mu_0 = 4\pi \times 10^{-7}$ N/A-m)

($1{\cdot}41 \times 10^{-4}$ T)

27. A cosmic ray's proton enters the earth's magnetic field in the direction normal to the field at a speed of $10^7\ ms^{-1}$. Calculate the force acting on the proton. Charge on the Proton$=1{\cdot}6\times10^{-19}$C and Earth's magnetic field$=13\times10^{-7}$T.

($20{\cdot}8\times10^{-19}$ N)

28. Alpha particles of mass 6.68×10^{-27} kg and charge 3.2×10^{-19} coulomb are accelerated in a cyclotron in which the uniform magnetic field is $1{\cdot}25$ T. How rapidly should the electric field between the dees be reversed ? What is the velocity and kinetic energy of an alpha particle when it moves with radius 25 cm ?

($1{\cdot}91\times10^7$ p s^{-1} ; $1{\cdot}50\times10^7\ ms^{-1}$; $7{\cdot}52\times10^{-13}$ joule)

29. An electric bulb is rated as 200 V, 500 W. What is the resistance of the filament ? How much will it cost to light this bulb for 8 hours if the price of electricity is 75 paise per unit ?

(80 ohm ; Rs. 3.00)

[*A.I.S.S.C. 1986*]

UNIT 8

UNIVERSE

> *"The astronomer's problem is not a lack of information but an embarrassing excess of it. His is often a problem of disentanglement rather than one of synthesis."*
>
> **FRED HOYLE**

8·1 INTRODUCTION

Science and humanities are often presented as contrast of ideas but in reality nothing is more genuinely humane than science. The desire to understand and explain the world around them has been one of the most characteristic traits of wise and hard working human beings and as the method has changed from creation of myths to analysis of starlight and creation of models the demand upon imagination and creative human thought has increased rather than diminished. Surely the concepts used to explain the universe now—black holes: white holes; curved space, an expanding universe, etc. – are more bizarre and make more demands upon the imagination than the products of the myth-makers of ancient times. The structure of the universe does not present itself to be perceived by the casual observer, nor does the evolution of the universe reveal itself at a glance. What we know about the structure and evolution of the universe is learnt by imagining models, studying the models and then comparing their properties with what can be seen of the real world. In the end we can never know whether we are correct in our description; we can only know sometimes that we are wrong.

Since ancient times the view of the universe held by educated people has undergone a radical revision. The universe is no longer a realm of caprice, moved by the whim of gods, but a world governed by laws which can be discovered. With the opening of space age a new picture of the universe is emerging out. For astronomers this is one of the most exciting period – observatories revolving round the earth, material brought from the moon, experiments done on the surface of Mars (Viking Findings) and on

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Note :—Unit 8—Universe is not in the Hr. Sec. Syllabus of Orissa. It may be left by these students.

the surface of venus, discovery of pulsars and Quasars are all telling new stories about the universe. From a closed box of small size the universe has become an almost limitless expanse of space. The elements in it have changed from the few planets and few thousand stars which can be seen by the naked eye to the billions of billions of stars which make up uncounted galaxies.

Fundamental changes in the prevailing view of the nature of the universe have occurred several times in the past. In the 17th century the fact was discovered that the same laws of mechanics which operate on the surface of the earth also govern the motion of planets. In the 19th century the distances to stars first began to be measured and the compositions of stars were determined, with the result that for the first time we knew that the stars are made of the same elements as the earth. In the early 20th century the fact that the sun is not at the centre of the galaxy was first recognised, the expansion of the universe was discovered, and the theory of general relativity was revealed as a better explanation of gravitational phenomena than Newton's theory of gravitation. Now in the last half of the 20th century discoveries have been made which may force further significant changes in our view of reality. Probably nothing so dramatic as displacing the earth and our solar system from the centre of the universe will occur, but tentative explanations are being offered for the behaviour of the nuclei of some galaxies and of quasars which are fully as dramatic as that.

The aim of the present unit in this book is to generate interest in the minds of the young students so that they may take up further study of this challenging observational science—astronomy.

The part of the universe in which we live is called the Solar System. It includes the sun, the earth and the moon, other planets (Mercury ; Mars ; Venus ; Saturn ; Pluto; Jupiter, Neptune, Uranus) and their satellites, asteroids, comets, meteoroids, meteors and meteorites. The outermost planet Pluto is at a distance of 6×10^{12} metres from us. Farther away are stars very much like our sun, which occur singly, in pairs as binary stars, or in clusters like Pleiades (Krittika). The sun is a typical star and a member of a large system called the Milky way (Akashaganga) which is estimated to contain as many as 100 billion stars similar to the sun. The Milky way, like all other galaxies, contains huge quantities of interstellar gas, mainly hydrogen, and interstellar dust. It is estimated that the total mass of interstellar gas in the universe is about equal to the mass of the stars. Probably the mass of the interstellar dust constitutes about 1 percent of the total mass of the universe. The universe is made up of hundreds of millions, perhaps 10 billions, of galaxies similar to the Milky Way. The galaxies form the major building blocks of the universe.

The distances between stars are very large ; the nearest star Alpha centauri is at a distance of 4 light years. The external galaxies are scattered in space to at least about 5 billion light years in all directions, and probably beyond. *One light year is the distance travelled by light in one year* with a speed of 3×10^8 ms^{-1}. It is equal to $9{\cdot}46 \times 10^{15}$m.

In this chapter our aim is to present the factual information concerning stars and galaxies ; quasars and pulsars ; recent findings about solar system using space probes and about the theories of the origin of the universe, etc. Much of the material given in this chapter is rather qualitative and further details on many of the observations presented here can be had from the various books on astronomy.

8·2 ASTRONOMICAL INSTRUMENTS (*Elements of optical and Radio Astronomy*)

Most of the information that we have about astronomical objects comes to us by light or other electromagnetic waves (Fig. 8·1). The *two windows* in the atmosphere indicated are the two regions in which radiation from outside the atmosphere can reach the surface of the earth. *The atmosphere is opaque to all other wave lengths.* In the electromagnetic spectrum, the two regions (windows) in the spectrum, where the atmosphere is transparent are from about 290 nm (in the ultraviolet) to the near infrared, and from 8 mm to 17 m. The visible region, and the ultraviolet and the infrared near to the visible, lies in the first window ; all radio astronomy must be done through the second window which is the true window involving no atmospheric absorption. Since the development of rockets and satellites which carry instruments

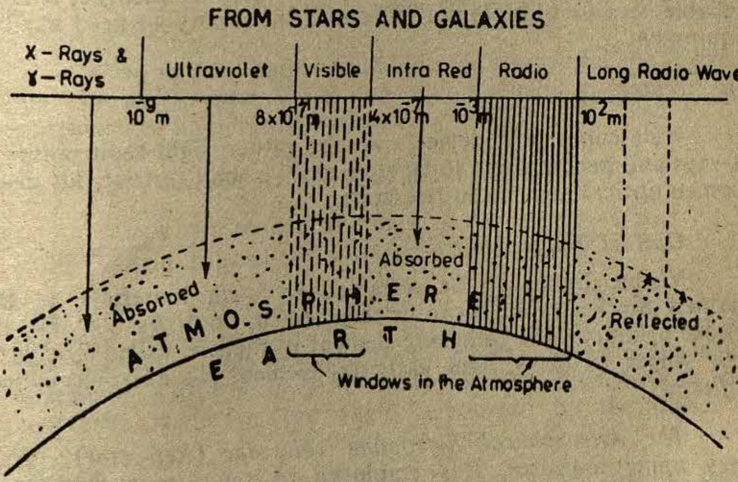

Fig. 8·1

above the atmosphere, our knowledge of the appearance of the universe as seen by radiation of other wavelengths, notably the shorter radiations, ultraviolet and X-ray has increased tremendously.

Man's curiosity has always driven him to search out the unknown and has led him to devise the means of exploring it. As man achieved the technological advances which permitted him to detect the electromagnetic radiations emitted out by the heavenly bodies—the link between the distant star and the earth, it was inconceivable that he should not explore it.

A modern astronomer uses the following instruments in astronomical research :

(A) **Telescopes**

(B) **Spectrograph**

(C) **Space observatories**

These instruments can be conveniently divided into two groups—instruments for *collecting* light or radiowaves and other are the *recording* instruments of various kinds. Our eye serves both as collector and recorder of light. But as collector it is not efficient on account of its small size and as recorder it is not reliable because of biological effects. That is why we need auxiliary equipment.

(A) Telescopes

Telescopes are devices which gather light-radio-waves or X-rays and permit them to be studied. We shall restrict this discussion to optical and Radio telescopes.

Optical Telescopes :

Visible light can be refracted by such materials as glass formed into lenses, or it can be reflected by mirrors. A telescope whose primary light gathering element is a lens is called a *refractor*; one whose primary light gathering element is a mirror is called a *reflector*.

The Astronomical Refracting Telescope (*Refractor*) *:* As its very name indicates, it is employed to see celestial bodies, like stars and planets etc., and was invented by *Kepler* in 1611, and first used by the Huyghens in 1665.

Essentially, it consists of (i) *a convex lens of large focal length* called the *object-glass*, the function of which is to form a real, inverted and diminished image of the distant object at its focus, and (ii) *a convex lens of short focal length*, called the *eye-piece*, the function of which is to magnify the inverted image formed by the object-glass, and lying within its focus.

For normal adjustment of the telescope, however, the final image must be formed at infinity, so as to be seen by the unaccommodated eye, and hence the inverted image of the distant object formed by the object-glass must lie at the focus of the eye-piece.* In other words, it must lie at the focus of both, the object glass and the eye-piece.

Thus, in Fig. 8·2, AB is the real, inverted and diminished image of the distant object star or planet, formed by the object-glass, lying at the focus of the object-glass as well as that of the eye-piece, so that the rays from it are rendered parallel on passing through the eye-piece and the final image is seen by the unaccommodated eye at infinity ; in other words, the telescope is in normal adjustment.

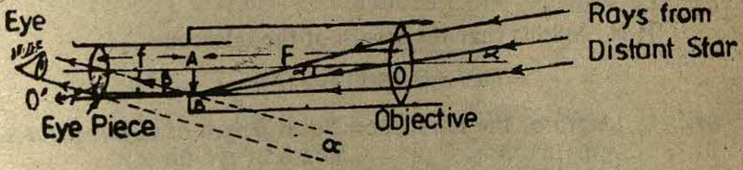

Fig. 8·2

Astronomical bodies are all so far away that the light reaching the earth from them comes in virtually parallel rays, so the operation of a telescope is simple.

Now, magnifying power of the telescope

$$=\frac{\text{angle substended by image at the eye}}{\text{angle subtended by object at the eye}}=\frac{\angle\beta}{\angle}$$

where β is the angle subtended by the image at the eye-piece, or at the eye, (which is close to it), and α, the angle subtended by it at the object-glass, but since the length of the telescope is negligible compared with the huge distance of the object from it, we may also take α to be the angle subtended by the object at the eye.

Now $\angle\beta=\angle AO'B$, and $\angle\alpha=\angle AOB$ (being vertically opposite angles).

Magnifying power of the telescope $=\frac{\angle AO'B}{\angle AOB}$.

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*In fact, an astronomical telescope is provided with a number of eye-pieces of varying focal lengths, so that the observer may choose the one that gives him the magnifying power convenient to him, some objects requiring greater magnification than others.

Again, since the angles are small, we may measure them by their tangents ; so that,

$$\text{magnifying power of the telescope} = \frac{\tan AO'B}{\tan AOB} = \frac{AB}{O'A} \Big/ \frac{AB}{OA} = \frac{OA}{O'A}$$

But OA=F, the focal length of the object-glass, and O'A=*f*, the focal length of the eye-piece.

$$\therefore \text{ magnifying power of the telescope} = \frac{F}{f}$$

$$= \frac{\text{Focal length of object-glass}}{\text{focal length of eye-piece}}.$$

Thus, *the greater the focal length of the object-glass and the smaller the focal length of the eye-piece, the greater the magnifying power of the telescope.*

It is obvious that the length of the telescope

$$= OO' = OA + O'A = (F + f).$$

or, length of the telescope = Focal length of object-glass + focal length of eye-piece.

Since the objective is a lens and it forms an image by the process of refraction, this kind of telescope is called a refractor.

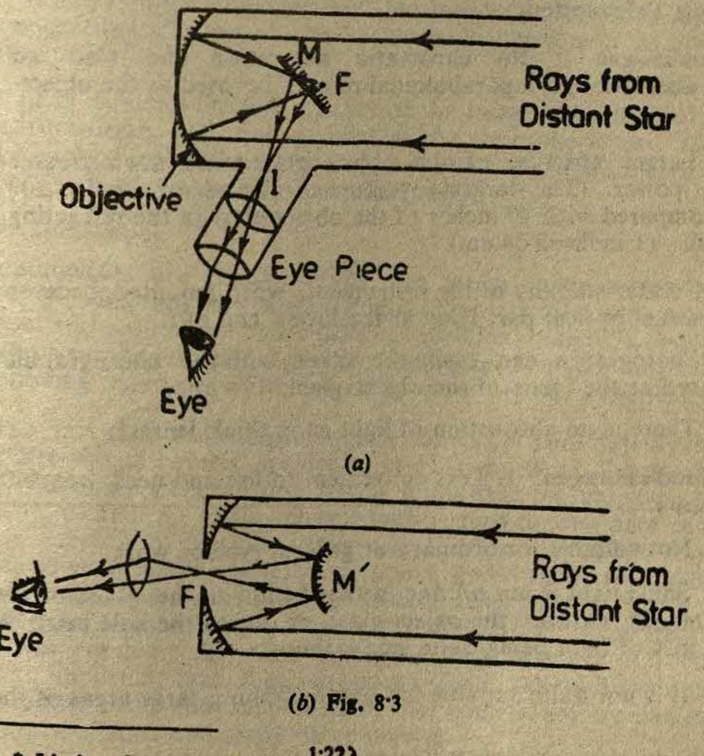
Many very small telescopes are refractors, but large refractors have so many disadvantages that very large telescopes are never of this type. The largest refractor in the world, in the Yerkes Observatory, has an objective 40 inch (=102 cm) in diameter.

The Astronomical Reflecting Telescope : (*Reflector*) :

The first reflecting telescope was constructed by Newton. Thinking that it was impossible to have a compound lens which could be achromatic, he used a concave mirror as the object glass. Parallel rays from a distant object (star or planet) were reflected at the mirror and before they could meet at its focus F Fig. 8.3 (*a*), they were made to fall on a plane mirror M, placed at an angle of 45° with the axis of the instrument. The rays were thus reflected on to one side as shown, forming an image of the distant object at I. This image was then seen magnified through an eye-piece in the usual manner. Since the objective is a mirror of large focal length so this telescope is called a *Reflector.*

Newton was greatly disappointed with the performance of his first reflecting telescope, due to much of the incident light on the object glass being lost by reflection. For ordinary purposes, therefore, where the aperture of the object glass need not be large, a refracting type of telescope is found to be more suitable, there being less loss of light by absorption in it. But, for large-aperture telescopes, with *high** *resolving powers*, such as those used for observing heavenly bodies, the lens-object-glass would be a rather thick one, the absorption of light in which would far exceed that lost by reflection at the mirror-object-glass. A reflecting telescope is thus better for this purpose than a refracting one.

Further, *chromatic aberration* (colouration in image) is inherently absent in the case of mirror-images, and spherical aberration (*i.e.*, light striking regions near the centre is brought to focus at a different point from that striking near the edge) too is eliminated by using a paraboloidal mirror as the object glass, instead of a concave mirror. So that, the reflecting telescope is free of these two defects in the images formed by it. Yet another point in their favour is that whereas it is not possible to make large-aperture lenses**, large-aperture mirrors can be easily obtai-



(*a*)

(*b*) Fig. 8·3

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* Limit of Resolution $\theta = \frac{1 \cdot 22\lambda}{d}$ where λ is the wavelength of light and d is the diameter of the objective. To have better resolving power, the diameter d should be as large as possible. θ is the angular separation between two points which are very close to each other.

** This is so because in the first place, it is extremely difficult to find large masses of glass, free from imperfections, and secondly, being supported only at the rim, the lens is bound to bend a little under its own weight.

ned, because all they need is a smooth surface. That is why in the giant reflecting telescope at the California Institute of Technology of the U.S.A., the object glass is a pyrex mirror, of 200 inches ($\approx$500 cm.) diameter, coated with aluminium.*

Cassegrain, in his telescope, made a small hole around the pole of the concave mirror (or the object glass) and received the reflected beam from it on a convex mirror M, instead of on a plane one, as shown in Fig. 8.3(b), which was thus brought to focus at F. The image at F is then observed through an eye-piece, as in other cases. The arrangement thus results in an effective increase in the focal length and hence the magnifying power of the telescope, which, as in the case of the refracting telescopes, is given by the ratio between the focal lengths of the object glass and the eye-piece, *i.e.*, by F/f.

Advantages and Disadvantages of Reflecting Telescopes over Refracting Telescopes.

Advantages 1. No chromatic aberration and also no spherical aberration, if a paraboloidal mirror be used as the object-glass.

2. Larger aperture of the object glass and hence a greater resolving power. (The largest aperture of the mirror used is 200 inches, compared with 40 inches of the object glass in the refracting telescope). (1 inch=2·54 cm).

3. Greater stability of the instrument, when mounted, because of the heaviest optical part lying at the lower end of it.

4. Photographs can easily be taken, with the photographic plate placed at the focus of the object glass.

5. There is no absorption of light as in thick lenses.

Disadvantages. 1. Less convenient to use and need frequent adjustments.

2. Not suitable for ordinary or general type of work.

3. Some light is cut off due to the shadow of the plane or the convex mirror cast on the object-glass, or due to the hole made in it—this loss of light being quite small, though.

4. It is not quite suitable for photographing large areas of the sky

Due to these reasons reflectors are far more widely used than refractors, especially for large telescopes. Hence largest telescopes of the world such as the 200 inch telescope of U.S.S.R are all reflectors. The 48 inch telescope of Japal-Rangapur observatory of Osmania University, which is the largest telescope in south east

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*Aluminium is found to be more suitable for the purpose than silver.

Asia, is also a reflector. A 90 inch reflector is being made by the Indian Institute of Astrophysics, Bangalore for its Kuvalpur observatory.

Observers using telescopes to look at objects a block away are most interested in magnification — how much larger things look through the instrument than without it. If one is studying the planets or the moon the *magnification* of a telecope is also important, but for most astronomical work — looking at stars and other objects in deep space — *light gathering power* is more important than magnification. The light gathering power of the telescope, which determines our ability to see fainter objects, increases in proportion to the area of the objective $\frac{\pi D^2}{4}$ where D is the diameter or aperture of the objective. The advantage of a telescope of diameter 200 inch over one 100 inch is not that the former makes things look larger, but that it gathers 4 times as much light, permitting photographs to be made in one-fourth as much exposure time.

But in practice the fine quality of the image is finally limited by the turbulence and transparency of the atmospheric air. That is why astronomical observatories are normally located on remote mountain tops where the air is calm and clear.

Radio Telescopes

Radiowaves are fundamentally the same phenomenon as visible light, differing only in wavelength. Both are electromagnetic radiation, and both obey similar laws of reflection. Like light waves, radiowaves can be refracted, but glass has negligible effect on them, and refracting telescopes which use radio-waves are not practical.

The fact that radio signals reach the earth from outside the atmosphere was discovered accidently in 1931 by K.G. Jansky of the Bell Telephone Laboratories. Jansky found that the antennas he was using picked up radio noise which seemed to originate in a particular region of the sky, later identified as the Milky Way. The first antenna made specially to receive Radio waves from outside the atmosphere was built by Grote Reber in 1936, but not until after the Second World War did the study of the radio frequency waves reaching the earth from other objects in the universe became an important branch of astronomy. Such waves have now been received from the sun, some of the planets, some galaxies, certain types of stars and many other objects

A radio telescope consists of a metallic reflector [Fig. 8·4 (a) & (b)] which focuses the incident radiation into an antenna at its focus connected to a radio-receiver. Since neither the human eye nor photographic film responds to radio waves, one does not "see" with a radio telescope in the same sense as with a visible-light instrument. The output is a tracing on a sheet of paper showing the *intensity* of the radiation reaching the telescope. If the receiver were connected to a speaker rather than to a recorder, the radio signals being received would sound like static or random

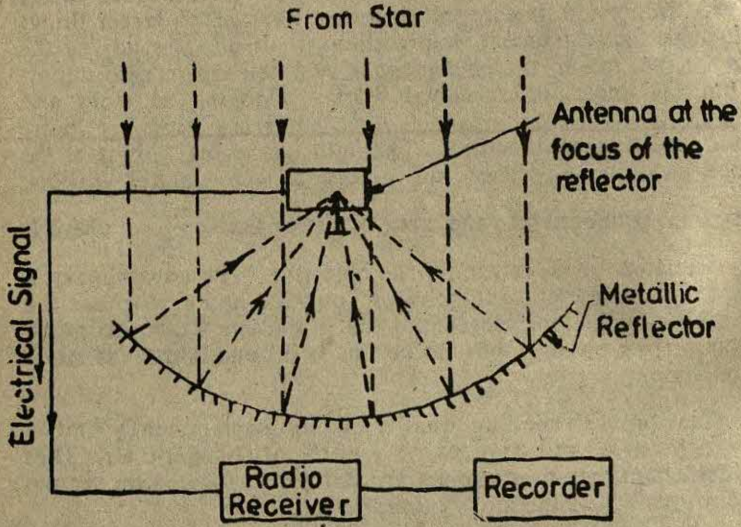


Fig. 8.4 (a)

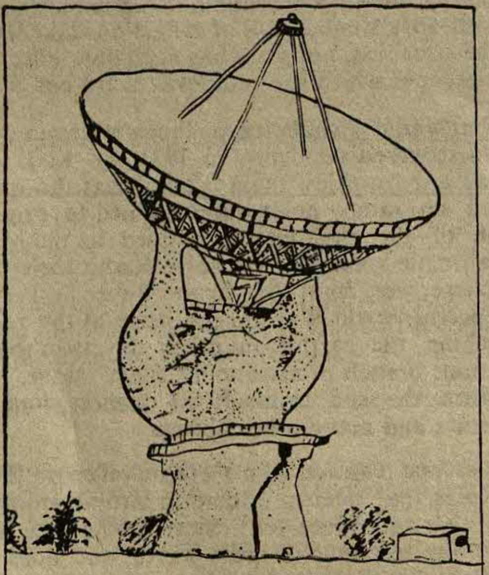


Fig. 8.4. (b)

noise; even though they are called signals, they do not carry information of the sort on commercial broadcasts.

With a few exceptions, radio telescopes can be divided into three classes : steerable dishes; fixed dishes, and large arrays. Steerable dishes are *parabolic reflectors* which can be pointed and turned to compensate for the earth's rotation so that they always point at the same object. The best known of this type is probably the 250 ft (=75 m) diameter dish at JODRELL BANK in England. The National Radio Astronomy Observatory in Greenbank, West Virginia, U.S.A. has a 300-ft (90 m) dish. *These reflectors can be made much larger than optical reflectors because the accuracy with which they must be shaped depends on the wavelength, of the radiation to be recevied, and the deviations of perhaps an inch (2·54 cm) can be tolerated in a radio telescope reflector* whereas the mirror of an *optical telescope must be accurate to within two-millionths of an inch.* Since the amount of energy gathered by a mirror is proportional to its area, the advantages of making the reflector large are obvious. On the other hand, the mechanical problems of mounting and moving such a large structure and building it to withstand the wind sets limits on the size. The best known telescope is at Arecibo, Puerto Rico, and has a diameter of 3 300 ft (=990 m). In India, the objective of the big radio telescope in Ootacamund is a cylinder of 500 m in length and 50 m in width.

Instead of using a reflector and one detector and directing the reflector toward a source, it is possible to use many detectors and to combine their outputs with appropriate time delays so that the effect is very much the same as if a movable reflector had been used. Arrays of detectors with a "directing" done electronically can be spread out over distances of a kilometer or more. Large arrays like this are in use in many parts of the world.

Radio telescopes are not obstructed by clouds of dust as radio waves can pass through them. Because of large aperture, radio telescopes can pick up feeble signals and they can detect galaxies even at twice the distance as far away as the faintest and most distant galaxy which an astronomer can see through an optical telescope. Radio telescopes can see galaxies as well during the day as they can see in night. Finally for the same area of the reflector, radio telescopes are cheaper to construct.

(B) Spectrograph

The light which comes from stars and other heavenly bodies tells us not only the *direction to its source*, but also the *composition* of the star, its *temperature* (using Stefan-Boltzmann Law and Wien displacement Law); its *velocity* toward or away from us (Doppler-effect) and in some cases such data as the *intensity of magnetic fields* and *the pressure* of the gases in the atmosphere. We can often infer something about a *star's rotational speed* from a study of the light we receive from it. All this information is derived from a study of the *spectrum* of light emitted by the star or any other heavenly body.

For analysing light in the visible range and near the ultraviolet or near infrared regions coming from a star or heavenly body, a spectrograph is used. Two kinds of spectrograph are generally used (1) Prism type, (2) Grating type.

Whatever may be the type of the spectrograph it is mounted at the end of the optical telescope as an attachment.

Light from the telescope entering the spectrograph first strikes a narrow slit, which blocks all but a very narrow beam. Beyond the slit is a lens whose focal point lies at the slit so that the light coming through the slit and diverging to the lens moves in parallel rays after going through the lens. A prism or a diffraction grating separates light of different colours and bends light of different colours to different amounts. Another convex lens focuses the light leaving the prism or grating on to a *photographic film* or some other *detector*. For work in the visible or ultraviolet regions, photographic film is almost always used (actually photographic plates, thin plates coated with emulsion, are more often used than film), but in the infrared region the detector is likely to be of a different sort, perhaps a thermocouple which is heated by the infrared radiation striking it.

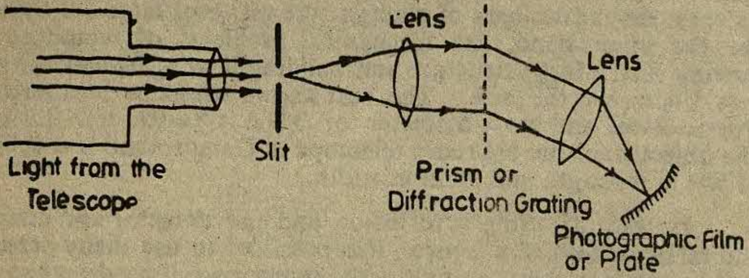


Fig. 8·5

Suppose that light is passed through a spectrograph and the spectrum is observed. What does one see ? The spectrum will have one of the three appearances : It will consist of a series of bright lines; It will be a continuous smear of colour from the violet to the red; It will be a continuous smear except that it will be crossed by a series of dark lines. The appearance of the spectrum tells us something about the source of the light.

Photographic plates have certain defects of their own; in particular the darkening of the plate is not proportional to the amount of light. Photocells are found to be very good for this purpose; because the photocurrent is directly proportional to the intensity of light. But in their case we can only observe one star at a time. Hence a judicial combination of photo-graphic and photoelectric techniques is used in practice.

(*c*) Space Observatories (New Windows)

Astronomers have been hindered in their observations by the earth's atmosphere that like a veil, blots out or distorts electromagnetic radiation.

About 99 percent of the atmosphere's air molecules are concentrated between the Earth's surface and an altitude of about 34 km. By placing telescopes and other equipment for studying the heavens in observatories above this altitude, man can see the universe from a vantage point above the haze of the atmosphere. For example, the first of a series of orbiting solar observatories has provided tremendous quantities of new information about our sun ; about how variations in solar activity influence earth's atmosphere and magnetic field ; and about the amount of cosmic radiation that reaches earth's surface.

By means of future observatories of this type and by means of artificial satellites, men will be able to acquire more definite information about the nature and origin of the solar system. He will be able to view in greater detail the physical features of earth's moon, of mars, and of other planets.

With his earth-based telescopes, man has gathered data about his own Milky way galaxy and other galaxies. He has found evidence of planets revolving around other stars. Mounted on a platform in space, man's telescopes will be able to reveal hitherto unavailable information about stars and galaxies. Perhaps, they may enable him to learn about the existence of planets that resemble earth.

Thus new branches of astronomy (X-ray and ultraviolet) for the study of the universe are taking birth.

8·3 THE STUDY OF THE SOLAR SYSTEM

The solar system, man hopes to explore, is tiny in relation to the universe as a whole, but it is an area of tremendous magnitude in earth terms. Its primary our sun, is a star located at the centre of the system with nine planets revolving around it in near circular orbits (Fig. 8·6). Some of the planets, like earth, have natural satellites of their own (Jupiter has 12), and there are thousands of other bodies (planetoids which lie between Mars and Jupiter, comets : dust etc.) moving within the system.

The planets are held in their orbits by the Sun's gravity. They all move in the same direction around the sun and their orbits lie in nearly the same plane, with Pluto the exception. Their orbital speeds are higher near the sun. Mercury, the planet nearest the sun, makes a circuit in 88 days. Earth's period of revolution is 365¼ days. Distant Pluto, more than 5·6 billion kilometres from the sun takes 248 earth-years to make one circuit.

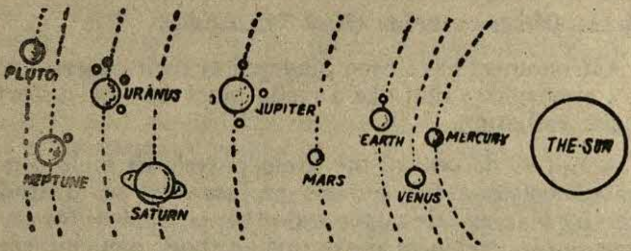


Fig. 8·6. The Solar System

In the early 17th century, Johannes Kepler formulated three laws (see unit 3), which described the motions of the planets about the sun. These laws together with Sir Issac Newton's laws of gravitation and motion, are important to space research. They make it possible to deduce mathematically the motions of the planets and other bodies in the solar system and to calculate flight paths to these bodies.

We shall now discuss how the physical properties of the objects in the solar system are obtained from observations.

(*a*) **Distances within the solar system**

It is possible to determine the periods of the planets with high accuracy, and then, from Kepler's Third law, to calculate the relative values of the semi-major axes of their orbits. The relative sizes of the orbits in terms of astronomical units can also be determined by direct observation as shown in Fig. 8·7 and Fig. 8·8. *An astronomical unit (A U) is the mean distance of the earth from the sun.* It is found to be $1{\cdot}496 \times 10^{11} m$. The sizes of the orbits of Mercury and Venus can be computed from observations of the planets at maximum elongation (greatest distance from the sun) as shown in Fig. 8·7. The angle θ can be measured and the distance (r) from the sun to the planet can be computed from the right triangle shown.

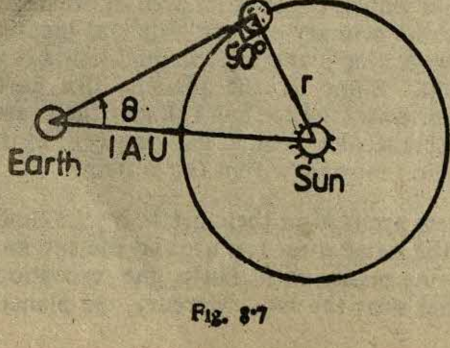


Fig. 8·7

Computation of the distance to an outer planet can be made by observing the planet at least twice when it is at the same position on its orbit. The earth is not at the same position on its orbit when the two observations are made (Fig. 8·8) and hence angles θ_1 and θ_2 can be measured and θ can be computed from the datas on which the observations are made. Distance from the sun to the planet can then be computed. It was by computations such as this, based on observations made by Brahe, that Kepler determined the orbit of Mars.

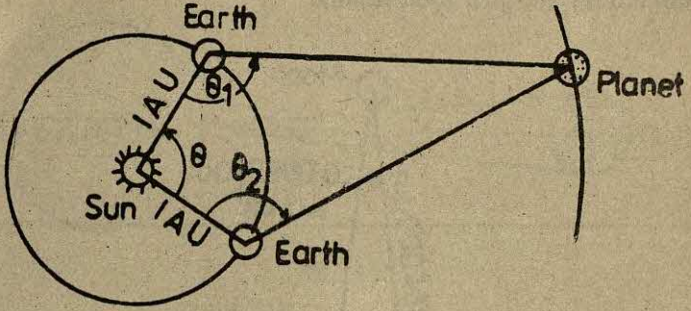


Fig. 8·8

Table 8·1 give the semi-major axes (in astronomical units) of all the planets. With these values, it is easy to layout the solar system to scale. For earth, of course, the value is 1·00 A U by definition.

As an example of measurement of a distance by *triangulation*, consider the problem of determining the distance to the moon. Suppose that two observers, situated 3200 km apart simultaneously measure the distance between the vertical and the line of sight to a particular crater on the moon (Fig. 8·9). To make the example simple, we shall assume that they choose a time when the angles they measure are the same, and each measures 14·59°. Since the arc of the earth's surface between them has a length 3200 km and the radius of the earth is 6400 km, the angle at the centre of the earth subtended by that arc is 0·5 × 57·3° or 28·7°. Since the interior angles of a quadrilateral must add to 360°, the angle at the apex of the triangle must be 0·48°. If the arc along the surface of the earth is 3200 km, the straight line through the earth joining the two observers is approx. 3184 km, but the difference is not important since the angular measurements introduce more uncertainly than that difference. If the distance to the moon is now computed from

$$\text{Distance} = 57{\cdot}3 \times \text{base of triangle} \times \frac{1}{\text{angle in degrees}}$$

it is

$$\text{Distance} = \frac{57{\cdot}3 \times 3184}{0{\cdot}48} = 384000 \text{ km}$$

The distance to the moon can be measured with considerable accuracy by triangulation, but distances to the planets are so great that this method does not give good results.

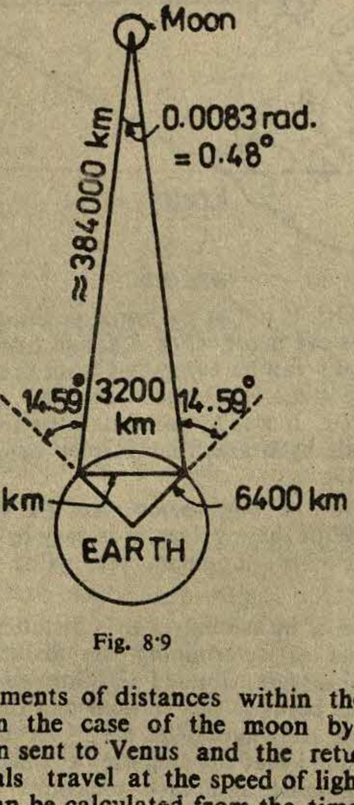


Fig. 8·9

The best measurements of distances within the solar system are made by *radar* or in the case of the moon by visible light. Radar signals have been sent to Venus and the return echo received. Since radar signals travel at the speed of light, 3×10^8 ms^{-1}, the distance to venus can be calculated from the time required for the signal to go from the Earth to the planet and back with an uncertainty of only a few kilometres.

The first American astronauts to land on the moon left a corner reflector, a device with a number of reflectors which send light back in the same direction from which it arrives. A laser pulse directed toward the moon through a telescope was reflected by the corner reflector and received on the earth again, and measurement of the travel time permitted calculation of the distance from the telescope to the reflector. It is hoped that this sort of measurement will permit determination of the earth-moon distance to within a few centimetres.

(*b*) Size

The objects of the solar system when observed through telescope give rise to disc images. We can, therefore, easily measure the angular diameter θ of the planet or any other heavenly body. From the definition of angle (Fig. 8·10), it follows that

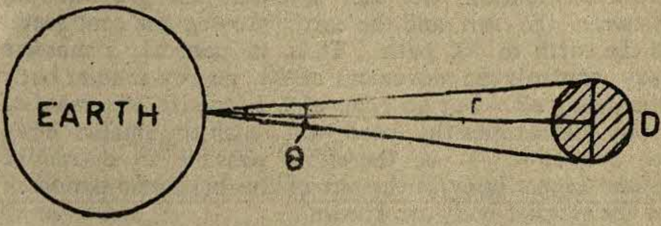


Fig. 8·10

$$\boxed{D = \theta \times r} \qquad .. (1)$$

where D = size in m

θ = angular size in radians

r = distance of the object from the earth in m.

For example sun's angular radius is 960″ or 0·00465 radians. Distance of the sun from the earth = I A U = $1{\cdot}496 \times 10^{11}$ m.

∴ Solar radius = $6{\cdot}956 \times 10^{8}$ m. It is about 109 times the radius of the earth. The diameters of the planets in units of the radius of the earth are given in Table 8.1. Similarly multiplying the angular separation of a satellite from the parent planet by the distance of the planet, we can calculate the distance of the satellite from the planet in kilometres.

This method cannot be applied to find the diameters of stars because stars appear as points in the telescope.

(*c*) Rotation

We know that our earth revolves round the sun in one year and at the same time it rotates round its north-south axis from west to east in 23 hours and 56·1 minutes. The planets also exhibit the phenomenon of rotation round an axis passing through their respective centres. The period of rotation of a planet is determined by observing the movement of surface markings or cloud patches over the disc of the planet. The rotation periods are given in Table 8·1. In most cases the sense of rotation is from west to east as in the case of the earth. Mercury does not show any surface markings and venus is always covered with a thick blanket of clouds. Their rotation periods were determined recently by using the radar echo technique. Both of these planets are found to rotate very slowly and the rotation of venus is *retrograde* *i.e.*, from east to west.

(*d*) **Mass :**

If we think of the earth moving in a circular orbit in order to simplify the calculation, we can say that the gravitational attraction between the sun and the earth provide the centripetal force to hold the earth in its path. Thus, in general, a massive heavenly body controls the movement of the nearby smaller body by its gravitational pull. For an orbit of a given size, the mass of the bigger body determines the speed with which the smaller body must revolve around it. It is, therefore, possible to determine the mass of the bigger body, if the size of the orbit and period of revolution of the smaller body are known.

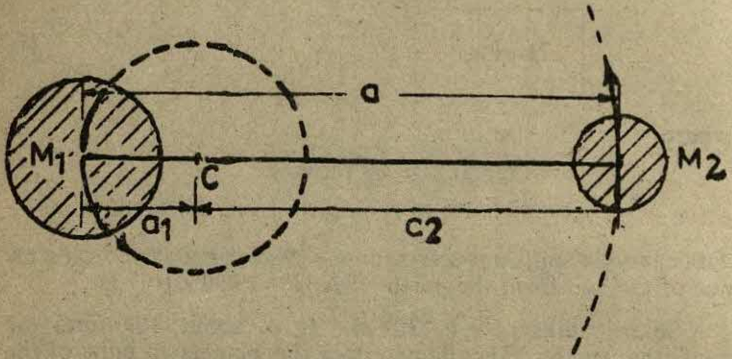


Fig. 8·11

Let M_1 be the mass of a large central body like the sun and let another smaller body, say a planet of mass M_2 revolve round it in a circular orbit of radius '*a*' (Fig. 8·11). Actually both bodies revolve round a common centre of mass C. By the *principle of balance* we have

$$\boxed{M_1 a_1 = M_2 a_2}$$

where a_1 and a_2 are the radii of revolution from the centre of mass C *i.e.*, $a_1 + a_2 = a$

$$\therefore \frac{a_1}{a_2} = \frac{M_2}{M^1}$$

which gives $\frac{a_1 + a_2}{a_2} = \frac{M_2 + M_1}{M_1}$

or $$a_2 = \frac{M_1 (a_1 + a_2)}{(M_1 + M_2)} = \frac{M_1 a}{M_1 + M_2}$$

Similarly, $a_1 = \frac{M_2 a}{M_1 + M_2}$.

Now if V_1 and V_2 are the circular velocities of the two bodies in their orbits around the centre of mass C, we have

$$V_1=\frac{2\pi a_1}{T}$$

and
$$V_2=\frac{2\pi a_2}{T}$$

where T is the period of revolution.

Now considering the force acting on one of the bodies, say M_2, we can write,

Gravitational force = mass × centripetal acceleration

or
$$\frac{GM_1M_2}{a^2}=\frac{M_2V_2^2}{a_2}=\frac{4\pi^2M_2a_2}{T^2}$$

Dividing both sides by M_2 and substituting the value of

$$a_2=\frac{M_1a}{M_1+M_2},$$ we have

$$\frac{GM_1}{a^2}=\frac{4\pi^2}{T^2}\cdot\frac{M_1a}{M_1+M_2}$$

or
$$\boxed{M_1+M_2=\frac{4\pi^2}{G}\cdot\frac{a^3}{T^2}} \quad ...(2)$$

By knowing the period of revolution T, the sum of masses *i.e.*, (M_1+M_2) can be obtained from equation (2). Further, if we know the position of the centre of mass C *i.e.*, if we know $\frac{a_1}{a_2}$ we can find $\frac{M_1}{M_2}$ or M_1 as well as M_2.

In case one of the bodies say M_1 is very massive (as in the case of sun-planet system) as compared to the other then

$$\boxed{M_1=\frac{4\pi^2}{G}\cdot\frac{a^3}{T^2}} \quad ...(3)$$

(since $M_2 \ll M_1$)

where T is the time period of revolution of the smaller body.

Equation (2) can also be applied to a planet-satellite system in which M_1 will be the mass of the planet and M_2 that of the satellite.

TABLE 8·1

PHYSICAL PROPERTIES OF THE OBJECTS IN THE SOLAR SYSTEM

g (Earth) = 9·82 ms^{-2} ; d=days ; h=hour ; m=minutes ; ++ Total No. of Asteroids 1600

Object	*Period of Revolution in Year*	*Semi-Major axis (a) in A U*	*Radius R (In Earth Units)*	*Rotation Period (T)*	*Mass M in (Earth Unit)*	*Mean density kgm^{-3}*	*g in Earth Units*	*Temperature (Degree C)*	*Albedo*	*Surface Pressure in atmosphere*	*Atmospheric Chemcomposition*	*No. of Satellites*	*Velocity of Escape kms^{-1}*
Mercury	0·241	0·387	0·38	58·6 (d)	0·056	$5·4 \times 10^3$	0·367	+340 day −120 night	0·06	0	Vacuum	0	4
Venus	0·615	0·723	0·96	−243 (d) (Retrograde)	0·815	$5·1 \times 10^3$	0·886	+480 (Surface) −40 clouds	0·85	100	CO_2 (95%)	0	10·5
Earth	1·000	1·000	1·00	23 h 56·1 m	1·000	$5·52 \times 10^3$	1·000	+45 (Equator) −50 Poles	0·40	1	N_2 (80%) O_2 (20%)	1	11·2
Mars	1·881	1·524	0·53	24 h 27·4 m	0·107	$3·97 \times 10^3$	0·383	+30 Equator −130 Poles	0·15	0·006	CO_2 (97%) N_2 (3%)	2	5

Ceres (*Largest Asteroid*)	4·603	2·767	0·055	90 h 05 m	0·0001	3·34×10³	0·18	—	0·07	0	Nil	++	—
Jupiter	11·864	5·203	11·23	9 h 50·5 m	317·9	1·33×10³	2·522	−140 clouds	0·45	—	H_2, He, CH_4, NH_3	14	60
Saturn	29·46	9·540	9·41	10 h 14 m	95·2	0·70×10³	1·074	−175 clouds	0·61	—	H_2, He, CH_4, NH_3	10+ ring	37
Uranus	84·01	19·18	3·98	−10 h 49 m (Retrograde)	14·6	1·33×10³	0·922	−220 clouds	0·35	—	H_2, He CH_4	5+ ring	21
Neptune	164·1	30·07	3·88	15 h	17·2	1·66×10³	1·435	−230 clouds	0·35	—	H_2, He, CH_4	2	22·5
Pluto	247	39·44	0·5	6·39 (d)	0·11	4·9 ×10³	0·440	−240 ?	0·14	?	?	Nil	24?
Moon	—	—	0·27	27·32 d	0·0123	3·34×10³	0·170	+110 day −150 night	0·07	0	Vacuum	—	—

1 year=365·257 days ; 1 A U =1·496×10^8 km ; Earth Radius (R)=6378 km ; Earth Mass (M)=5·977×10^{27} g

The masses of the planets in terms of earth mass are given in Table *8·1*. On knowing the mass and radius we can easily calculate the mean density $\frac{3M}{4\pi R^3}$ and acceleration due to gravity $g=\frac{GM}{R^2}$ They are given in table 8·1 above

(*e*) **Surface Temperature :—**

Planets shine by the reflected light of the sun. Only a fraction of the solar radiation is reflected by the planet. The remainder is absorbed and it heats up the surface of the planet, which then radiates energy in the *infrared* and *radio regions* of the electromagnetic spectrum. On measuring the total *self-radiation* of the planet and applying Stefan's Law $E=\sigma T^4$ where $\sigma=5{\cdot}67\times 10^{-8}\ Jm^{-2}\ deg^{-4}\ s^{-1}$ and Wien's Law $\lambda_m T=b$ where $b=0{\cdot}2897$ cm-K (see unit 5), we can determine the surface temperature T of the planet; the sun etc. Table 8·1 also gives the temperature of the planets obtained in this way. The observed temperature decrease as we go away from the sun, because the planets receives less and less solar energy according to inverse square law. Venus is an exception, because it has a very thick atmosphere of carbon dioxide which acts as a blanket and keeps the surface hot.

The temperature of the sun has been found to be about 5800 K, confirming that the sun is approximately a black body.

(*f*) **Atmosphere** :

We known that our earth has an atmosphere consisting of 80% nitrogen and 20% oxygen with some carbon dioxide and water vapour. We can determine whether a planet or satellite has an atmosphere in two ways :—

(*i*) to observe meteorological phenomena of clouds and storms

(*ii*) to detect atmospheric gases with the help of a spectrograph.

As clouds are good reflectors of light, they increase the reflecting power of the planet which is called *albedo* (*i.e., the ratio of sun's energy reflected by a planet to that absorbed by it*). Table 8·1 also gives the albedos of various planets and the moon. The albedo of venus is 0·85 *i.e.*, it reflects 85% light ; venus is known to be covered with dense clouds. The dull rocky surface of Mars reflects only 15% of sunlight. Moon and Mercury reflect only 6 to 7% of sunlight very much like the rocks on earth and hence they have practically no atmosphere.

The chemical composition of the atmosphere of a planet is derived from the study of its spectrum which would show the characteristic absorption bands of the gaseous molecules in the atmosphere. This is how it was found that the atmospheres of

Venus and Mars contain mostly carbon-dioxide. Atmospheres of Jupiter and Saturn contain hydrogen, helium ; methane and ammonia, while those of Uranus and Neptune have hydrogen, helium and methane only.

The ability of a planet to hold an atmosphere depends primarily on two factors, *the gravity* (g) and *temperature.* Small bodies like the moon have very low gravity and consequently small escape velocity (see unit 3). Therefore the atoms of the atmospheric gases escape one by one leaving nothing behind. Mercury, Mars and Pluto, for which g is somewhat larger, should retain some atmosphere. Only in the case of Mercury which is very hot on account of its closeness to sun, the atmosphere has completely escaped. Since high temperature is associated with larger kinetic energy (see unit 4) and hence with larger mean velocities of atoms and molecules, which are sufficient to overcome the effect of gravity. Again since the velocity of lighter atoms is more by the law of equipartition of energy, they can escape much more easily than the heavier ones. That is why the planets, Venus, Earth and Mars have lost all their hydrogen and helium which are the lightest of all atoms. Heavier planets Jupiter and Saturn have, however, retained all gases including hydrogen and helium.

Possibility of Life on Other Planets of the Solar System
(Recent Findings Using Space Probes)

In this day of space travel, we should consider the possibility of life on other planets, some of which man may visit in a few decades.

The following conditions must hold in order for plant and animal life, of the kind with which we are familiar, to exist :

(*i*) The planet must have a suitable temperature range. The coldest temperatures must not be much colder and the hottest not much hotter than those reached on the earth. This in turn means that the planet's distance from the sun must not differ too much from that of the Earth. Of course, in the case of a planet of a much hotter sun the critical distance for a proper temperature range would be greater

(*ii*) The planet must have a sufficient amount of atmosphere. A planet much smaller than the earth would have such a low "velocity of escape" that a gaseous atmosphere would escape molecule by molecule. Thus leaving an airless planet or at least one with an atmosphere too rare to support life as we know it.

(*iii*) The planets must have the right kind of atmosphere. In terms of our own carbondioxide-oxygen cycle, animal life needs oxygen and exhales carbon dioxide ; plant life needs carbon dioxide and gives out oxygen. There must be sufficient quantities of these two important gases. Our kind of life could not exist in an atmosphere of "poisonous" gases such as ammonia gas, carbon monoxide, or methane, which are known to exist on certain planets.

(*iv*) The planet must have a considerable amount of water on its surfaces.

Applying these conditions to the other planets of the solar system, we have to conclude that the Earth is the only planet on which life as we know it can exist naturally.

(*a*) **MERCURY** :

It is a planet closest to the sun and about three times the size of the moon. Its same side always faces the sun as it moves in its orbit. Its sunlit side has a temperature of about 370°C, and the dark side about —240°C. Due to extremes of temperature and absence of any atmosphere, no life is possible on this planet. Photographs taken by Mariner 10 space vehicle launched by U.S.A. in February 1973 show that the surface of Mercury is marked by craters of all sizes.

(*b*) **Venus**

It is a brilliantly shining planet because it reflects as much as 85% of the sunlight falling upon it. It is earth's neighbour and as regards its radius, mass and density, it has been rightly called the earth's twin. It is covered by a thick layer of dense clouds which account for the large amount of sunlight reflected by it. The surface temperature of Venus is about 480°C. It is partly due to the fact that it is close to the sun and partly because the atmosphere and clouds prevent heat from radiating away from its surface. *Venera space probes* sent by U.S.S.R. between 1965 to 1971 showed that Venus is covered with a thick atmosphere containing 95% carbon-dioxide. The surface pressure is about 100 times the atmospheric pressure on the earth. Under these circumstances, there is no possibility of life on this planet.

Venus is rotating very slowly and more remarkable yet, it is rotating backwards. Venus revolves about the sun and in a counterclockwise sense, and rotates clockwise about its axis. This is called retrograde rotation.

(*c*) **Mars**

Mars is perhaps the most interesting of the planets, when seen through a telescope it is seen to have reddish colour ; at least it has been the object for the most speculation. We get the best view of it ; its surface can be seen without much cloud obscuration; and of all the planets in the system other than earth, it seems the most likely to be hospitable to life.

The Martian day is almost the same as the terrestrial day : 24 hr, 37 min, 23 s. The Martian year is about two of our years. It is situated from the sun at a distance of about 1½ times as much as that of the earth. The planet exhibits white polar caps that grow large in winter and small in summer in step with the seasons. Other changes in certain greenish areas are also seasonal. Temperatures in the equatorial regions tend to run from about 30°C during the day to perhaps −75°C at night. The temperature at the poles is about −130°C. The presence of earth-like conditions on Mars suggested the possibility of existence of some life on Mars.

The Mars vehicles sent by Russia and Mariner space probes sent by U.S.A., however, revealed that the ice caps were made of solid CO_2 with a small percentage of water in them. The photographs taken by Mariner 9 during 1971-72 showed that Mars has craters like those on the moon but the boundaries of these craters have been smoothened by the eroding action of strong winds which causes huge dust storms. Viking-I and Viking-II were sent by U.S.A. to find out if any life existed on Mars Viking-I began on July 20, 1976, man's first ever search for life outside the third planet. Forty five days later, the second craft, Viking-II landed further north about 7000 kilometres away. Between them the two crafts have sent back valuable data and remarkable pictures. But *Martian life, if it is there, still remains elusive.*

Viking Scientists argue that the failure to detect organic material in Martian soil does not necessary prove that there is no life on the planet. "The search may not have been sufficiently thorough or we may have been looking at the wrong places, "says Vikings chief project scientist, Mr. G.A. Soffen. Others like astronomer Zdenek Kopal of Manchester University, feel that "the results obtained so far leave no doubt that Mars has no life."

Even if the vikings have disappointed many who expected to see at least some form of living matter (lichens etc) on the planet, they have yielded an enormous body of important information. Many of the findings have radically changed our ideas about Mars.

The Martian atmosphere turned out to be completely different from what was known earlier. Besides carbon-dioxide and oxygen, viking instruments found as much as 3 percent nitrogen which was thought to be virtually absent on the planet.

A major viking discovery was that the ice of the polar caps is made of frozen water and not solid carbon dioxide as many scientists suspected. If that is so, there would be enough water to fill a lake one kilometre deep spread over 33,000 square kilometres ! There was also evidence of water frozen into the ground as "permafrost", possibly a few kilometres thick.

Another unexpected thing came just before Mars went behind the sun and out of communication with earth. The Seismometer of viking-II detected a "Marsquake", the first tremor on the planet recorded by man's instruments. After all, Mars was not completely dead :

The viking missions also threw new light on the origin of two tiny Martian moons-Phobos and Deimos. Close up pictures taken by viking cameras showed them to be of meteoritic origin and not besaltic as thought earlier. If so, that would mean that the two moons were captured by the planet and not created from it.

(*d*) Moon :—

From the stand point of the universe as a whole, the moon is entirely insignificant (as, of course, is the earth), but residents of the earth, cannot ignore it. Exploration of the moon by manned and unmanned vehicles sent from the earth is providing information about both the moon itself and the early stages of formation of the solar system.

The moon always keeps the same face towards the earth because its period of rotation is equal to its period of revolution round the earth. The rear side of the moon was first photographed by the Russian space probe Luna-3 in 1960. It also shows craters, dark planes and lighter shaded mountains like the visible front side. From telescopic studies it was already known that the lunar landscape is bleak and foreboding. It is an *airless* and *waterless* expanse of desert and rocks *without vegetation, life and sound.* Lack of atmosphere, and long duration of the day, which lasts for 15 earth days there, makes the temperature soar to 110°C at mid-day and drop to −150°C at night. Hence the U.S.A. first sent out surveyor and orbiter space crafts to observe the moon from close range before the astronauts Armstrong, Aldrin and Collins finally conquered it in July 1969 in their Apollo 11 space vehicle. They and their successors brought back lunar soil and rocks for examination. The Russians also sent an unmanned robost vehicle Lunakhod in January 1973 and brought back Lunar Samples to earth all by remote control. Analysis of Lunar material has shown that it is similar to the rocks in the earth's crust. The age of lunar rocks agrees with the age of the earth and meteorites all being about 4½ billion year old. Evidently the moon was formed at about the same time as the earth.

The magnetic field of the moon is very small and one infers that the core of the moon is not molten ; although why it has a magnetic field at all is a mystery.

Data on Moon

Mean distance from the earth = 3 83 000 km

Mean diameter = 3480 km

Mass = $7{\cdot}35 \times 10^{22}$ kg

Period of revolution = $27\frac{1}{3}$ days

Period of rotation = $27\frac{1}{3}$ days

Mean density = 3400 kg/m^3

Accleration due to gravity = 1·96 ms^{-2}

Temperature +110°C (day)
−150°C (night)

(*e*) Outer Planets

All planets beyond Mars are so cold that living organisms are not likely to survive there. Further their atmospheres are full of poisonous gases like methane and ammonia. Pioneer and Voyager space crafts have been launched by U.S.A. for a close study of the outer planets with the hope that we could at least understand the origin of our solar system.

Thus we reach the conclusion that life as we know it probably does not exist elsewhere in our solar system.

But what about other planets of other solar systems out in space ? As can be read in books dealing with cosmology, there is no reason to doubt that a large number of the billions of suns out in space have families of planets, and it seems reasonable that one or more planets of a large number of these system will be of the right size and right distance from their sun to have proper atmosphere and temperature for the development of life as we know it. Remember, though that even the nearest of these other possible planetary systems is 32 million million kilometres from our own solar system. We are not likely to exchange visits with them very soon.

Other Objects in the Solar System

Besides the planets and their satellites, the solar system contains

(*i*) Asteroids
(*ii*) Comets and
(*iii*) Meteors and Meteorites

(*i*) Asteroids

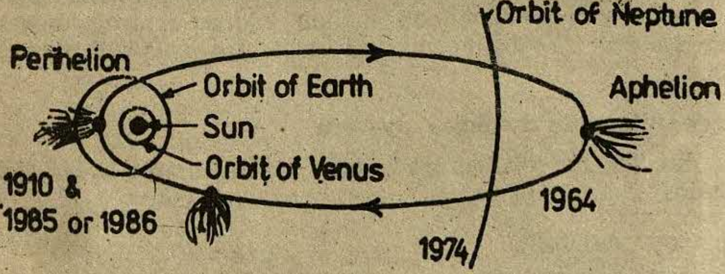
As early as 1801 Piazzi, an Italian astronomer, discovered with his telescope a body less than 800 km in diametre and having an orbit around the sun. Within a few years several similar but still smaller bodies were found. Because all these were much smaller than the planets and had orbit between those of Mars and Jupiter, it was decided to call them *asteroids* or *Plantoids* to distinguish them from the nine major planets. Since 1891 photography has been employed in the search for asteroids, and more than 2000 have been discovered for which orbits have been computed. The largest along them is *Ceres* with a radius of 350 km while the smallest might be only 100 metres across. They are believed to be the pieces of a much larger planet which broke up due to the gravitational effect of Jupiter. The density and chemical composition of asteroids are very much like that of the moon.

(*ii*) Comets

Comets are composed of small particles of rock like material surrounded by large masses of easily vaporised substances like ice, water, ammonia and methane. They move in highly elliptical orbits and spend most of their time for away from the sun. A comet has a nucleus of about 10 km diameter which is not visible most of the time. When a comet is far away from the sun, it has no tail. As it moves along its path and heads towards the sun, it begins to become elongated in the direction **away** from the sun, thus forming a tail. The tail always points away from the sun regardless of the direction in which the comet travels.

The formation of tail is due to what is called *radiation pressure*. As the comet approaches the sun, some of its substances get vaporised due to solar radiation, which are then forced away from the comet by the pressure of light from the sun.

Figure (8·12) shows the orbit of Halley's comet which revolves around the sun in a period of about 75 or 76 years and was last seen from the earth in 1910 as the comet passed near its perihelion. The student may look forward to seeing it in 1985 or 1986.



Orbit of Halley's Comet
Fig. 8·12

(*iii*) Meteors and Meteorites

In addition to the planets and asteroids that are revolving around the sun, millions of very small bodies sometimes called *planetesimals* and weighing few grams to hundreds of metric tonnes are likewise moving around the sun throughout the space occupied by the solar system. Occasionally the earth in its orbital motion comes so close to one of these bodies that the body is attracted towards the earth by the earth's large gravitational force. As the object comes hurtling toward the earth's surface, it moves through the extensive layer of gases that we call atmosphere. The friction of this motion produces heat that causes the outside of the small fast-moving body to become incandescent, and we see a *meteor* or shooting star. Thus a meteor is a relatively small body that has become incandescent because of the heat of friction in passing through the earth's atmosphere.

Most meteors are so small that during their brief passage through the earth's atmosphere they vaporize completely, owing to the intense heat of friction. Occasionally, however, a meteor that is so large, it does not completely vaporize actually reaches the earth's surface and is then called a *meteorite*. The biggest of meteorites form craters on the surface of the earth. In an authenticated case in 1954 an Albana Housewise in U.S.A. was injured by a glancing blow from a small meteorite that fell through the roof of her home.

The cause of craters on Moon, Mercury and Mars is also the same.

Space missions to asteroids and comets are being planned by America during 1980's.

The Sun :—

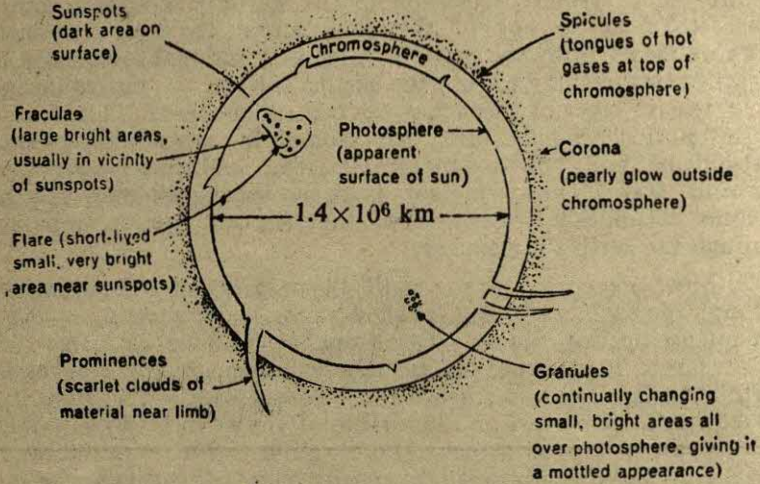
(*a*) **The Sun** is important to us for three reasons :—

(*i*) The Sun is our main source of light and heat. Most of our energy comes directly from the sun in the form of radiation.

(*ii*) The Sun is by far the most massive body of the solar system and hence controls the motions of the other bodies of the system. 99 percent of the mass of the solar system resides in the sun itself.

(*iii*) The sun is the nearest of the stars, and by studying the sun with telescopes and other instruments we can indirectly learn a great deal about stars in general even though they may be millions of light years away. All other stars appear as points of light in even the largest telescopes and hence we cannot observe details about their surface.

For measuring the distance of the sun and its mass, refer to Distances with in the solar system, size and Mass explained earlier in *Article 8·3 in this unit*. The distance of the sun from the earth comes out to be equal to $1·496 \times 10^{11}$ m which is called astronomical unit (A U.). The mass of the sun is found to be $1·989 \times 10^{30}$ kg. Combining the distance to the sun with its apparent diameter permits the computation of its actual diameter, $1·4 \times 10^6$ km. The sun's diameter is 109 earth diameters. Its volume is therefore approximately 1 million times the volume of the earth. From its volume and mass one can compute the density, which is 1400 kgm^{-3}, about ¼th the density of the earth.


Sunspots (dark area on surface)
Chromosphere
Spicules (tongues of hot gases at top of chromosphere)
Fraculae (large bright areas, usually in vicinity of sunspots)
Photosphere (apparent surface of sun)
Corona (pearly glow outside chromosphere)
1.4×10⁶ km
Flare (short-lived small, very bright area near sunspots)
Prominences (scarlet clouds of material near limb)
Granules (continually changing small, bright areas all over photosphere, giving it a mottled appearance)

Layers and features of the sun
Fig. 8·13

The sun does not have a solid surface. The gases of the sun extend thousands or millions of kilometres into space around the apparent surface. The upper atmosphere of the sun is almost transparent to visible light. There is a narrow-zone, (approx. 200 km) in which the gases of tne sun become opaque. That zone called the *photosphere*, is the surface which we see. Light leaving the upper part of the photosphere travels to us with little impediment ; light from the bottom of the photosphere is likely to be absorbed and remitted before it escapes and light from below the bottom of the photosphere is certain to be stopped before it leaves the sun. The layer of atmosphere directly above the photosphere is the *chromosphere* which extends for about 12000 km beyond the photosphere.

The other important properties of the sun are given below :

(*b*) Surface Temperature

The sun radiates heat in all directions and the earth receives only a small fraction (about 0·00005%) of this energy. On its way to the earth some energy is scattered by air, clouds, dust particles, etc. in the sun's atmosphere. *This amount of heat energy received per minute by 1 square centimetre of a perfectly black body placed at a mean distance of the earth from the sun, the surface being held perpendicular to the sun's rays is called*

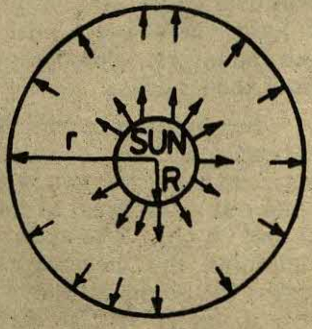


Fig. 8·14

SOLAR CONSTANT. Its value is about 2 cal per cm^2 per minute or $(2\times10^4)/60$ cal per m^2 per second or $(2\times10^4)\times4{\cdot}18/60$ Jm^{-2} s^{-1} which amounts to $1{\cdot}39\times10^3$ watts m^{-2}.

Let us imagine a sphere of radius r equal to 1 astronomical unit (A U) with sun at its centre and S the solar constant (Fig. 8.14). The total amount of heat energy received by the sphere of radius r is given by $4\pi r^2$. S watts. This is the total heat energy emitted by the sun per second. If R is the radius of the sun, the heat energy emitted by one square metre of the solar surface per second is

$$E=\frac{4\pi r^2.S}{4\pi R^2}=\frac{r^2}{R^2}.S$$

Now $$r=1{\cdot}496\times10^{11}\text{ m}$$

and $$R=6{\cdot}928\times10^8\text{ m and }S=1{\cdot}39\times10^3\text{ watts m}^{-2}$$

$$E=\frac{(1{\cdot}496\times10^{11})^2}{(6{\cdot}928\times10^8)^2}\times1{\cdot}39\times10^3$$

$$=6{\cdot}41\times10^7\text{ watts m}^{-2}$$

Now according to Stefan's—Boltzmann law,

$$E=\sigma T^4$$

where $\sigma=5{\cdot}735\times10^{-8}$ watts/m^2/(K)^4

$$T=\left(\frac{E}{\sigma}\right)^{\frac{1}{4}}=\left(\frac{6{\cdot}41\times10^7}{5{\cdot}735\times10^{-8}}\right)^{\frac{1}{4}}$$

$$=5800\text{ K}$$

$$=5527^\circ\text{C}$$

which gives the surface temperature of the sun, supposing it to be a *black body radiator.*

(*c*) Conditions at the centre of the sun

As we go deep below the surface layers of the sun, the density, pressure and temperature go on increasing due to the compression by the weight of the layers above. Calculations show that the temperature at the centre of the sun is of the order of 14×10^6K. It shows that the sun must be gaseous all the way to the centre because of this high temperature. About 70 percent of the sun's mass is hydrogen, about 28 percent helium and the remaining 2 percent is composed of other heavier elements from lithium to uranium. The density of the sun is about 10^{-4} kg/m^3 at the surface and it increases to 10^4 kg/m^3 at the centre. The pressure at the centre of the sun is of the order of 2×10^{16} Nm^{-2}.

(d) Source of Solar Energy

The cause of the solar energy from the sun and other stars has long been a mystery which is not clearly understood even today. The sun has been radiating energy at the rate of about 4×10^{16} Js^{-1} for several billion years without showing any sign of cooling off. Chemical reactions cannot possibly be the source of this energy, because even if the entire mass of the sun is assumed to consist of pure carbon, its complete combustion would maintain the radiation at the above rate for only a few thousand years. The process of fission cannot also be expected to be the source of the sun's energy, because the abundance of heavy elements in the sun is too small to account for its high rate of emission for so long a period. Hydrogen and helium constitute more than 95% of the sun's mass. It is thus probable that certain nuclear processes involving hydrogen and helium may be the actual source of solar energy.

Prof. Hans. A. Bethe proposed in 1939 a set of thermonuclear reactions to account for the energy produced in the sun and it is known as *carbon-nitrogen cycle.*

*The cycle consists of a chain of nuclear transformation in which hydrogen is converted into helium, with the aid of carbon and nitrogen as catalysts, the highly energetic thermal protons constituting the projectiles in the successive reactions. A proton collides with ordinary carbon (C^{12}) and forms the lighter isotope of nitrogen (N^{13}). The nucleum N^{13} being radioactive, emits a positron (counterpart of electron) and is transformed into stable heavier isotope of carbon (C^{13}) Being struck by another proton, this carbon is transmuted into ordinary nitrogen N^{14} again. The nucleus N^{14} collides with still another proton and produces the unstable oxygen isotope O^{15} which decays with positron emission to form the stable nitrogen isotope N^{15}. Finally, this N^{15} reacting with a fourth proton breaks up giving a carbon nucleus (C^{12}) and a helium nucleus (He^4) ;

$$C_6^{12}+H_1^1\rightarrow{}^*N_7^{13}+h\nu \text{ (Energy)}$$

$$^*N_7^{13}\rightarrow C_6^{13}+e^+ \text{ (Positron)}$$

$$C_6^{13}+H_1^1\rightarrow N_7^{14}+h\nu \text{ (Energy)}$$

$$N_7^{14}+H_1^1\rightarrow{}^*O_8^{15}+h\nu \text{ (Energy)}$$

$$^*O_8^{15}\rightarrow N_7^{15}+e^+$$

$$N_7^{15}+H_1^1\rightarrow C_6^{12}+He_2^4$$

It will be noted that these reactions form a *closed chain* or *cycle*, returning to the starting point after every six steps. Carbon and nitrogen, therefore, remain unaffected, enabling the cycle to continue. The period of a complete cycle is estimated to be about 5 million years.

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*For bright students only.

Summing up all the above equations,

$$\boxed{4H_1^1 \rightarrow He_2^4 + 2e^+ + \text{Energy}}$$

According to modern views of stellar evolution, the above mentioned carbon-nitrogen cycle comes at a rather late stage in the life of the stars. A star is formed by the condensation of a large amount of matter at a point in space. Under the influence of gravitational attraction the amassed matter gradually contracts, resulting in a rise of temperature until the central part is about 200 000°C. At this stage, there is sufficient thermal energy for a *proton-proton interaction* to form a deutron as given by

$$\underset{\text{(Proton)}}{H_1^1} + H_1^1 \rightarrow \underset{\text{(Deutron)}}{D^2_1} + e^+$$

which is followed by two additional proton reactions resulting in an α-particle (helium nucleus), as

$$D_1^2 + H_1^1 \rightarrow He_2^3$$

$$He_2^3 + H_1^1 \rightarrow \underset{\text{(α-particle)}}{He_2^4} + e^+$$

Summing up these equations ; we have

$$\boxed{4H_1^1 \rightarrow He_2^4 + 2e^+ + \text{Energy}}$$

In both these cycles, four hydrogen atoms fuse together to form a helium atom (He_2^4). Now the atomic mass of four hydrogen atoms (H_1^1) is 4·032 while that of helium atom is 4·003. Thus there is a loss of 0·7 percent of mass. This mass is converted into energy according to Einstein's mass energy relationship.

$E = mc^2$ where E represents energy, m is the *mass* and c is the velocity of light (3×10^8 ms^{-1}). The loss of mass gives rise to 5×10^{14} Jkg^{-1} of energy which is enough to maintain the radiation of energy from the sun for a long time.

(*e*) Other Surface Phenomena (Sun spots and Solar activity) :

Photographs of the sun taken in white light show many dark spots which are called *sun spots.* Sunspots appear dark by contrast with the solar surface, because they have a somewhat lower temperatures of about 4500 K. Some of the sunspots are so big that they can easily swallow several earth like bodies. Sunspots are characterised by intense magnetic fields of 2 to 3 thousand gauss. The number of sunspots are found to vary from year to year with a period of about 11·1 years. This periodicity is known as sunspot cycle. The sunspots are found to move across the disc from day to day ; from this it is inferred that the sun rotates with a period of about 25 days at its equator. Some regions in the vicinity of sunspots sometimes become suddenly very much brighter than usual ; the regions are called *flares*. During periods of high sunspot activity, as many as 5 to 10 flares may be seen in one day. When a flare is seen at the edge of the disc of the sun it is a jet of gas extending upward several thousand kilometres and it may rise hundreds of kilometres per second. A flare is of short duration—perhaps an hour. During flares X-rays ; ultraviolet radiation, and radio waves are emitted in unusual quantities, and particles are ejected into space at high speed. Both the radiation and the particles affect the earth changing radio communication over great distances and disturbing earth's magnetic field ; *aurorae* are visible effects of the particles.

In contrast to the flares, which are of short duration, *prominences* may persist for weeks or months. Photographs taken with only one colour of light often show dark lines near sunspots. The dark lines are called *filaments*. When they are seen at the outer edge of the sun's disc, they often appear as great arches, and then they are called *prominences*. Prominences can be seen with coronagraphs and many have been photographed. Solar astronomers study the face of the sun constantly and astrophysicists try to understand the origin and convolutions of the magnetic fields within the sun which can produce the effects seen on the surface. Meteorologists are also interested in some aspects of the sun's activity, for the earth's magnetic field and possibly to some extent the earth's weather, are affected by the outbursts on the sun. During the time astronauts have been in space, special watch has been kept for flares, for the X-ray and particle emission from violent flares could be hazardous for men exposed in thin space suits.

Example 8·1. The lens of our eye has a diameter of 8 mm. How much fainter objects can be seen through a telescope of 120 cm aperture as compared to the faintest naked eye stars ?

Solution.

Diameter of the eye lens = 0·8 cm

$$\text{Area of the eye lens} = a_1 = \frac{\pi(0{\cdot}8)^2}{4}\ \text{cm}^2$$

Diameter of the telescope objective = 120 cm

$$\text{Area of the objective} = a_2 = \frac{\pi(120)^2}{4}$$

$$\therefore \qquad \frac{a_2}{a_1} = 22500$$

The telescope of 120 cm aperture, therefore, can see stars 22500 times fainter than those seen by naked eye.

Example 8·2. Calculate the aperture of the objective of a telescope which may be used to resolve stars separated by 1 second arc ($4{\cdot}84\times10^{-6}$ radian) for light of wavelength 5×10^{-7} m.

Solution.

Here $\lambda=5\times10^{-7}$ m

$0=4{\cdot}84\times10^{-6}$ radian

Aperture of the objective$=$D$=$?

Now the angular limit of resolution of telescope

$$\theta=\frac{1{\cdot}22\lambda}{D}$$

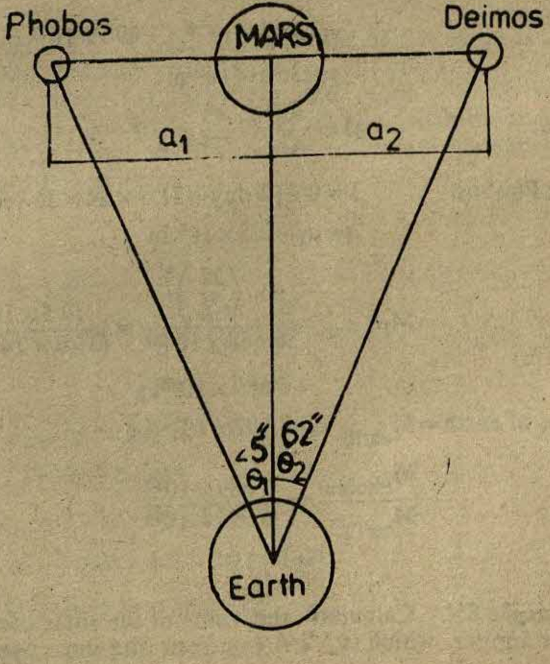
$$D=\frac{1{\cdot}22\lambda}{\theta}=\frac{1{\cdot}22\times5\times10^{-7}}{4{\cdot}84\times10^{-6}}$$

$$=0{\cdot}126 \text{ m}$$

Example 8·3. The distance of the satellites of Mars are 25″ for Phobos and 62″ for Deimos at mean opposition when Mars is 0·524 A U from earth. Calculate distance of the two satellites for Mars in A U and in meters.

If the period of revolution is 0·319 day for Phobos, find the mass of Mars in units of the earth's mass which is $5{\cdot}977\times10^{24}$ kg ($G=6{\cdot}668\times10^{-11}$ Nm^2 kg^{-2}).

Solution



For Phobos

Angular separation $\theta_1 = 25''$

$$= \frac{25}{3600} \times \frac{\pi}{180} \text{ rad}$$

Distance of Mars from Earth $= 0{\cdot}524$ A U

Distance of Phobos from Mars $(a_1) = 0{\cdot}524 \times \theta_1$ A U

$$a_1 = \frac{524}{1000} \times \frac{25}{3600} \times \frac{\pi}{180} = 6{\cdot}35 \times 10^{-5} \text{ A U}$$

$$= 6{\cdot}35 \times 10^{-5} \times 1{\cdot}496 \times 10^{11}$$

$$= 9{\cdot}5 \times 10^{6} \text{ m}$$

For Deimos

Angular separation $\theta_2 = 62''$

$$= \frac{62}{3600} \times \frac{\pi}{180} \text{ rad.}$$

Distance of Deimos from Mars $(a_2) = 0{\cdot}524 \times \theta_2$ A U

$$= 0{\cdot}524 \times \frac{62}{3600} \times \frac{\pi}{180}$$

$$= 157{\cdot}4 \times 10^{-6} \text{ A U}$$

$$= 157{\cdot}4 \times 10^{-6} \times 1{\cdot}496 \times 10^{11} \text{ m}$$

$$= 236{\cdot}1 \times 10^{5} \text{ m}$$

Since $\quad M = \dfrac{4\pi^2}{G} \dfrac{a^3}{T^2}$

For Phobos, $\quad T = 0{\cdot}319 \text{ day} = 31{\cdot}9 \times 24 \times 36 \text{ sec}$

$$a = a_1 = 9{\cdot}5 \times 10^{6} \text{ m}$$

$$M_{\text{Phobos}} = \frac{4 \times \left(\frac{22}{7}\right)^2}{6{\cdot}668 \times 10^{-11}} \times \frac{(9{\cdot}5 \times 10^{6})^3}{(31{\cdot}9 \times 24 \times 36)^2}$$

$$= 6{\cdot}689 \times 10^{23} \text{kg}$$

Mass of earth $= M_{\text{earth}} = 5{\cdot}997 \times 10^{24}$ kg

$$\frac{M_{\text{Phobos}}}{M_{\text{earth}}} = \frac{6{\cdot}689 \times 10^{23}}{5{\cdot}997 \times 10^{24}}$$

$$= 0{\cdot}111$$

Example 8·4. Calculate the value of the solar constant at a distance or Jupiter which is 5·2 A U from the sun. Hence calculate the temperature of a black body on Jupiter by Stefan's Law (solar constant at the earth's surface $=1{\cdot}388\times10^3$ watts m^{-2}; Stefan's constant $\sigma=5{\cdot}669\times10^{-8}$ Wm^{-2} deg^{-4}).

Solution.

Solar constant at the Earth's surface

$$=1{\cdot}388\times10^3 \text{ watts } m^{-2}$$

Distance between sun and earth $=1$ A U

Dista[illegible]een sun and Jupiter $=5{\cdot}2$ A U

Sinc[illegible] radiation obeys inverse square law, the radiation received by 1 m^2 surface of the Jupiter or solar constant at Jupiter

$$\frac{1{\cdot}388\times10^3}{(5{\cdot}2)^2}=51{\cdot}33 \text{ watts } m^{-2}$$

According to Stefan's Law

$$E=\sigma T^4$$

$$T^4=\frac{E}{\sigma}$$

$$T=\left(\frac{E}{\sigma}\right)^{\frac{1}{4}}=\left(\frac{51{\cdot}33}{5{\cdot}669\times10^{-8}}\right)^{\frac{1}{4}}$$

$$=173{\cdot}5 \text{ K}$$

Example 8·5. Period of revolution of the moon is 27·32 days and its mean distance from earth is 384 400 km. Calculate the sum of the masses of the earth and moon. Further, using the known fact that the centre of mass of the earth-moon system lies at $4{\cdot}75\times10^6$ m from earth's centre, calculate the mass of each. $(G=6{\cdot}668\times10^{-11}$ SI units)

Solution.

We know that

$$M_1+M_2=\frac{4\pi^2}{G}\times\frac{a^3}{T^2}$$

Here,

$$a=384400 \text{ km}=3844\times10^5 \text{ m}$$

$$T=27{\cdot}32 \text{ days}$$

$$=27{\cdot}32\times24\times60\times60 \text{ sec}$$

$$G=6{\cdot}668\times10^{-11} \text{ Nm}^2\text{ kg}^{-2}$$

Let M_1 = mass of the earth

and M_2 = mass of the moon

$$M_1+M_2=\frac{4\times\left(\frac{22}{7}\right)^2}{6{\cdot}668\times10^{-11}}\times\frac{(3844\times10^5)^3}{(27{\cdot}32\times24\times60\times60)^2}$$

$$=6{\cdot}046\times10^{24}\text{ kg}$$

Let a_1 and a_2 be the distances of the earth and the moon from the centre of mass of the earth-moon system and let $a_1+a_2=a$

Now $a_1=\frac{M_2a}{M_1+M_2}$ and $a_2=\frac{M_1a}{M_1+M_2}$

$$M_2=\frac{(M_1+M_2)\,a_1}{a}$$

Now $a_1=4{\cdot}75\times10^6$ m

and $a=384{\cdot}4\times10^6$ m

$$M_2=\frac{(6{\cdot}046\times10^{24})\times4{\cdot}75\times10^6}{384{\cdot}4\times10^6}$$

$$=7{\cdot}45\times10^{22}\text{ kg}$$

Again, $M_1+M_2=6{\cdot}046\times10^{24}$

$$M_1=6{\cdot}046\times10^{24}-7{\cdot}45\times10^{22}$$

$$=5{\cdot}97\times10^{24}\text{ kg}$$

8·4 GENERAL IDEAS ABOUT STARS

On a clear night one can count as many as 2500 stars in one hemisphere, the total being 5000 in the whole sky. With the aid of telescopes millions of stars can be seen. It will be useful to try to gain some perspective on stars and on our relation to them. How bright are they ? How far away are they ? Are they uniformly distributed or are they organized into groups ? We shall describe now some of the methods available to the astronomers for making such studies.

(*a*) Stellar distances

So far distances within the solar system were discussed in astronomical units and in terms of light travel time. As we shift our attention to the stars, the distances we must work with become so much greater that only light travel time is useful. *The light*

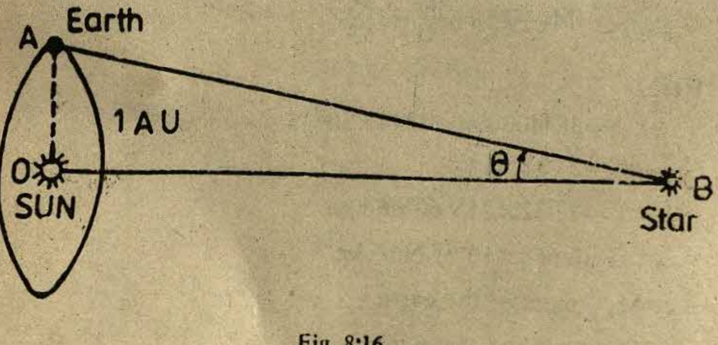


Fig. 8·16

year, which is the distance light travels in one year, is a common unit used in discussing distances to the stars. One light year is $9{\cdot}48 \times 10^{15}$ m. Note that light year is a distance—not a time.

The nearest star is Alpha centauri. It is 4·3 light years away. This means that when we look at this particular star, we see light which left there 4·3 years ago.

There are two methods of determining stellar distances :

(*i*) Annual Parallax Method

(*ii*) Spectroscopic Method

(*i*) Annual Parallax Method

A necessary consequence of the facts that the earth moves around the sun and that not all stars are the same distance from the earth is the possibility of seeing annual parallax, an apparent motion of nearer stars (closer than 100 light years) against the background of more distant stars which are motionless. Then as the earth moves around the sun, a nearby star which lies in the plane of earth's motion should move back and forth along a line ; a star which lies in a direction perpendicular to the plane of the orbit should appear to move in an elliptical path which is a duplicate of the path of the earth and a star lying somewhere between the perpendicular and the plane of the orbit should appear to move in a path which is a flattened ellipse (Fig. 18·17). If the stars are motionless, the parallax of a nearby star should be observable in *one* year, if the stars are in motion, more than one year's observations will be required to separate parallactic motion from true motion. The latter is the actual situation.

Annual parallax was not actually observed until 1838 although William Herschel had made a careful attempt to detect it somewhat earlier. In that yeat F. Bessel detected the parallax of the star 61 Cygni, and F. Struve detected the motion of Vega. Both men used specially equipped telescopes to measure the separation between the star being observed and someback ground object, the work was tedious and difficult. Now photographs are taken of the same area of the sky throughout the year, and from measurements of the movements of the images of near by stars their parallaxes can be determined.

If the parallax of a star can be measured, the distance to the star can be computed by the method of annual parallax or trigonometric parallax. The trigonometric parallax of a star is defined as the angle sub-

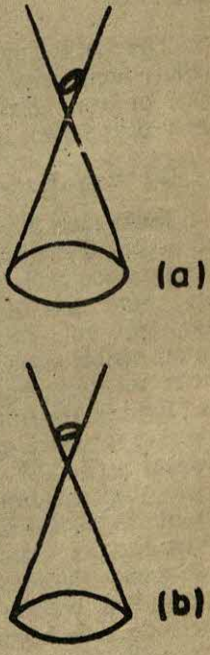


Fig. 8·17

tended at the star by one astronomical unit. That is, we use the displacement of the star during *half a year*. Fig 8·16 shows the triangle in which the base is the astronomical unit (A.U.) and the angle (θ) at the star is the trigonometric parallax, if that angle is known, the distance to the star can be computed easily.

$$\theta=\frac{AO}{AB}$$

where $AO=1$ A U

and $AB=$distance of the star.

This method has its limitations. Parallaxes for almost 7000 stars have been measured ; for only the 700 are the distances known to within a 10 percent accuracy.

(*ii*) Spectroscopic Parallax Method :

The method of spectroscopic parallax is used to determine the distance to stars too far away (more than 100 light years) for annual parallax to be observed and it is used to help calibrate other methods which measure even greater distances. It is most reliable when applied to a cluster of stars so that many points can be fitted to the main sequence line in the Hertzsprung-Russell diagram (Fig. 8.18), the best common distance for all the stars determined.

The method involves determination of the apparent and the absolute luminosity of the star and then figuring out how far an object of given absolute luminosity would have to be to give us the observed apparant luminosity.

(*b*) Magnitudes

More than 2000 years ago Hipparchus a Greek astronomer who prepared the first star map, divided the naked eye stars into six magnitude classes according to their brightness. The brightest stars were put in the first magnitude class. Successively fainter stars were put in 2nd, 3rd, 4th, 5th magnitude classes, the faintest visible stars being put in the 6th magnitude class. On precise measurement with photometers attached to telescopes it was found in the 19th century that the ratio of the brightness between successive magnitude classes is 2·512, so that a 1st magnitude star is $(2{\cdot}512)^5=100$ times brighter than a 6th magnitude star. On using this relation it was found that some of the stars originally put in the 1st magnitude class are actually brighter than the average for that class. Hence zero and negative magnitudes were introduced. On the other hand more powerful telescopes made fainter stars visible. Therefore it was necessary to extent the magnitude scale beyond the 6th. Thus the stars 2·512 times fainter than the 6th magnitude were assigned 7th magnitude and so on. The 200 inch telescope can show us stars of 22nd or 23rd magnitude. It is expected that the 90 inch space telescope to be put in orbit around the earth by the space shuttle in 1980's would be able to show us stars of 28th magnitude, which would be 10^{22} times fainter than our sun.

(*c*) Absolute brightness of stars

So far we have been talking of the brightness of a star as it appears to us or in other words its *apparent magnitude*. A star looks bright either because it is intrinsically bright or because it is nearer to us. The sun looks brightest of all the stars because it is closest to us, although it is really fainter than many other stars like Betelgeuse which look fainter because of their large distance. *The true or absolute magnitude of a star corresponds to the brightness of the star if it were* taken to some standard distance (32·6 light years from the sun **from its actual position)**

The *apparent brightness* of an object varies inversely as the square of the distance. So we can compute the true brightness of a star from its observed apparent brightness and distance by bringing it to a standard distance.

$$I = \frac{L}{4\pi R^2}$$

where I = Intensity of radiation reaching the earth measured in joules/second/metre^2 in the same wavelength band

L = Luminosity of the star measured in joules / sec of energy emitted in a particular wavelength region.

R = distance to the star in metres.

On this basis we find that Sirius and Betelgeuse are respectively 22 and 19000 times brighter than our sun intrinsically. They look faint only on account of their large distances. The absolute brightness of stars ranges between 60000 times to $\frac{1}{400000}$th of the brightness of the sun.

(*d*) Stellar Spectra

Just like the sun, the stars also show a continuous spectrum on which dark absorption lines are superimposed. The continuous spectrum is produced by the surface layers of the star which form what is called the *photosphere* of the star. The photosphere emits continuous radiation in all wavelengths like a black body. The dark absorption lines are produced when the continuous radiation of the photosphere passes through the top most cooler layer which is known as the reversing layer. The wavelengths of the dark absorption lines depend upon the composition of the material of the reversing layer. So we can identify the atoms and molecules in the stars by noting the positions of Fraunhofer lines in their

TABLE 8·2
IMPORTANT CHARACTERISTICS OF SPECTRAL CLASSES OF STARS

Spectral Class	*Colour*	*Temperature (approximate) K*	*Salient spectral features*	*Example*	*Mass in terms of solar mass*	*Radius in terms of solar Radius*	*Absolute Magnitude MV*	*Luminosity in terms of sun's luminosity*
O	**Blue**	**Above 25 000**	Strong lines of ionised helium; Lines of doubly ionised oxygen, nitrogen & carbon	10 Lacertea	40	20	−6·0	10^5
B	**Blue**	25 000-11,000	Lines of neutral helium are most prominent. Hydrogen lines stronger than in O class	Rigel, Spica	15	7	−3·1	10^4
A	White	11 000-7,500	Hydrogen lines are more prominent. Singly ionised magnesium, Silicon, iron. calcium and titanium appear	Sirius; Vega	2·3	1·8	+1·4	25
F	Green	7500-6000	Hydrogen lines are weaker than in A stars and neutral metals are stronger.	Procyon	1·4	1·2	+2·7	3
G	Yellow	6000-5000	Lines of ionised calcium are strongest feature. Hydrogen lines are weak. Lines of many neutral and singly ionised metals are present.	Sun, Capella	1·0	1·0	+4·8	1
K	Orange	5000-3500	Neutral metal lines are more prominent. Hydrogen lines are almost non-existent. Molecular bands are becoming important.	Arcturus	0·7	0·8	+6·1	0·4
M	Red	Below 3500	Molecular bands are most prominent feature. Titanium oxide bands are very prominent.	Antares, Betelgeuse	0·3	0·4	+11·8	1/40

spectra. It is found that stars contain the same 92 natural and about a dozen artificial elements that are found on earth.

The spectra of all stars are not alike ; the spectrum is related to the colour of the star. There are seven principle classes of stellar spectra denoted by letters, O, B, A, F, G, K and M. Their properties are given in *Table 8·2*.

We see that O and B type stars are blue and their spectra show helium lines. The A type stars are white and their spectra are dominated by hydrogen lines. F and G type stars of green and yellow colours show lines of ionized and neutral metals. Finally the spectra of the orange K and the red M stars contain molecular bands of titanium-oxide in their spectra.

The colour of a star is actually related to the temperature of the photosphere. This can be understood from Wien's displacement law.

$$\lambda_{max}\ T = \text{constant}$$

As the temperature T is increased, λ_{max}, the wavelength at which the intensity is maximum becomes shorter and the colour changes ,accordingly. For example, if we heat an iron ball, which is black when cold, it first becomes red hot and on further heating changes its colour to white and then to blue. Similarly the coolest stars look red or orange, the hotter ones appear yellow or green while the hottest stars are white or blue. Thus we see that the spectral sequence O, B, A, F, G, K, M arranges the stars according to decreasing surface temperature.

In earlier days it was thought that the differences in the spectra of stars were caused by the differences in their chemical composition. For example O and B stars were called heluim stars, A stars were thought to be made of hydrogen and F and G stars of metals. The Indian Astro-physicist M.N. SAHA showed in 1922 that all stars have the same chemical composition and the differences in their spectra are principally due to different surface temperatures of stars. At low temperatures of a few thousand degrees, the gases occur in the form of molecules as is found to be the case for the M type stars. As we increase the temperature the molecules first break into neutral atoms and on further increasing the temperature the atoms becomes ionised. Metals have low ionization potentials, hence they are ionised at low temperature. This is what happens as we go from M and K to G and F type stars. By the time the temperature is raised to about 10000 degrees in A type stars all metals lose several valence electrons and hydrogen is excited giving rise to strong Balmer lines. By further increase of temperature to 20 000 degrees, hydrogen is completely ionised and helium is excited. Thus in B stars neutral helium lines dominate the spectrum. Finally in the hottest O type stars with temperatures of 35 000 degrees even helium is ionised. Thus

Saha's Theory of ionisation has helped us to determine the surface temperatures of stars from their spectra as given in Table 8.2.

After knowing the temperature it is easy to find the true chemical composition of the stellar atmospheres. *H.N. RUSSEL* was the pioneer in this field. He and later workers found that all stars have essentially the same composition as that of the sun. Not only stars but the gas and dust that exist in the interstellar space are also made of the material of the same composition. This cosmic mixture has hydrogen, the lightest of all elements, as the most abundant constituent. 88 percent of all the atoms in the universe are hydrogen atoms; 11·8 percent are helium atoms and the remaining heavier elements make up only 0·2 per cent population of atoms.

(*e*) The Hertzsprung – Russell Diagram

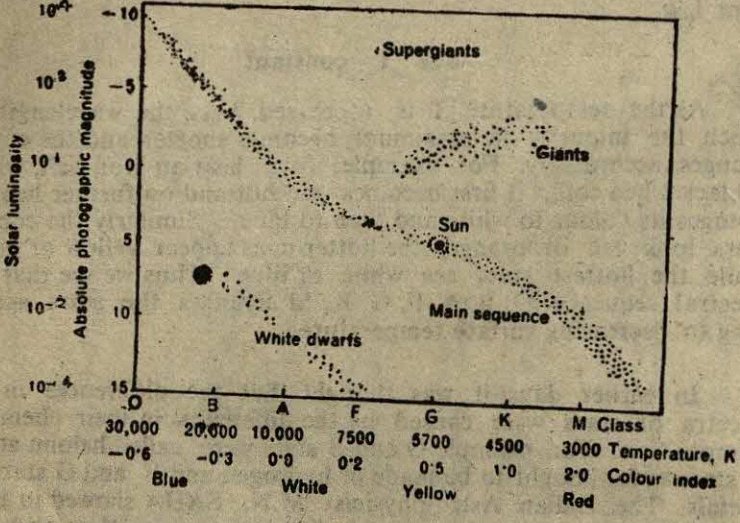


Fig. 8·18

In the early years of the 20th century the Danish astronomer Ejnar Hertzsprung and American Henry Norris Russell, independently found a way to correlate two kinds of information about stars. The result, known as the Hertzsprung-Russell diagram (more commonly, the H-R diagram) is one of the most useful devices yet developed for organizing information about the stars.

In the H-R diagram, each star is represented by a point. The horizontal axis of the diagram is labelled with the spectral classes (O, B, A, F, G, K, M), and the vertical axis is labelled with absolute magnitudes. Thus a star which is spectral class O and is very luminuous is represented by a point at the upper left corner of the diagram. The sun which is spectral class G and of moderate luminosity, is represented by a point near the centre of the diagram. Colour index or any other function of surface temperature can be used on the horizontal scale instead of stellar class to generate essentially the same graph. In general terms, then, *the H-R diagram is a plot of intrinsic luminosity* (or *absolute magnitude*) *against surface temperature.*

Fig 8·18 shows such a diagram. Most of the stars fall on a broad line running roughly from upper left to lower right, that band is called the *main sequence*. In the lower left are *white dwarfs.* They are intensively hot but not bright. This implies that their surface areas and hence their volumes must be exceedingly small. Most of these stars are slightly larger than the earth but their masses are comparable with the mass of the sun. At present a few hundred white dwarfs are known. To the right of the main sequence and above it are *giants* and *supergiants*. Stars at the left side of the diagram are hot and blue or white ; those in the centre are yellow and those at the right are red. Stars near the bottom of the diagram are intrinsically faint and those near the top are very luminous.

A star cannot be plotted on the H-R diagram unless its absolute magnitude is known, which means that its distance from the sun must be known.

(*f*) Sizes or Radii of Stars :

Since the stars are seen as point sources even with the largest telescope their angular diameters cannot be measured. So the method of determining size described in Art. 8.3 above cannot be applied to them and we have to use some other indirect method.

If we assume that stars emit radiation like *black bodies* then each unit area of its surface would emit energy per second $E=\sigma T^4$ where T is the surface temperature. The surface area of the star is $4\pi R^2$ where R is the radius of the star. Hence the total luminosity of the star will be $L=(4\pi R^2)\times(\sigma T^4)$.

This equation will give the radius R when temperature T is known from the spectrum and luminosity L from the absolute magnitude.

The radii of various spectral types of stars are given in Table 8·2. For most stars they range from about 20 solar radii to about 1/10th of the radius of the sun. There are a few exceptions : for example Capella and Betelgeuse have radii equal to 50 and 220 times the radius of the sun. Such stars are called *giants* and *supergiants* on account of their large size. By contrast the other most common stars like the sun are called *dwarfs*. At the other extreme we have stars like the companion of Sirius which is only twice as big as the earth in diameter. These very small stars are called *white dwarfs*.

(g) Masses of Stars –

About half of the stars in the Milky way are binary systems. A binary system contains a pair of stars which revolve around their common centre of mass. Alpha centauri, Sirius, Spica are all binaries. Masses of stars are obtained by the study of the bianaries and through the application of Kepler's third law incorporated in equation,

$$M_1+M_2=\frac{4\pi^2}{G}\cdot\frac{a^3}{T^2}\text{(derived in Art 8.3 above)}$$

M_1 and M_2 are the masses of stars and a_1 and a_2 are the radii of the orbits and T is the period of revolution where $a=a_1+a_2$. For convenience let us take sun's mass as the unit of mass, the astronomical unit as the unit of distance and the year as the unit of time. In these units $G=4\pi^2$ and hence

$$M_1+M_2=\frac{a^3}{T^2}=\frac{(a_1+a_2)^3}{T^2}$$

Thus from the knowledge of the motion of stars about their common centre of mass, it is possible to find their masses. Masses of about 100 stars have been measured by the above method. The masses of stars range from about 40 times the mass of the sun for the hottest 0 stars to a tenth of the solar mass for the coolest M stars.

For all stars except the white dwarfs it is found that the luminosity increases with mass roughly according to the 3½th power of mass. For example Sirius, which is 2·3 times as massive as the sun, is 25 times brighter than the sun. (See table 8·2).

Example 8·6. The two components of Procyon have a separation of 4·55" and period of 40·6 years. If the distance of the binary is 11·3 light years, calculate the sum of masses of the two stars in solar units. If the ratio of their masses is 3 : 1, find the mass of each star.

If both components of Procyon have the same spectra while one component is 15 000 brighter than the other, calculate the ratio of their radii.

Solution.

Angular separation between the components $=\theta=4{\cdot}55''$

$$=\frac{4{\cdot}55}{60\times60}\times\frac{\pi}{180}$$

$$=2{\cdot}206\times10^{-5}\text{ rad}$$

Now the distance of the binary from Procyon (r)

$$=11{\cdot}3\text{ light years}$$

$$=\frac{11{\cdot}3\times9{\cdot}46\times10^{12}}{1{\cdot}496\times10^{8}}$$

$$=7{\cdot}15\times10^{5}\text{ A U}$$

because 1 light year $=9{\cdot}64\times10^{12}$ km

and $\quad 1\text{ A U}=1{\cdot}496\times10^{8}$ km

Hence distance between the components

$$=a=\theta\text{ (in rad)}\times r\text{ (in A U)}$$

$$a=2{\cdot}206\times10^{-5}\times7{\cdot}15\times10^{5}$$

$$=15{\cdot}77\text{ A U}$$

The two components revolve round the common centre in 40·6 years. Taking sun's mass as the unit of mass, astronomical unit as the unit of distance. and the vear as the unit of time, **therefore, the sum of the masses of the two heavenly objects revolving round a common centre is**

$$M_1+M_2=\frac{a^3}{T^2}=\frac{(15{\cdot}77)^3}{(40{\cdot}6)^2}$$

$$=2{\cdot}38\text{ M (Sun)}$$

Since $$\frac{M_1}{M_2}=\frac{3}{1}$$

$$M_1=2{\cdot}38\times\tfrac{3}{4}=1{\cdot}785\text{ M (Sun)}$$

and $$M_2=2{\cdot}38\times\tfrac{1}{4}=0{\cdot}595\text{ M (Sun)}$$

For all stars, the luminosity increases with mass roughly according to 3½th power of mass. If L_1 and L_2 stand for the spectral brightness of the components of mass M_1 and M_2 respectively, we have

$$\frac{L_1}{L_2}=\left(\frac{M_1}{M_2}\right)^{7/2}$$

$$\frac{M_1}{M_2}=\left(\frac{L_1}{L_2}\right)^{2/7}=(15000)^{2/7}=15{\cdot}58$$

If R_1 and R_2 are the radii of the components and the density ρ is supposed to be the same for both, we have

$$\frac{M_1}{M^2}=\frac{\frac{4}{3}\pi R_1^3\cdot\rho}{\frac{4}{3}\pi R_2^3\,\rho}=\frac{R_1^3}{R_2^3}$$

$$\frac{R_1}{R_2}=\left(\frac{M_1}{M_2}\right)^{\frac{1}{3}}=(15{\cdot}58)^{\frac{1}{3}}=2{\cdot}49$$

(*h*) Interiors of Stars

On dividing the mass of a star by its volume we get its mean density. The mean densities of dwarf stars range from 10 kg m^{-3} for the hottest O stars to 50000 kg m^{-3} for the coolest M stars.

The lowest densities are found in supergiants *e.g.*, Antares (α Scorpii) is only three thousandth as dense as air The densest stars are white dwarfs like the companion of Sirius which is 5000 times denser than the densest element platinum.

As in case of the sun, temperature and density increase towards the centres of stars. The *Central* densities of dwarf stars range from a few thousand kg m^{-3} for 0 type stars to 10^4 kg m^{-3} for F, G, K and M type stars. The central temperature is maximum (about 30 million degrees) for early type O and B stars and minimum (about 10 million degrees) for M type stars.

(*i*) The Motion of Stars

The stars appear motionless, but they do so only because they are far away.

One direct method of detection of stellar motions is to take a series of photographs over a period of years and then compare them carefully. After applying due correction for the parallax the remaining change in position represents the proper motion.

A better method is to study Doppler shifts in their spectral lines. It can be proved that the light waves from those objects which are *speeding away* from the earth are stretched just as the sound waves from a railway engine that is speeding away from a listener are stretched out to create the familiar change in the pitch. The change in wavelength of light $\triangle\lambda$ produced by the motion of a star moving with a velocity V is approximately given by

$$\frac{\triangle\lambda}{\lambda}=\frac{V}{c}$$

where λ is the unstretched wavelength, c is the velocity of light and $V < c$.

If the star is moving towards the earth, the wavelengths of light reaching us are *shortened*. But if a star is moving away from the earth, the wavelength of light are stretched. Such shifts in the wavelengths can be detected and measured by a spectrograph. From such studies it has been found that *most of the stars seem to move away from the earth* and their black absorption lines which cross their continuous spectra are thus shifted towards the red end of the spectrum. The Doppler shift is, therefore, known as *red shift*.

Stellar Evolution

Like other things in the universe, a star is born, lives for a certain length of time and eventually dies.

Birth of a Star. According to one theory the large space between the stars is filled with particles of thin dust and gas. The study of the spectra of the star light after it passes through these gases shows that these gases contain as much as 75 percent hydrogen. The remaining is mostly helium with minor amounts of some heavier elements. The weak gravitational attraction between these particles draws them closer together to form a cloud. As the particles get closer, the gravitational attraction increases so that the original cloud becomes a rapidly shrinking and thicker mass. For this to occur, the original cloud should have a mass of at least one thousand solar masses. Increasing pressure in the centre of this shrinking mass causes the temperature to rise. At a certain stage during contraction, the cloud breaks up into a large number of fragments of stellar size and each fragment continues to shrink. When the temperature of a fragment reaches millions of degrees, nuclear fusion begins to take place so that it begins to radiate light from its surface. Thus each fragment becomes a source of energy and cluster of stars in born.

The amount of matter which accumulated to form the star decides its mass. The size is determined by the *equilibrium* between gravitational forces trying to pull the material inside and the forces due to electro-magnetic radiation pushing it out.

Death of a Star. The source of energy of each star is the fusion process involving carbon-nitrogen cycle or proton-proton reaction already discussed in Art. 8·3. During these reactions, hydrogen is converted into heluim and energy liberated during these reactions keeps the star shining for millions of years. The life span of a star is determined by the amount of hydrogen it had at its birth and the rate at which it is consuming it. When the stars have exhausted a certain proportion of their available hydrogen, the core begins to contract while the outer regions expand. Their surface temperatures drop and star becomes *giant* or *supergiant*. The life span of a star during its giant and supergiant phase is small as compared to that of dwarf phase. The end of this giant and supergiant phase is small as compared to that of dwarf phase. At the end of this giant or supergiant stage which lasts for millions of years, the star produces energy on such a large scale and a rapid rate that it explodes in the form of a *nova* or *supernova* throwing out a major portion of its enveloping mass into interstellar space. The core finally cools to a stage that no more energy is radiated. Thus the star finally becomes a *dead ball* of highly compressed matter, lost from sight for ever.

The average life of a star is about 10 billion years. The sun which has lived through about half of its dwarf phase is thus about 5 billion years old.

Within the past few years new radio and infrared telescopes have made it possible for many astronomers to probe the birth places of massive stars. Recent observations seem to indicate that the birth of such stars is sequentially triggered by *shock waves* travelling through gigantic clouds of intersteller gas and dust.

8·5 THE MILKY WAY : SIZE AND SHAPE

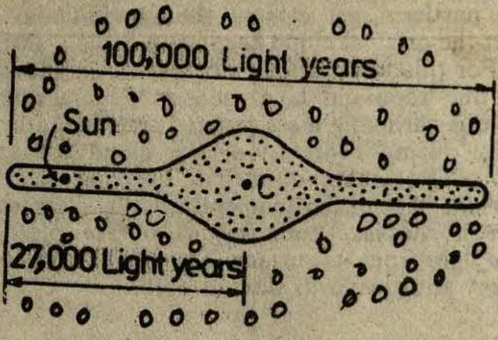


Fig. 8·19

Milky way is a very large band of stars, having a misty cloud like appearance, extending itself across the sky like part of a great circle. It is thicker in the middle and thins out at the edges (Fig. 8·19). The diameter of the milky way is about 100,000 light years. The sun which belongs to this galaxy is situated in its central plane at a distance of about 27 000 light years from the centre. The thickness of the disc is about 5000 light years at the centre and about 1000 light years near the sun and diminishes as we reach the edges. When seen through a telescope, it is found to consist of billions of stars that give it a cloudy appearances.

The sun is one of about 150 billion stars in the Milkway. All the stars revolve about the centre. The sun moving at a speed of 250 km/s revolves about the centre of the galaxy in about 220 million years. The Milkyway is not unformly bright everywhere. Most of the visible stars lie in the plane of the flat circular disc. When one looks in a direction at right angles to the plane of the Milkyway, the sky appears much darker because of thin distribution of stars as compared to that in the central plane of the Milkyway.

Interstellar matter. The space between the stars in the Milkyway is filled with dust clouds, spread more or less evenly. The dust obstructs the light of the stars behind them. Sometimes, the dust gets illuminated by a hot star and begins to shine as a result of the light it reflects, giving rise to a bright nebula like Orion nebula. Hydrogen is the most abundant of all the elements filling the intersteller space in our galaxy.

Star clusters. There are two types of star in our galaxy, *i.e. galactic clusters and globular clusters*. The galactic clusters are a group of 100 to 1000 stars bound by mutual gravitational forces. The cluster moves as a whole round the galaxy and each individual member star orbits around the common centre of the cluster. Globular clusters are a group of about 100 000 stars closely packed. About 100 globular clusters are known to exist in the milky way, all of them about 20 000 light years from the sun.

Structure. Recent studies have revealed that the disc of Milkyway has a *spiral structure*. Normally our view of the Milkyway is cut off by the interstellar dust clouds. But radio waves which can penetrate through dust clouds have been used to study the galactic structure. The central part of our galaxy is a huge spheroid surrouned by spiral arms. The total width of spiral arms is about 1200 light years and they are separated from each other by about 500 light years.

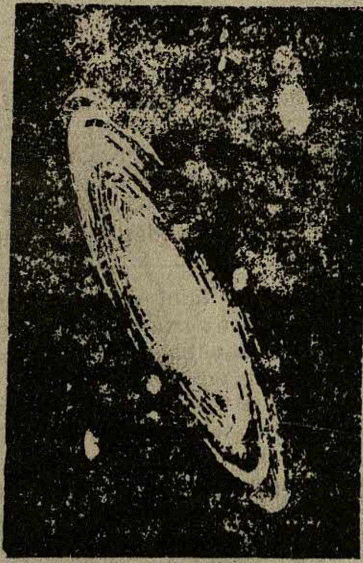


Fig. 8.20 (Stars and Galaxies)

8·6 CLASSIFICATION OF GALAXIES

(1) *Normal galaxies* :—A galaxy is a large band of stars, gas and dust held together by gravitational forces. The galaxies have been photographed and identified by powerful microscopes. The properties of normal galaxies are mentioned below :

(*i*) A galaxy is a band of billions of stars, travelling together through the vast expanse of the universe.

(*ii*) The size of each galaxy is tremendous. For example, the galaxy (Milkyway) to which our solar system belongs is about 100 000 light years in diameter and about 5 000 light years thick.

(*iii*) All the galaxies can be divided into three groups as regards their shape. Most common of them consist of central spheroid surrounded by spiral arms and they are called *spiral galaxies*. Other show elliptical discs on photographs and are called *elliptical galaxies*. A small per cent have no regular shape and are called *irregular galaxies*.

(*iv*) They are very bright near the centre but gradually get dim towards the edges.

(*v*) Normal galaxies emit a comparatively small amount of radio radiation as compared to the total radiation emitted.

(2) *Radio galaxies*— These are the galaxies which emit million times more radio radiation compared to normal galaxies. The radio radiation does not come from the galaxy itself. This radiation is found to originate from two large regions situated symmetrically on either side of the galaxy. According ~~to~~ one belief, a great explosion occurred in the central galaxy which ejected two clouds of charged particles on opposite sides.

8·7 QUASARS AND PULSARS

Quasars stands for *quasi-stellar radio source* and is another powerful source of radio radiation. The quasars are found to be receding with a velocity equal to 90 per cent of the velocity of light. The most distant quasars have been located by Doppler shift method at a distance of about five to ten billion light years from the solar system. They are the farthest objects discovered. The cause of the tremendous energy outout of quasars is still not known.

They appear optically as faint point images. About 150 quasars are known. Some quasars exhibit red shifts that are much larger than normal. Some scientists have interpreted such an unusually large red shift as being due to gravitational effects *i.e.* light will lose energy as it passes through the gravitational field which results in a change in wavelength towards the red or longer wavelength. Quasars may, in fact, be very massive bodies in a state of gravitational collapse. A very massive object emits radiation as the collapse begins, this continues until the body collapsing on itself becomes so dense, the resulting gravitational field is large enough to keep all radiation from escaping, from the massive body. When this occurs, the *object will emit no electromagnetic* radiation, can be observed only by its gravitational field. *Do not confuse a black body with a black hole.* A black body absorbs all the radiation striking it, but it also emits radiation. A black hole absorbs all the radiation striking it, but it radiates nothing. Neither matter nor radiation can escape from it. A black hole cannot be approximated by anything from familiar experience. Perhaps we can gain some idea of the position of black holes on the scale of size by considering the sun or stars of the same mass as the sun. At present the sun has a radius of about 7×10^5 km. But if the mass of the sun formed a black hole, the radius would be 3 km. If earth were compressed until it formed a black hole, it would have a radius of 9 mm ! Even the mass of Mount Everest can fit into a hole not wider than the head of a needle.

Our only hope of recognizing black holes, if they exist, seems to be to find binary systems in which one member has collapsed to a black hole or to find certain x-ray sources—particularly known as Cygnus X-1, which has closely black hole like character. It is theoretically suggested that black holes may serve something like entrances to tunnels connecting our space with adjacent spaces in the universe. In our space matter falls into black holes while in another space it is absorbed in *WHITE HOLES*".

Another rather recent groups of objects observed by their emitted radio waves is the so-called *PULSAR*. A pulsar is a source of radio waves that varies in intensity at regular intervals. At the present time the origion of pulsars is thought to be a *neutron star*. The neutron is the neutral particle of mass about the same as a proton. A neutron star is pictured as an immensely dense object made of neutrons. The pulsating radio emmission presumably is produced by the rotation of such a star. The neutron star is supposed to result from the burn out or last stage of an ordinary star.

Scientists continue to discover different objects in the universe— for example, the *Quasar* type object discovered in 1972. Five objects were observed which had rapid variation in output of radio, infrared, and visible radiation. These objects have been described as *compact extragalactic non-thermal sources*. These *Quasi-Quasars* differ from Quasars in several ways. First the radio singals come from the core rather than from throughout the core and surrounding gas core, as is the case with Quasars. The light and radio waves are polarized; that is, there is a particular orientation of the electromagnetic waves. These newly observed objects have no characteristic spectra, which makes them unique among objects observed in the universe. Because no characteristic line spectra are observed, *no red shift* can be determined and consequently the distance of these rather dim objects is unknown.

A deeper understanding of the nature of Quasar, Black Holes, Pulsars and Quasi-Quasars may be helpful in determining the size and shape of the universe.

8.8 COSMOLOGICAL THEORIES OF THE UNIVERSE

The most striking feature of the universe is its expansion. There are millions of galaxies in the universe. When the galaxies are observed through a spectrograph, the light from all of them is found to shift towards the longer wavelength or the red end of the spectrum which is also termed as *red shift*. It indicates that *all the galaxies are receding away from us*. Further, the speed of recession V increases proportionately with their distance R, *i.e.* $V \propto R$ or $V = HR$ where H is called *Hubble's constant* and the relation is known as the Velocity Distance Law or Hubble's Law.

If we divide the distance of any galaxy by its velocity of recession, we get the same number for all galaxies. This number is a time. According to the most recent work its value is about 10 000 million years and represents the characteristic time of the universe, called the age of the universe. The Hubble's Constant, H can be expressed as

$$H = \frac{1}{\text{age of the universe (10 000 million years)}}$$

This relation helps us to determine the distance of distant galaxies and quasars. The recession of galaxies shows as if the entire universe were expanding from some central point. Based on this idea of expanding universe, three main theories have been put forward to account for the origin and evolution of the universe.,

(a) THE BIG BANG THEORY :

Astronomers looking into space have noticed that on the large scale things look about the same in all directions that is, ignoring the presence of local effects such as our being in the plane of a particular galaxy and thus seeing a Milky Way across the Sky, the universe has no direction that is special. Further it seems that there is no change with distance either, except insofar as time and distance are linked. This notion has been codified as the *Cosmological principle*. The universe is homogeneous and isotropic throughout the space. The assumption of homogeneity says that the distribution of matter does not vary with distance from the Sun, and the assumption of isotropy says that it is about the same no matter in which direction one looks.

A set of theories of the universe known as *big bang theories* satisfies the cosmological principle. Basically, these cosmologies say that once upon a time there was a great bing bang that began the universe. From that moment on, the universe expanded, and as the galaxies formed they shared in the expansion.

Many ask whether the fact that there was a big bang means that there was a centre of the universe from which everything expanded. The answer is no; first of all, the big bang may have been the creation of space itself. Furthermore, the matter of this primordial cosmic egg was everywhere at once. There may be an infinite amount of matter in the universe, so it is possible that at the big bang an infinite amount of matter was compressed to an infinite density while taking up all space.

What was present before the big bang ? Versions of the big bang theory have been put forward to deal with this questions, but there is no real way to answer it. For one thing, we can say that time began at the big bang and that it is meaningless to talk about "before" the big bang because time did not exist. We do not now think that the universe can remain in a static condition, so it seems unlikely that the universe was always just sitting there in an infinitely compressed state, whatever "always" means. Of course, these possibilities do not answer the question of why the big bang happened.

If we consider Hubble's Law with Hubble's constant as the same for local and distant galaxies, we can extrapolate backward in time. We simply calculate when the big bang would have had to take place for the universe to have reached its current state at its current rate of expansion. This calculation indicates that the universe is some where between 13 billion and 20 billion years old. According to Hubble's Law, the Velocity of recession of a galaxy becomes equal to the Velocity of light at a distance equal tc 20 billion light years. No light from a galaxy at this distance would ever reach us so that this distance becomes the boundary of observable universe.

(*b*) THE OSCILLATING THEORY

Another possibility is that there had been a prior big bang and then a recollapse due to gravitational pull and that this most recent big bang was one in an infinite series of bangs. This version of the big bang is called the *oscillating universe*. It solves some questions, for if the universe is oscillating then we know that there are other cycles before the more recent big bang. However, the oscillating universe theory does not really tell us anything about a real origin of the universe because in this case there was no origin.

According to this theory the calculation indicates that the expansion and contraction repeats itself after every 8 billion years. Thus this theory suggests a universe with an outer boundary which moves in and out at regular intervals.

(*c*) THE STEADY STATE THEORY :

The cosmological principle, that the universe is homogeneous and isotropic, is very general in scope, but starting in the late 1940's three British Scientists began investigating a principle that is even more general. Hermann Bondi, Thomas Gold and Fred Hoyle considered what they called the *perfect cosmological principle*, that the universe is not only homogeneous and isotropic in space but also unchanging in time.

The theory that follows from the perfect cosmological principle is called the *steady state theory*. It has certain philosophical differences from the big bang cosmologies. For one thing, according to the Steady State Theory, the universe never had a beginning and will never have an end. It always looked just about the way it does now and always will look that way. Some scientists are glad to see the question of what happened before the big bang or the need for finding a cause for the big bang, eliminated in the Steady State Theory.

But the steady state theory must be squared with the one major fact that we know about cosmology, that the universe is expanding. How can the universe expand continually but not change in its overall appearance ? Remember that in the basic formulation of the theory we consider the matter to be smoothed out throughout all space to an average density. If follows that for the density to remain constant new matter must be created at the same rate that the expansion would decrease the density. Thus the density remains the same. Now the matter that is created in the steady state theory is not simply matter that is being converted from energy according to Einstein's famous formula, $E=mc^2$. No, this is matter that is appearing out of nothing and is thus equivalent to energy appearing out of nothing.

Many scientists objected very strenuously to the idea that matter could appear out of nothing. After all the law of conservation of matter and energy seems very basic. Nevertheless, since we know the rate at which the universe is expanding we can calculate how much new matter would have to be created. It works out to be only one hydrogen atom per cubic centimeter of space every 10^{15} years. This is the rate necessary to maintain the overall value observed for the density of matter in space. This value is far too small for us to be able to measure on earth. No one knows, of course, in what form the matter would appear. It could appear uniformly distributed throughout space or perhaps it could preferentially appear in places where there was no matter already.

For many years a debate raged between proponents of the big bang theories and proponents of the steady state theory. The evidence that came in consistently seemed to favour big bang cosmologies over the steady state theory. Since the time of the discovery of quasars, many astronomers have felt that the distribution of quasars—all far away from us in space, which corresponds to their having been more numerous at an earlier time is strong evidence against the steady state cosmology. Still stronger evidence against the steady state theory is the discovery in 1965 of the primordial background radiation (3K radiation) that is most readily explained as a remnant of the big bang itself.

SUMMARY

1. Astronomy is an observational science rather than an experimental science.
2. The link between distant stars and the earth is the electromagnetic radiations Our atmosphere blocks these radiations everywhere except in the visible and radio region.
3. The astronomers collect light through telescopes and analyse the light so collected by a spectrograph. Radio telescopes are used to see very distant and faint objects.
4. The solar system as now known consists of the sun at the centre, 9 planets (including the earth), large bodies that revolve around the sun.

   2000+asteroids, smaller bodies that revolve around the sun, with orbits mostly between those of Mars and Jupiter.

   32 natural satellites, bodies that revolve around planets.

   Several hundred comets, bodies of huge volume and small mass that travel in elliptical orbits about the sun.

   Millions of meteors, small particles that plunge into the earth's atmosphere daily and a few large objects called meteorites that reach the earth-surface.
5. The average distance between the sun and the earth is called Astronomical Unit (A U) 1 A U $=1.496\times10^{11}$m.
6. The objects in the solar system are not far and their distances can be obtained easily by the method of triangulation. Mass of a planet having a satellite is easily determined from the knowledge of the orbit of the satellite. The composition of the atmosphere and the surface temperature is obtained from the careful study of the spectrum of the self-radiation of the planet.
7. The sun is a typical star. It is one of about 100 billion stars in the Milky Way, which occupies about a trillionth of known space. The universe made up of perhaps 10 billion similar galaxies which are scattered in space to at least about 5 billion light years in all directions and probably beyond.
8. Light year is a unit of distance and not a unit of time. One light year $= 9.46\times10^{12}$km.
9. Galaxies are grouped as spiral, elliptical and irregular. Most galaxies are of spiral type as is our own galaxy. In addition to about 100 billion stars the Milky Way contains much interestellar gas and dust The latter may cause bright and dark nebulae.

10. Stars have been classified according to their spectra. The hotter stars give more light at the blue end of the spectrum. Stars vary in temperature and brightness. A star that suddenly becomes much brighter is called a nova. A star that appears to the unaided eye as one star may be multiple. Many stars are multiples (Binary system).
11. Cosmology may be thought of as the theory of the origin, structure, and development of the universe as a whole. The models for rival theories of cosmology are basically either evolutionary or steady state.
12. There is some evidence based on Doppler effect that we live in an expanding universe. This evidence led to the explosion theory.
13. *Quasars* are celestial sources which radiate energy intensely in the form of radio waves and also in the optical region of the spectrum. About 150 are known. A deeper understanding of Quasars may be useful in determining the size and shape of the universe.
14. A *Pulsar* is a source of radio waves that varies in intensity at regular intervals. It is believed to be a rotating, exceedingly dense neutron star.
15. It is suggested that the gravitational field around some dense stellar objects is so great that electromagnetic radiation from it and its vicinity cannot escape ; the result is a "*black hole*"

QUESTIONS

1. What is astronomy ? Why do we call it an observational science ?
2. Give a brief description of some of the important constituents of the universe.

   (**Hint** : Solar system ; stars and galaxies)
3. Describe the various kinds of telescopes used for astronomical observations. Mention the advantages of reflecting type telescopes.
4. What are Radio telescopes ? What are their advantages over optical telescopes ?
5. Distinguish between a planet, a moon, and a star.
6. Distinguish between a comet, a meteor, a meteorite ; an asteroid and a satellite.
7. Explain briefly how will you determine
   - (*i*) the distance
   - (*ii*) the mass
   - (*iii*) the size
   - (*iv*) period of rotation and
   - (*v*) surface temperature

   of the objects in solar system ?
8. How can you ascertain the presence of atmosphere around a planet ? How will you study the chemical composition of the atmosphere.
9. Explain why some bodies in the solar system have atmospheres and others do not ?
10. Discuss the possibility of life on the various planets of the solar system.
11. Mention some of the important physical properties of the sun.
12. How sun is able to produce its energy ?

13. What are sun spots ? Distinguish between flares and Prominences.
14. The rate at which solar energy strikes the earth, called the solar constant is $1{\cdot}37 \times 10^3$ J m^{2}s. How can that number be used to compute the temperature of the sun's photosphere ?
15. Write short notes on :

    (*i*) Asteroids (*ii*) comets (*iii*) Meteors
16. Narrate the complete life history of a star.
17. Describe the structure and contents of the Milky Way.
18. Give the properties of normal galaxies and explain how radio galaxies and Quasars differ from them ? [*A.I.S.S.C. 1986*]
19. Briefly discuss the three main cosmological theories of the universe.
20. How has the distance to the moon been measured ?
21. What causes the tail of a comet to always point away from the sun ?
22. (*a*) What do you mean by red shift ?

    (*b*) On what scientific discovery is the steady theory based ?
23. How do astronomers distinguish, in general, between hotter stars and cooler stars on the basis of colour ?
24. How red shift is used by some scientists as evidence that we live in an expanding universe ?
25. Which one of the rival theories—explosion, expansion—contraction, steady state—do you tend to favour ? Describe that theory, and tell why you think it is the best one ?
26. Define Quasars. Describe them. What do astronomers hope to learn by studying them ?
27. What is Hertzsprung-Russell diagram (H-R diagram) ?
28. When we see an event such as the explosion of a star, we are actually, "looking back into history." Explain this statement.
29. Describe the ways optical spectra from stars are used to obtain information on the nature of stars and their motion.
30. What is the difference between (*i*) a red giant and a white dwarf (ii) Quasars and Pulsars and (*iii*) black holes and Quasi-Quasars (*iv*) Neutron stars and comets ?(*Delhi Senior Sec 1980*
31. When astronomers propose a model of how a star transforms during its life-time, do they base their model on the observation of changes occurring in a single star over a period of time or on the different type of stars observed in the universe ? Explain.
32. Can you think of reasons why we have not observed planets moving about some other star besides our sun ? Assume that other planetary systems would be similar to our solar system. How reasonable is it for us to visit such planetary systems in the life time of a human ?

33. (*a*) Write down some reasons to justify the expensive astronomical work such as an orbiting space observatory.

    (*b*) Which would appear more important to you, further observation of stars, galaxies, pulsars and similar objects or visiting Mars or Jupiter ? Discuss.

34. Explain briefly how would you determine

    (*i*) the distance (*ii*) size (*iii*) surface temperature (*iv*) mass

    (*v*) magnitude (*vi*) Absolute brightness, of a star ?

35. Distinguish between a white dwarf, a neutron star and a black hole. [*A.I.S.S.C. 1986*]

PROBLEMS

1 The eyelens has a mean aperture of 7·5 mm. A distant star is viewed through a telescope of aperture ·75 m. How much brighter the star will appear through a telescope in comparison to the naked eye ?

(10,000)

2. A distant star is viewed with a telescope of aperture 0·1 m. Now, it is viewed with a telescope of aperture 0·4 m. How much fainter will the star appear in the first telescope in comparison to the second telescope ?

$\left(\frac{1}{16}\right)$

3. The angular radius of the sun is $16'$. Find the radius of the sun (*a*) in AU and (*a*) in metres.

$[(a)\ 4{\cdot}654 \times 10^{-3}$ AU $(b)\ 6{\cdot}962 \times 10^{8}$ m]

4. The period of revolution of Phobos around Mars is 0·319 days and its mean distance from the Mars is $9\,519 \times 10^{6}$ m. Calculate the mass of Mars (*a*) in units of earth's mass (*b*) in kg. $G = 6{\cdot}668 \times 10^{-11}$ Nm^2 kg^{-2}. Distance of earth from Mars is 6×10^{10} m.

(Mass of earth $= 5{\cdot}977 \times 10^{24}$ kg)

[(*a*) 0·112 M (Earth)

(*b*) $6{\cdot}733 \times 10^{23}$ kg]

5. Calculate the surface temperature of the sun, if the total energy radiated per sec per unit area is $6{\cdot}41 \times 10^{7}$ watts m^2. Stefan's constant $= 5{\cdot}669 \times 10^{-8}$ SI units.

(5800 K)

6. The total luminosity of the sun is $3{\cdot}9 \times 10^{26}$ watts. The mean distance of the sun from the earth is $1{\cdot}496 \times 10^{11}$ m. Calculate the value of the solar constant.

($1{\cdot}388 \times 10^{3}$ watts m^{-2})

7. The total luminosity of the sun is $3\ 9 \times 10^{26}$ watts. The mean distance of the sun from the Jupiter is 5.2 AU. Calculate the value of solar constant on Jupiter. (1 AU $= 1{\cdot}496 \times 10^{11}$ m).

(51·05 watts m^{-2})

8. The two components of a binary star system have a period of 50 years. The separation between them is 4·5″. The distance of the binary is 10 light years. Calculate the sum of the masses of the two stars in solar units.

[1·051 M (sun)]

9. A satellite of Jupiter has a period of 1·77 days and orbital radius of $4\cdot22\times10^8$ m. Find the mass of the Jupiter. ($G=6\cdot668\times10^{-11}$ $Nm^2\,kg^{-2}$).

($1\cdot904\times10^{27}$ kg)

10. A radar signal is sent towards Mars from the earth and its echo is received 6 minutes and 40 seconds later. How far is Mars from the earth at this time ?

(6×10^{10} m)

11. The mass of the moon is $7\cdot342\times10^{22}$ kg and its radius is $1\cdot738\times10^6$ m. Calculate the mean density and acceleration due to gravity at its surface. What shall be the weight of a man on the moon if he weighs 72 kg on the earth ?

($3\cdot34\times10^3$ kg m^{-3} : 1·62 ms^{-2}, 12 kg)

12. The average diameter of the eye pupil in night is 7 mm. How much fainter stars can be seen through a telescope of 50 cm diameter as compared to the faintest naked eye stars ?

(5,100)

13. The moon subtends an angle of 57′ at the base line equal to the radius of the earth. Compute the distance. Given that radius of the earth$=6\cdot378\times10^6$ m.

($3\cdot844\times10^8$ m)

14. Calculate the mass of the sun, given that the distance between the sun and the earth is $1\cdot49\times10^8$ km and $G=6\cdot66\times10^{-11}$ SI units. Take the year to consist of 365 days.

($1\cdot972\times10^{30}$ kg)

15. The period of a double star system (Binary system) is 40 years and the semi-major axis of the orbit of the fainter star about the brighter is 22 AU. Calculate the sum of the masses of the two stars.

(6·65 solar masses)

16. The distance of Phobos from Mars is $9\cdot519\times10^6$ m. The mass of the Mars is $6\cdot723\times10^{23}$ kg. Calculate the time period of revolution of the satellite around the planet. Neglect the mars of Phobos. ($G=6\cdot66\times10^{-11}$ SI units).

(0·319 day)

17. The distance of the Phobos is 25″ as observed from the earth. The distance of Mars from the earth is 0·524 AU. Calculate the distance of Phobos from Mars in AU and in metres.

($6\cdot351\times10^{-5}$ AU ; $9\cdot519\times10^6$ m)

MATHEMATICAL NOTES

The mathematical background in differentiation and integration that is necessary for your understanding of discussions in most units of this book is summarized below.

Differential Calculus

Differential calculus deals with the way in which one quantity varies with the other, on which it depends, and such other allied topics.

Throughout calculus, we come across two types of quantities, viz. (*a*) *Constants*—those which retain the same value throughout a set of mathematical investigation, *e.g.* the mass of the body, the value of g at a place, the symbol etc. (*b*) *Variables*—those which take on different values or to which any desired values may be given, *e.g.* the radius of a circle, the side of a square or a cube, for they are not fixed quantities and any values may be given to them.

A variable which can take every numerical value (or all numerical values from one given number to another) is called a *continuous variable.* Thus if a train starts from rest, is observed to be moving with speed of 20 km/hr ten minutes later, it must have assumed every possible velocity between 0 and 20 km/hr during these ten minutes. Its speed is, therefore, a continuous variable. We are concerned here only with continuous variables.

Dependent and Independent variables — **Functions** : If a quantity x assumes a set of different values and its value does not depend upon that of any other quantity, it is said to be an *independent variable.* On the other hand, if a quantity y bears a certain relation to x, it is said to be a *dependent variable.* Thus since the area of a circle depends upon its radius, we say that the area is a dependent variable, and the radius, an independent variable mathematically speaking, that *the area of a circle is a function of its radius.* Denoting the area of a circle by y, and its radius by x, we express the relationship by the expression.

$$y=\pi x^2$$

Here x is the independent variable, y the dependent variable, and π, a constant.

Therefore, a variable y is said to be a function of the variable x, when its value depends on the values that x assumes. In general it is written mathematically as

$$y=f(x)$$

It must clearly be understood, however, *that f(x) does not mean f into x, but is only a symbolic way of representing 'some function of x'.*

We can know the value of $f(x)$, if we know the value of x. For example, if $f(x)$ denotes the function, x^2+2x+3, its value is 11, when $x=2$, and is 3, when $x=0$, being different for different values of x. Similarly, the value of $f(x)$ is $f(a)$, if x has the value a, and so on.

Limiting Value of a Function :

In applications, the situation that usually arises is this. We have a variable v, and a given function z of v. The independent variable v assumes values such that $v \rightarrow l$. We then have to examine the values of the dependent variables z, and, in particular, determine if z approaches a limit. If there is a constant a such that $\lim z=a$, then the relation described is written

$$\boxed{\underset{v\rightarrow l}{\text{Lt}} \quad z=a}$$

read, "the limit oi z, as v approaches l, is a".

Theorems on Limits :

In calculating the limiting value of a function, the following theorems may be applied.

Suppose u,v and w are functions of a variable x, and suppose that

$$\underset{x\rightarrow a}{\text{Lt}} \quad u=\text{A}, \qquad \underset{x\rightarrow a}{\text{Lt}} \quad v=\text{B}, \qquad \underset{x\rightarrow a}{\text{Lt}} \quad w=\text{C}$$

Then the following relations hold

(1) $\underset{x\rightarrow a}{\text{Lt}} \quad (u+v-w)=\text{A}+\text{B}-\text{C}$

(2) $\underset{x\rightarrow a}{\text{Lt}} \quad (uvw)=\text{ABC}$

(3) $\underset{x\rightarrow a}{\text{Lt}} \quad \dfrac{u}{v}=\dfrac{\text{A}}{\text{B}}$, if B is not zero

Briefly, in words, the limit of an algebraic sum, of a product, or of a quotient is equal, respectively, to the same algebraic sum, product, or quotient of the respective limits, provided, in the last named, that the limit of the denominator is not zero.

If C is a constant (independent of x) and B is not zero, then, from the above

$$\operatorname*{Lt}_{x\to a} (u+c)=A+C,$$

$$\operatorname*{Lt}_{x\to a} cu=CA,$$

$$\operatorname*{Lt}_{x\to a} \frac{c}{v}=\frac{C}{B}$$

Consider some examples.

Ex. 1 Prove

$$\operatorname*{Lt}_{x\to 2} (x^2+4x)=12$$

Solution : The given function is the sum of x^2 and $4x$. We first find the limiting values of these two functions.

$$\operatorname*{Lt}_{x\to 2} x^2=4. \text{ since } x^2=x.x$$

$$\operatorname*{Lt}_{x\to 2} 4x=8$$

Hence

$$\operatorname*{Lt}_{x\to 2} (x^2+4x)=4+8=12$$

Ex. 2 Prove

$$\operatorname*{Lt}_{z\to 2} \frac{Z^2-9}{Z+2}=-5/4$$

Solution :

Considering the numerator,

$$\operatorname*{Lt}_{z\to 2} (z^2-9)=-5$$

For the denominator,

$$\operatorname*{Lt}_{z\to 2} (z+2)=4$$

Hence

$$\operatorname*{Lt}_{z\to 2} \frac{z^2-9}{z+2}=-5/4.$$

Continuous and Discontinuous Functions :

In Ex 1 of the preceding article, where it was shown that

$$\underset{x \to 2}{\text{Lt}} \; (x^2+4x)=12,$$

we observe that the answer is the *value* of the function for $x=2$. That is, the limiting value of the function when x approaches 2 as a limit is equal to the value of the function for $x=2$. The function is said to be *continuous* for $x=2$. The general definition is as follows :

Definition :

A function $f(x)$ is said to be *continuous* for $x=a$ if the limiting value of the function when x approaches a as limit is the value assigned to the function for $x=a$. In symbols, if

$$\underset{x \to a}{\text{Lt}} \; f(x)=f(a),$$

then $f(x)$ is continuous for $x=a$.

The function is said to be *discontinuous* for $x=a$ if this condition is not satisfied.

Attention is called to the following two cases of common occurrence.

Case 1.

As an example illustrating a simple case of a function continuous for a particular value of the variable, consider the function

$$f(x)=\frac{x^2-4}{x-2}$$

For $x=1, f(x)=f(1)=3$. Moreover, if x approaches 1 as a limit, the function $f(x)$ approaches 3 as a limit. Hence the function is continuous for $x=1$

Case 2.

The definition of a continuous function assumes that the function is already defined for $x=a$. If this is not the case, however, it is sometimes possible to assign such a value to the function for $x=a$ that the condition of continuity shall be satisfied. The following theorem covers these cases.

Theorem :

If $f(x)$ is not defined for $x=a$, and

$$\text{if } \underset{x \to a}{\text{Lt}} \; f(x)=B,$$

then $f(x)$ will be continuous for $x=a$ if B is assumed as the value of $f(x)$ for $x=a$.

Thus, the function

$$\frac{x^2-4}{x-2}$$

is not defined for $x=2$ (since then there would be division by zero).

But for every other value of x,

$$\frac{x^2-4}{x-2}=x+2;$$

and $$\operatorname*{Lt}_{x\leftarrow 2}(x+2)=4;$$

therefore $$\operatorname*{Lt}_{x\to 2}\frac{x^2-4}{x-2}=4$$

Although the function is not defined for $x=2$, if we arbitrarily assign to it the value 4 for $x=2$, it becomes continuous for this value.

A function $f(x)$ is said to be continuous in an interval when it is continuous for all values of x in this interval.

Infinity ($\propto$) :

If the numerical value of a variable v ultimately becomes and remains greater than any preassigned positive number, however large, we say v becomes infinite. If v takes on only positive values, it becomes positively infinitive; if negative value only, it becomes negatively infinitive.

$$\operatorname{Lt} v=\propto,\quad \operatorname{Lt} v=+\propto,\quad \operatorname{Lt} v=-\propto$$

The notation $\operatorname{Lt} v=\propto$, or $v\to\propto$, must be read "v becomes infinite", and not "v approaches infinity"

We may now write, for example,

$$\operatorname*{Lt}_{x\to 0}\frac{1}{x}=\propto,$$

meaning that $\frac{1}{x}$ becomes infinite when x approaches zero.

It appears that if $$\operatorname*{Lt.}_{x\to a} f(x)=\propto,$$

that is if $f(x)$ becomes infinite as x approaches a as a limit, then $f(x)$ is discontinuous for $x=$a.

A function may have a limiting value when the independent variable becomes infinite. For example,

$$\operatorname*{Lt}_{x\to\propto}\frac{1}{x}=0$$

And, in general, if $f(x)$ approaches the constant value A as a limit when $x \rightarrow \propto$, we write

$$\mathop{Lt}\limits_{x \rightarrow A} \quad f(x)=A$$

Certain special limits occur frequently. These are given below. The constant C is not zero.

$$(1) \quad \mathop{Lt}\limits_{v \rightarrow 0} \quad \frac{C}{v}=\propto \qquad (2) \quad \mathop{Lt}\limits_{v \rightarrow \propto} \quad cv=\propto$$

$$(3) \quad \mathop{Lt}\limits_{v \rightarrow \propto} \quad \frac{v}{c}=\propto \qquad (4) \quad \mathop{Lt}\limits_{v \rightarrow \propto} \quad \frac{c}{v}=0$$

$$(5) \quad \mathop{Lt}\limits_{\theta \rightarrow 0} \quad \frac{\text{Sin } \theta}{\theta}=1$$

$$(6) \quad \mathop{Lt}\limits_{n \rightarrow \propto} \quad \left(1+\frac{1}{n}\right)^{n}=e$$

$$=1+n.\frac{1}{n}+\frac{n(n-1)}{2!}\left(\frac{1}{n}\right)^{2}+\text{------}+\propto$$

If u and v are functions of x, and if

$$\mathop{Lt}\limits_{x \leftarrow a} \quad u=A, \qquad \mathop{Lt}\limits_{x \leftarrow a} \quad v=0$$

and if A is not xero, then

$$\mathop{Lt}\limits_{x \rightarrow a} \qquad \frac{u}{v}=\propto$$

DIFFERENTIATION :

Increments. The *increment* of a variable in changing from one numerical value to another is the *difference* found by subtracting the first value from the second. An increment of x is denoted by the symbol $\triangle x$, read "delta x". The student is warned against reading this symbol "delta time x".

Evidently this increment may be either positive or negative according as the variable in changing increases or decreases. Similarly,

$\triangle y$ denotes and increment of y,

$\triangle \phi$ denotes an increment of ϕ,

$\triangle f(x)$ denotes an increment of $f(x)$ etc.

If in $y=f(x)$ the independant variable x takes on an increment $\triangle x$, then $\triangle y$ will denote the corresponding increment of the function $f(x)$ (or dependent variable y).

The increment $\triangle y$ is always to be reckoned from the definite initial value of y corresponding to the arbitrarily fixed initial value of x from which the increment $\triangle x$ is reckoned. For instance consider the function

$$y=x^{2}$$

Assuming $x=10$ for the initial value of x fixes $y=100$ as the initial value of y.

Suppose x increases to $x=12$, that is, $\triangle x=2$; then y increases to $y=144$, and $\triangle y=44$

Suppose x decreases to $x=9$, that is, $\triangle x=-1$, then y decreases to $y=81$, and $\triangle y=-19$.

In the above example, y increases when x increases and y decreases when x increases. The corresponding values of $\triangle x$ and $\triangle y$ have like signs. It may happen that y decreases as x increases, or the reverse; in either case $\triangle x$ and $\triangle y$ will then have opposite signs.

Comparison of Increments :

Consider the function

$$y=x^2 \tag{1}$$

Assuming a fixed initial value for x, let x take on an increment $\triangle x$. Then y will take on a corresponding increment $\triangle y$ and we have

$$y+\triangle y=(x+\triangle x)^2$$

Or

$$y+\triangle y=x^2+2x.\triangle x+(\triangle x)^2 \tag{2}$$

Subtracting (1) from (2), we have

$$\triangle y=2x.\triangle x+(\triangle x)^2 \tag{3}$$

We get the increment $\triangle y$ in terms of x and $\triangle x$.

To find the ratio of the increments, divide both members of (3) by $\triangle x$, giving

$$\frac{\triangle y}{\triangle x}=2x+\triangle x.$$

If the initial value of x is 4, it is evident that

$$\underset{\triangle x \to 0}{\text{Lt}} \frac{\triangle y}{\triangle x}=8$$

Let us carefully note the behaviour of the ratio of the increments of x and y as the increment of x diminishes.

Initial Value of x	New Value of x	Increment $\triangle x$	Initial Value of y	New Value of y	Increment $\triangle y$	$\frac{\triangle y}{\triangle x}$
4	5.0	1.0	16	25	9	9
4	4.8	0.8	16	23.04	7.04	8.8
4	4.6	0.6	16	21.16	5.16	8.6
4	4.4	0.4	16	19.36	3.36	8.4
4	4.2	0.2	16	17.64	1.64	8.2
4	4.1	0.1	16	16.81	0.81	8.1
4	4.01	0.01	16	16.0801	0.0801	8.01

It is apparent that as $\triangle x$ decreases, $\triangle y$ also diminishes, but their ratio takes on the successive values 9, 8.8, 8.6, 8.4, 8.2, 8.1, 8.01, illustrating the fact that the value of $\frac{\triangle y}{\triangle x}$ can be brought as near to 8 as we please by making $\triangle x$ sufficiently small.

Therefore

$$\underset{\triangle x \to 0}{\text{Lt}} \frac{\triangle y}{\triangle x} = 8$$

Derivative of a function of one Variable :

The fundamental definition of the differential calculus is as follows :

The derivative (or differential coefficient) of a function is the limit of the ratio of the increment of the function to the increment of the independent variable, when the latter increment varies and approaches zero as a limit.

When the limit of this ratio exists, the function is said to be *differentiable*, or to possess a derivative.

The above definition may be given in a more compact form *symbolically* as follows. Given the function

$$y=f(x) \qquad \dots (i)$$

and consider x to have a fixed value.

Let x take on an increment $\triangle x$; then the function y takes on an increment $\triangle y$, the new value of the function being

$$y+\triangle y=f(x+\triangle x) \dots (ii)$$

To find the increment of the function, subtract (i) from (ii), giving

$$\triangle y=f(x+\wedge x)-f(x) \dots (iii)$$

Dividing both members of (iii) by the increment of the variable. $\triangle x$,

$$\frac{\triangle y}{\triangle x}=\frac{f(x+\triangle x)-f(x)}{\triangle x} \dots (iv)$$

The limit of the right-hand member when $\triangle x \to o$ is, from the definition, the derivative of $f(x)$, or by (i), of y and is denoted by the symbol $\frac{dy}{dx}$. Therefore

$$\frac{dy}{dx}=\underset{\triangle x \to o}{\text{Lt}} \frac{f(x+\triangle x)-f(x}{\triangle x} \dots (A)$$

defines the derivative of y [or $f(x)$] with respect to x.

From (iv) we get also

$$\frac{dy}{dx} = \underset{\Delta x \to o}{\text{Lt}} \frac{\Delta y}{\Delta x}$$

Similarly, if u is a function of t, then

$$\frac{du}{dt} = \underset{\Delta t \to o}{\text{Lt}} \frac{\Delta u}{\Delta t} = \text{derivative of } u \text{ with respect to } t.$$

The process of finding the derivative of a function is called differentiation.

Symbols for Derivatives:

Since Δy and Δx are always finite and have definite values, the expression

$$\frac{\Delta y}{\Delta x}$$

is really a fraction. The symbol

$$\frac{dy}{dx}$$

however, is to be regarded *not as a fraction* but as the *limiting value of a fraction.* It is to be considered as a whole.

Since the derivative of a function of x is in general also a function of x, the symbol $f'(x)$ is also used to denote the derivative of $f(x)$. Hence if

$$y = f(x)$$

We may write

$$\frac{dy}{dx} = f'(x),$$

which is read "the drivative of y with respect to x equals f prime of x". The symbol $\frac{d}{dx}$ when considered by itself is called the *differentiating operator*, and indicates that any function written after it is to be differentiated with respect to x. Thus,

$\frac{dy}{dx}$ or $\frac{d}{dx} y$ indicates the derivative of y with respect to x.

$\frac{d}{dx}(2x^2+5)$ indicates the derivative of $2x^2+5$ with respect to x,

y' is an abbreviated form of $\frac{dy}{dx}$

If, then, $y = f(x)$

we may write the identities

$$y' = \frac{dy}{dx} = \frac{d}{dx} y = \frac{d}{dx} f(x) = f'(x)$$

General Rule for Differentiation :

First Step.

In the function replace x by $x+\triangle x$, and calculate the new value of the function, $y+\triangle y$.

Second Step :

Subtract the given value of the function from the new value and thus find $\triangle y$ (increment of the function).

Third Step :

Divide the remainder $\triangle y$ (the increment of the function) by $\triangle x$ (the increment of the independent variable).

Fourth Step :

Find the limit of this quotient when $\triangle x$ (the increment of the independent variable) varies and approaches zero as a limit. This is the derivative required.

This is called *Four-step Rule*. The student should become thoroughly familiar with this rule.

Example 1 :

Differentiate $3x^2+5$

Solution :

Let $y=3x^2+5$

First step

$$y+\triangle y=3(x+\triangle x)^2+5$$
$$=3x^2+6x.\triangle x+3(\triangle x)^2+5$$

Second Step

$$(y+\triangle y)-y=3x^2+6x\triangle x+3(\triangle x)^2+5-(3x^2+5)$$

or $$\triangle y=6x.\triangle x+3(\triangle x)^2$$

Third Step

$$\frac{\triangle y}{\triangle x}=6x+3\triangle x$$

Fourth Step

$$\frac{dy}{dx}=\underset{\triangle x\to 0}{\text{Lt}}\frac{\triangle y}{\triangle x}=\underset{\triangle x\to 0}{\text{Lt}}(6x+3\triangle x)$$
$$=6x$$

Therefore,

$$y'=\frac{dy}{dx}=\frac{d}{dx}(3x^2+5)=6x$$

Interpretation of the derivative by geometry.

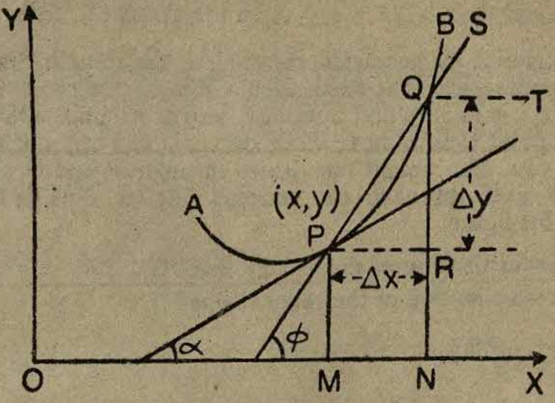


Fig (1)

We shall now consider a theorem which is fundamental in all applications of the differentiation calculus of geometry. It is necessary to recall the definition of the *tangent line* to a curve at a point *P* on the curve [Fig. (1)]. Let a secant be drawn through *P* and a neighbouring point *Q* on the curve. Let *Q* move along the curve and approach *P* indefinitely. Then the secant will revolve about *P*, and its limiting position is the tangent line at *P*. Let

$$y=f(x) \qquad (i)$$

be the equation of a curve AB. This curve is the graph of $f(x)$.

Now differentiate (1) by the General Rule and interpret each step geometrically from the figure (1). We choose a point P(x,y) on the curve and a second point Q $(x+\Delta x, y+\Delta y)$ near P, also on the curve.

First step

$$y+\Delta y=f(x+\Delta x)=\text{NQ}$$

Second step

$$\begin{aligned}(y+\Delta y)-y=f(x+\Delta x)-f(x)&=\text{NQ}-\text{MP}\\&=\text{NQ}-\text{NR}\\&=\text{RQ}\end{aligned}$$

Or $$\Delta y=f(x+\Delta x)-f(x)=\text{RQ}$$

Third step

$$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}=\frac{\text{RQ}}{\text{MN}}=\frac{\text{RQ}}{\text{PR}}$$

$$=\tan\angle\text{RPQ}=\tan\phi$$

$$=\text{Slope of Secant line PQ.}$$

At this point, therefore, we see that the ratio of the increments Δy and Δx equals the slope of the secant line drawn through the points P (x,y) and Q $(x+\Delta x, y+\Delta y)$ on the graph of $f(x)$.

Let us examine the geometric meaning of the Fourth Step. The value of x is now regarded as fixed. Hence P is a *fixed point* on the graph. Also, Δx is to vary and approach zero as a limit. Obviously, therefore, the point Q is to move *along the curve* and approach P as a limiting position. The secant line drawn through P and Q will then turn about P and approach the tangent line at P as its limiting position. In this figure,

ϕ=inclination of the Secant line PQ,
α=inclination of the tangent line PT.

Then $\underset{\Delta x \to 0}{\text{Lt}} \ \phi=\alpha$

Assuming that $\tan\phi$ is a continuous function, we have, therefore,

Fourth step : $$\frac{dy}{dx}=f'(x)=\underset{\Delta x \to 0}{\text{Lt}} \tan\phi=\tan\alpha$$

$$=\text{Slope of the tangent line at P.}$$

Thus we have derived the important theorem.

Theorem. *The value of the derivative at any point of curve is equal to the slope of the tangent line to the curve at that point.*

Example. Find the slopes of the tangents to the parabola $y=x^2$ at the vertex and at the point where $x=\frac{1}{2}$.

Solution. Differentiating by the General Rule, we get.

$\frac{dy}{dx}=2x$=slope of tangent line at any point (x, y) on curve.

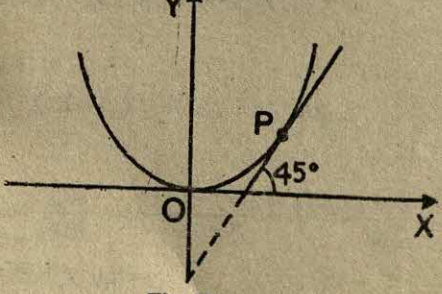


Fig. (2)

To find slope of tangent at the vertex, substitute $x=0$, giving.

$$\frac{dy}{dx}=0$$

Therefore the tangent at the vertex has the slope zero ; that is, it is parallel to the x-axis and in this case coincides with it.

To find the slope of the tangent at the point P, where $x=\frac{1}{2}$, we have

$$\frac{dy}{dx}=1\,;$$

that is. the tangent at the point P makes an angle of 45° with the x—axis.

Some of the important derivatives which are of use in the present course are listed below :

(*i*) $\frac{d\,(c)}{dx}=0$ where c is a constant quantity.

(*ii*) $\frac{d}{dx}\,(x)=1$

(*iii*) $\frac{d}{dx}(x^2)=2x$

(*iv*) $\frac{d}{dx}\,(x^n)=nx^{n-1}$

(*v*) $\frac{d}{dx}\,(cx^2)=c\frac{d}{dx}\,(x^2)=2\,c\,x$

(*vi*) $\frac{d}{dx}\,(\mathrm{UV})=\mathrm{U}\,\frac{d\mathrm{V}}{dx}+\mathrm{V}\frac{d\mathrm{U}}{dx}$ (U and V are functions of x)

(*vii*) $\frac{d}{dx}(\mathrm{U}+\mathrm{V})=\frac{d\mathrm{U}}{dx}+\frac{d\mathrm{V}}{dx}$

(*viii*) $\frac{d}{dx}(\mathrm{U}-\mathrm{V})=\frac{d\mathrm{U}}{dx}-\frac{d\mathrm{V}}{dx}$

(*ix*) $\frac{d}{dx}\,(\sin x)=\cos x$

(*x*) $\frac{d}{dx}\,(\cos x)=-\sin x$

(*xi*) $\frac{d}{dx}\,(\tan x)=\sec^2 x$

(*xii*) $\frac{d}{dx}\,(\cot x)=-\mathrm{cosec}^2 x$

(*xiii*) $\frac{d}{dx}\sin(wx+c)=w\cos(wx+c)$

(*xiv*) $\frac{d}{dx}\cos(wx+c)=-w\sin(wx+c)$

(*xv*) $\frac{d}{dx}(\log_e x)=\frac{1}{x}$

(*xvi*) $\frac{d}{dx}(\log_a{}^x)=\frac{1}{x}\log_e{}^a$

(*xvii*) $\frac{d}{dx}(a^x)=a^x\log_e{}^a$

(*xviii*) $\frac{d}{dx}(e^x)=e^x$

Integration. We have seen above that differentiation is the process of finding the differential coefficient of a given function. The inverse of this process of differentiation is called integration. *i.e.* the purpose of integration is to find the function whose differential coefficient is given.

For example, we know that $\frac{d}{dx}(x^3)=3x^2$, *i.e.* the differential coefficient of x^3 with respect to x is $3x^2$. Obviously, the function whose derivative is $3x^2$ is x^3. In other words, the integral of $3x^2$ is x^3.

Notation. Just as $\frac{d}{dx}(\ldots\ldots)$ is used to denote the differential coefficient of a function with respect to x, the symbol "$\int\ldots dx$" denotes the operation of finding the integral with respect to x.

Thus $$\int 3x^2dx=x^3$$

The function to be integrated is called *integrand.* In the above example, $3x^2$ is the integrand and x^3 is the integral.

Similarly, . as $\frac{d}{dx}(\sin x)=\cos x$, we have

$$\int \cos x dx=\sin x$$

Here $\cos x$ is the integrand and $\sin x$ is the integral.

Constant of Integration. We know that $\frac{d}{dx}(x^3+2)=3x^2$; Also $\frac{d}{dx}(x^3+4)=3x^2$. Again $\frac{d}{dx}(x^3+c)=3x^2$ where c is a constant. Thus the differential coefficient of all the above functions is the same and is equal to $3x^2$. The question naturally arises **which** of these functions should be taken as the integral of $3x^2$. **To** overcome this difficulty, the integral of $3x^2$ is taken as (x^3+c) where c is an arbitrary constant. (x^3+c) is called the indefinite integral of $3x^2$, since c can take any value it likes. In practice, however, c is not generally written, though it is always implied.

Fundamental Formulae. Since integration is the reverse of differentiation, the following formulae for integration can be easily written from our knowledge of different coefficients of certain functions.

(1) $$\int x^n dx=\frac{x^{n+1}}{n+1} \qquad (n\neq -1)$$

$$\text{If } n=-1,\ \int x^{-1}\,dx=\int \frac{1}{x}\,dx=\log x$$

(2) $$\int dx=x$$

$$(\because \int dx=\int 1\cdot dx=\int x^0 dx=\frac{x^{0+1}}{0+1}=\frac{x^1}{1}=x)$$

(3) $$\int \frac{1}{x}\,dx=\log_e x$$

(4) $$\int e^x dx=e^x$$

(5) $$\int a^x dx=\frac{a^x}{\log_e a}$$

(6) $$\int \sin x\,dx=-\cos x$$

(7) $$\int \cos x\,dx=\sin x \left[\therefore \frac{d}{dx}(\sin x)=\cos x\right]$$

The integral of the sum of a number of functions is **equal to** the sum of the integrals of individual functions.

$$\text{Thus } \int (u+v+w)\,dx=\int (u)\,dx+\int (v)\,dx+\int (w)\,dx$$

$$\text{e.g., } \int (x^3+\sin x)\,dx=\int x^3\,dx+\int \sin x\,dx=\frac{x^4}{4}-\cos x.$$

The Definite Integral :

Differential of the area under a curve :

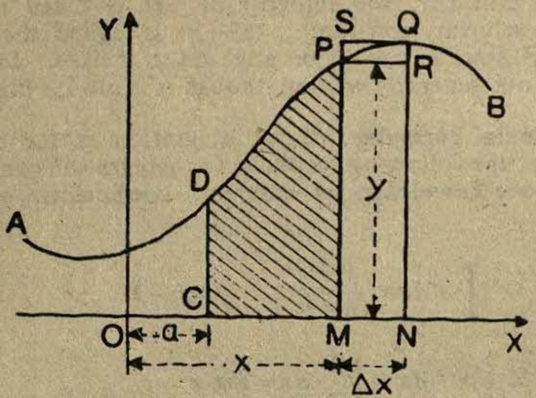


Fig (3)

Consider the continuous function $\phi(x)$, and let

$$y=\phi(x)$$

be the equation of the curve AB. Let CD be a fixed and MP a variable ordinate, and let u be the measure of the area CMPD When x takes on a small increment $\triangle x$, u takes on an increment Δu(=area MNQP). Completing the rectangles MNRP and MNQS, we see that Area MNRP<area MNQP<area MNQS, or MP. $\triangle x < \triangle u < \triangle$NQ$\triangle x$;

and, dividing by $\triangle x$,

$$\text{MP} < \frac{\triangle u}{\Delta x} < \text{NQ}.$$

Now let $\triangle x$ approach zero as a limit ; then since MP remains fixed and NQ approaches MP as a limit (since y is a continuous function of x) we get

$$\frac{du}{dx}=y(=\text{MP}),$$

or, using differentials,

$$du=y\,dx$$

Theorem. The differential of the area bounded by any curve, the x-axis, a fixed ordinate, and a variable ordinate is equal to the product of the variable ordinate and the differential of the corresponding abscissa.

The Definite Integral

It follows from the theorem in the last article that if the curve AB is the locus of

$$y=\phi(x),$$

then

$$du=y\,dx$$

or $du=\phi(x)\,dx$; where du is the differential of the area between the curve, the x-axis, and two ordinates. Integrating, we have

$$u=\int\phi(x)dx.$$

Denote $\int\phi(x)dx=f(x)+C$

$\therefore$ $u=f(x)+C$

We determine C by observing that $u=0$ when $x=a$.

Substituting these values, we get

$$0=f(a)+C$$

and hence $C=-f(a)$

Thus $u=f(x)-f(a)$ ...(*i*)

The required area CEFD is the value of u when $x=b$ in (*i*). Hence we have

$$\boxed{\text{Area CEFD}=f(b)-f(a)}$$

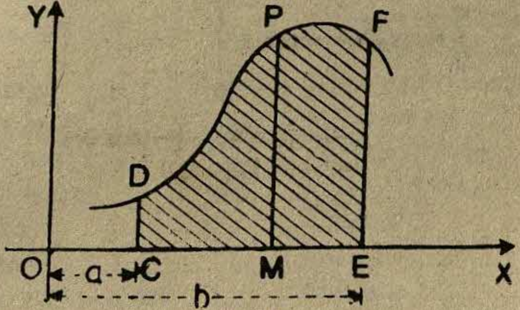


Fig. (4)

Theorem :

The difference of the values of $\int y\,dx$ for $x=a$ and $x=b$ gives the area bounded by the curve whose ordinate is y, the x-axis and the ordinates corresponding to $x=a$ and $x=b$. This difference is represented by the symbol

$$\int_a^b y\,dx \text{ or } \int_a^b \phi(x)\,dx\,; \qquad \text{...(ii)}$$

and is read "the integral from a to b of $y\,dx$". The operation is called *integration between limits*. a being the *lower* and b the *upper* limit.

Since (*ii*) always has a *definite value*, it is called *definite integral*.

$$\therefore \quad \int_a^b y\,dx=\int_a^b \phi(x)=f(b)-f(a).$$

We may accordingly define the symbol

$$\int_a^b \phi(x)\, dx \text{ or } \int_a^b y\, dx$$

as the numerical measures of the area bounded by the curve $y=\phi(x)$, the x-axis, and the ordinates of the curve at $x=a$ and $x=b$.

Rule :

First Step : Integrate the given differential expression.

Second Step : Substitute in this indefinite integral first the upper limit and then the lower limit for the variable, and subtract the last result from the first.

Example :

Find $\int_0^\pi \sin x\, dx$

Solution :

$$\int_0^\pi \sin x\, dx = \Big| -\cos x \Big|_0^\pi$$

$$= [-(\cos \pi)] - [-(\cos 0)]$$

$$= [-(-1)][-(1)]$$

$$= 2.$$

APPENDIX—B

PHYSICAL CONSTANTS AND USEFUL INFORMATION

Name	*Symbol*	*Value and Units*
Speed of Light in vacuum	c	$3{\cdot}00 \times 10^{8}$ ms^{-1}
Boltzmann's constant	k	$1{\cdot}38 \times 10^{-23}$ joule/molecule/K°
Gravitational constant	G	$6{\cdot}67 \times 10^{-11}$ newton-m^{2}/kg^{2}
Avogadro's number	N.	$6{\cdot}02 \times 10^{23}$ molecules/gram-mole
Planck's constant	h	$6{\cdot}63 \times 10^{-34}$ joules/sec
Permitivity constant	ϵ_0	$8{\cdot}85 \times 10^{-12}$ farad/m
Permeability constant	μ_0	$4\pi \times 10^{-7}$ weber/amp. m $= 1{\cdot}26 \times 10^{-6}$ henry/m
Electron charge	e	$-1{\cdot}60 \times 10^{-19}$ coulomb
Electron rest mass	m_e	$9{\cdot}1 \times 10^{-31}$ kg
Proton rest mass	m_p	$1{\cdot}6723 \times 10^{-27}$ kg
Neutron rest mass	m_r	$1{\cdot}6747 \times 10^{-27}$ kg
Absolute zero	K	$-273{\cdot}15$°C
1 atmosphere		$1{\cdot}01 \times 10^{5}$ Nm^{-2}
Standard Acceleration due to gravity	g	$9{\cdot}81$ m/sec^{2}
Uniuersal gas constant	R	$8{\cdot}314$ J mole/K°
Mechanical Equivalent of heat	J	$4{\cdot}18$ joules/calorie
One Horse Power		746 watts
First Bohr Orbit (Hydrogen radius)		$5{\cdot}29 \times 10^{-11}$ m
Specific charge of electron	$\frac{e}{m_e}$	$1{\cdot}76 \times 10^{11}$ coulomb/kg

Speed of Sound in dry air (STP)		$331\ ms^{-1}$
Density of air (STP)		$1{\cdot}29\ kg\ m^{-3}$
Density of water (4°C)		$1{\cdot}00 \times 10^3\ kg\ m^{-3}$
Mean radius of earth		$6{\cdot}371 \times 10^6\ m = 6371\ km$
Mean density of earth		$5522\ kg\ m^{-3}$
Mass of Earth		$5{\cdot}975 \times 10^{24}\ kg$
Mean orbital speed of earth		$29770\ ms^{-1}$
Electron Volt	eV	$1{\cdot}6 \times 10^{-19}\ J$
Million electron Volt	MeV	$1{\cdot}6 \times 10^{-13}\ J\ (=10^6\ eV)$
Atomic mass Unit	*u*	$931\ MeV = 1489{\cdot}6 \times 10^{-13}\ J$ $= 1{\cdot}66053 \times 10^{-27}\ kg$
Escape Velocity of earth		$11{\cdot}16\ km\ s^{-1}$
Ionization Potential for Hydrogen atom		$-13{\cdot}6\ eV = -21{\cdot}76 \times 10^{-19}\ J$

APPENDIX—C

THE GREEK ALPHABETS COMMONLY USED

Alpha	α	Beta	β
Gamma	γ	Delta	δ or △
Epsilon	ϵ	Zeta	ζ
Eta	η	Theta	θ
Iota	ι	Kappa	κ
Lambda	λ	Mu	μ
Nu	ν	Xi	ξ
Pi	π	Rho	ρ
Sigma	σ	Tau	τ
Upsilon	υ	Phi	ϕ
Psi	ψ	Omega	ω

APPENDIX—D

SI UNITS

S. No.	*Quantity*	*SI Unit*
1	Length	metre
2	Mass	kilogram
3	Time	second
4	Velocity	metre/second
5	Acceleration	metre/(second)^2
6	Force	newton
7	Work or Energy	joule (or newton metre)
8	Power	watt (or joule/second)
9	Torque	newton metre
10	Angular velocity	radian/second
11	Angular Acceleration	radian/(second)^2
12	Current	ampere
13	Charge (or Quantity of electricity)	coulomb
14	Electromotive force	volt
15	Potential difference	volt
16	Resistance	ohm
17	Resistivity (or Specific Resistance)	ohm metre
18	Conductance	mho
19	Magnetomotive force	ampere-turn
20	Magnetising force (H)	ampere-turn/metre
21	Magnetic flux (ϕ)	weber
22	Magnetic flux density (B)	tesla
23	Reluctance	ampere-turn/weber
24	Permeance	weber/ampere-turn

25	Permeability	henry/metre
26	Inductance	henry
27	Electric flux	coulomb
28	Electric flux density	coulomb/(metre)^2
29	Electric force (field strength)	volt/metre
30	Permittivity	farad/metre
31	Capacitance	farad
32	Specific heat	joule/kg/degree kelvin
33	Terminal conductivity	joule/metre/second/degree kelvin
34	Stress	newton/(metre)^2
35	Young's Modulus	newton/(metre)^2
36	Moment of Inertia	kilogram (metre)^2
37	Momentum	kilogram metre/second
38	Angular Momentum	kg m^2 rad/sec
39	Impulse	newton second
40	Coefficient of Viscosity	newton second/(metre)^2
41	Surface Tension	newton/metre
42	Pressure	newton/(metre)^2
43	Gravitational constant	newton (metre)^2/(kilogram)^2
44	Temperature	kelvin (K)
45	Amount of substance	mole
46	Intensity of illumination	Candela
47	Density	kilogram (metre)^3
48	Frequency	hertz (Hz) or cycles per second (c p s)

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						5	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	16	20	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	29	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	6	10	13	16	19	23	26	29
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	14	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	6	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	3	5	8	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	17	19	21
						2672	2695	2718	2742	2765	2	4	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	123	456	789
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 5 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2199	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS

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·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

APPENDIX E

FOR UNIT II—MOTION

Relative Velocity :

All motion is relative. A train standing at a station is at rest with respect to the earth, but it is in motion relative to the Sun. For ordinary purposes, however, we require only to determine the motion of a body relative to a second body which is at rest on the surface of the earth. Sometimes, however, it is necessary to determine the velocity of a body relative to another body which is itself in motion *i.e.*, its velocity with respect to the second body, or its *relative velocity* with respect to the latter.

Thus, the relative velocity of a particle A with respect to another particle B is the rate at which A changes its position relative to B.

If A and B are in motion and we impress the same velocity on both the particles, their relative motion remains unchanged. Thus, if two men are swimming it still water (say in a tank) along two parallel lines in the same direction, each with a velocity u, their relative position is not changed and hence the velocity of one relative to the other is zero. Now, if we suppose that they are swimming in a river along the current of water moving with a velocity v, even then there will be no change in their relative position, though a velocity v is added to both of them. On account of this, while finding the velocity of a body relative to another we do not take into account the velocity of the earth which is imparted in common to all bodies on its surface.

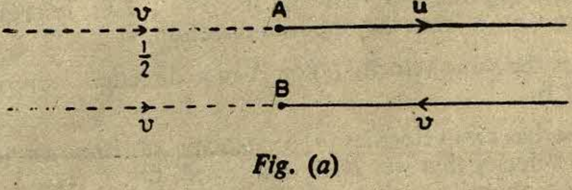

Fig. (*a*)

If the particle A [Fig. (*a*)] moves with a velocity u in the easterly direction and the particle B moves with a velocity v in the westerly direction as shown, then to find the velocity of A *relative* to B impress on B as well as on A, a velocity equal and opposite to that of B, that is $-v$. This will not affect their relative motion. The particle B will be reduced to rest while the particle A will possess two velocities v and u in the same direction. Therefore the velocity of A relative to B is equal to $-u-v=-(u+v)$, taking the direction of motion of B as positive.

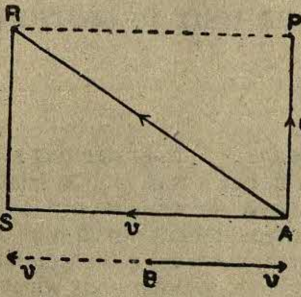

Fig. (*b*)

Again suppose that the particle A is moving towards north with the same velocity u while B is moving in the easterly direction with the velocity v. To find the velocity of A relative to B, impress on both, as before, a velocity $(-v)$ *i.e., equal and opposite to that of B*. The particle B will again be at rest, while A will have two velocities impressed upon it, at right angles to each other, one represented by AP and the other by AS [Fig. (*b*)]. Compounding these two we get the resultant velocity represented by AR, both in magnitude and direction. Thus AR represents in magnitude and direction, the velocity of A relative to B or the relative velocity of A, with respect to B.

Thus the following procedure is adopted for determining the relative velocity of the body A with respect to another body B when both of them are in motion with velocities, say u and v respectively :

(1) Make B stationary by impressing a velocity on B which is equal and opposite to that of its own ; *i.e.*, impress the velocity v on it from the opposite direction.

(2) Impress the same velocity (v) on A in a direction opposite to that of B.

(3) 'A' now has two velocities (*a*) a velocity of its own 'u' and (*b*) the velocity that we have impressed on it *i.e.*, v in a direction opposite to that of B. The resultant of the two velocities on A *i.e.*, u and v is known as the relative velocity of A with respect to B.

Example :–

Rain is falling vertically with a velocity of 1 km/hr and a man is walking due east at the rate of 2 km/hr. At what angle to the vertical must he hold his umbrella to make the rain drops at right angles ?

Solution :—

Impress upon both a velocity equal to 2 km/hr due west. This would bring the man to rest and the rain will have two velocities ; a velocity of 1 km/hr downwards, represented by AB [Fig. (*c*)] and a velocity of 2 km/hr due west represented by BC, so that the resultant velocity would be $\sqrt{(2)^2+(1)^2}=\sqrt{5}$ km/hr along AC. So, if the man wants that the drops should fall at right angles on the top of the umbrella, he should incline it at the angle CAB to the vertical.

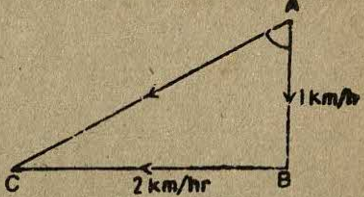

Fig. (*c*)

$$\tan CAB=\frac{2}{1}=2,$$

$$CAB=\tan^{-1}(2)$$